About the title
“Geometric Characterizations of Standard Normal Distribution
- Two Types of Differential Equations, Relationships with Square and Circle,
and Their Similar Characterizations -”,
we will modify some things as follows to contribute mathematics and sciences precisely on
November 14, 2018.

Important modification:

Present modified version: \( h_2(u) = h_P(u) \),
\[ \phi(u)^{-1}(h'_P(u) + uh'_P(u) - h_P(u)) = 0. \]

Previous misspelled version: \( h_2(u) = m_P(u) \),
\[ \phi(u)(m'_P(u) - um'_P(u) - m_P(u)) = 0. \]

The other equation in this paper: \( g(u) = g_P(u) \),
\[ g'_P(u) + ug_P(u) + g(u)^2 = 0. \]

If we think of three equations as follows
\[ h_P(u) = \phi(u) + u\phi(u), \]
\[ m_P(u) = \frac{\phi(u)}{\phi(u)} \]
\[ g_P(u) = \frac{\phi(u)}{\phi(u)} \]
these differential equations are expressed as
\[ \phi(u)^{-1}(h'_P(u) + uh'_P(u) - h_P(u)) = 0, \]
\[ \phi(u)(m'_P(u) - um'_P(u) - m_P(u)) = 0, \]
\[ g'_P(u) + ug_P(u) + g(u)^2 = 0. \]

The upper two of the equations are self-adjoint differential equations.
And we find that there is the following relationship between above three equations \( g_P(u) \),
\( h_P(u) \) and \( m_P(u) \). That is,
\[ \frac{g'_P(u)}{g_P(u)} = \frac{h_P(u)}{h'_P(u)} = \frac{m'_P(u)}{m_P(u)}. \]

© Shingo NAKANISHI, Osaka Institute of Technology, November 14, 2018.
I have a plan to speak about them on November 22, 2018 to be modified.
Self-adjoint differential equation: \( \phi(u)^{-1}(h'_2(u) + uh'_1(u) - h_2(u)) = 0 \)
\[
\begin{align*}
\phi(u)^{-1}h'_2(u) - \{\phi(u)^{-1}h_2(u)\} &= 0
\end{align*}
\]

Variable coefficient second order linear homogeneous differential equation for a standard normal distribution:
\[
\frac{d^2h_2(u)}{du^2} + u\frac{dh_2(u)}{du} - h_2(u) = 0, \\
\frac{dh_2(u)}{du} = -\Phi(-u), \\
\frac{dh_3(u)}{du} = \phi(u) + \phi(0).
\]

Equilibrium point between a banker, winners and losers: \( \phi(\lambda) = 2\lambda\Phi(-\lambda) \)
\[
\lambda = 0.612003
\]

Probability density function of a truncated normal distribution: \( \psi(u) = \phi(u) \Phi(-\lambda) \)

Cumulative distribution function of a standard normal distribution: \( \Phi(u) \)

Intercept form of a linear equation for winners:
\[
-\frac{1}{\lambda}u + \frac{1}{2\Phi(-\lambda)}\psi = 1
\]

Intercept form of a linear equation for losers:
\[
-\frac{1}{\lambda}u + \frac{1}{2\Phi(-\lambda)}\phi = 1
\]

Intercept form of a linear equation for a banker:
\[
-\frac{1}{\lambda}u + \frac{1}{2\Phi(-\lambda)}\text{ utility} = 1
\]

Utility function for winners:
\[
U_W(t) = (\phi(\lambda) - \lambda\Phi(-\lambda)\sqrt{t} = \lambda\Phi(-\lambda)\sqrt{t}
\]

Tangential equation: \( f(u) = \frac{-(1 - \Phi(-\lambda))u + 2\lambda\Phi(\lambda)}{\Phi(-\lambda)u + 2\lambda\Phi(\lambda)} \)
\[
(-2\lambda \leq u \leq 0) \quad \frac{-\Phi(-\lambda)u + 2\lambda\Phi(\lambda)}{\Phi(-\lambda)u + 2\lambda\Phi(\lambda)} \quad (0 \leq u \leq 2\lambda)
\]

Equilibrium point of an inverse Mills ratio: \( \frac{\phi(\lambda)}{\Phi(-\lambda)} = 2\lambda \)

Modified Version of our study,
“Geometric Characterizations of Standard Normal Distribution
- Two Types of Differential Equations,
Relationships with Square and Circle,
and Their Similar Characterizations -”

© Shingo NAKANISHI, Osaka Institute of Technology, November 4, 2018.