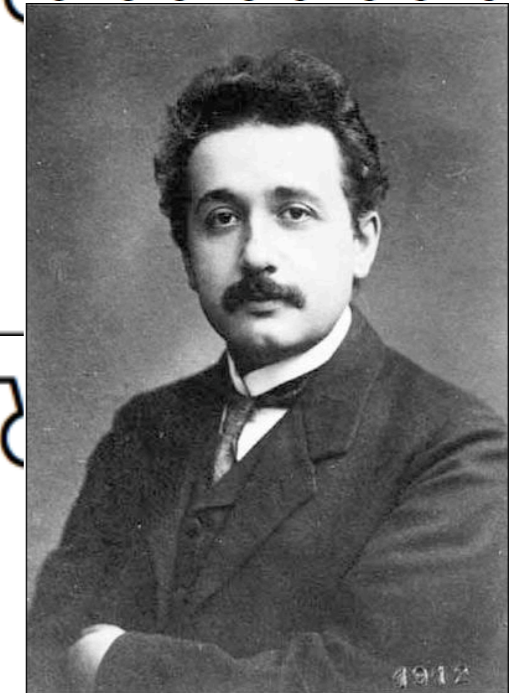


Why & How we know there are black-holes? Introduction to Einstein's Relativity

Hisaaki Shinkai

1 Special Relativity: Theory of time

2 General Relativity: Theory of gravity



真貝 寿明

Hisaaki Shinkai

<http://www.oit.ac.jp/is/~shinkai/>



General Relativity

Physics of heavy objects

「Gravity is produced by the warp of spacetime」

Special Relativity

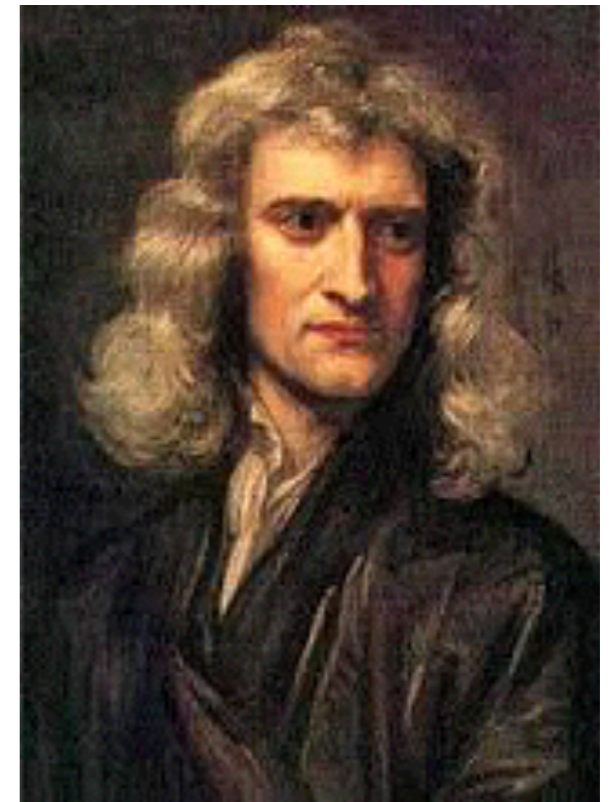
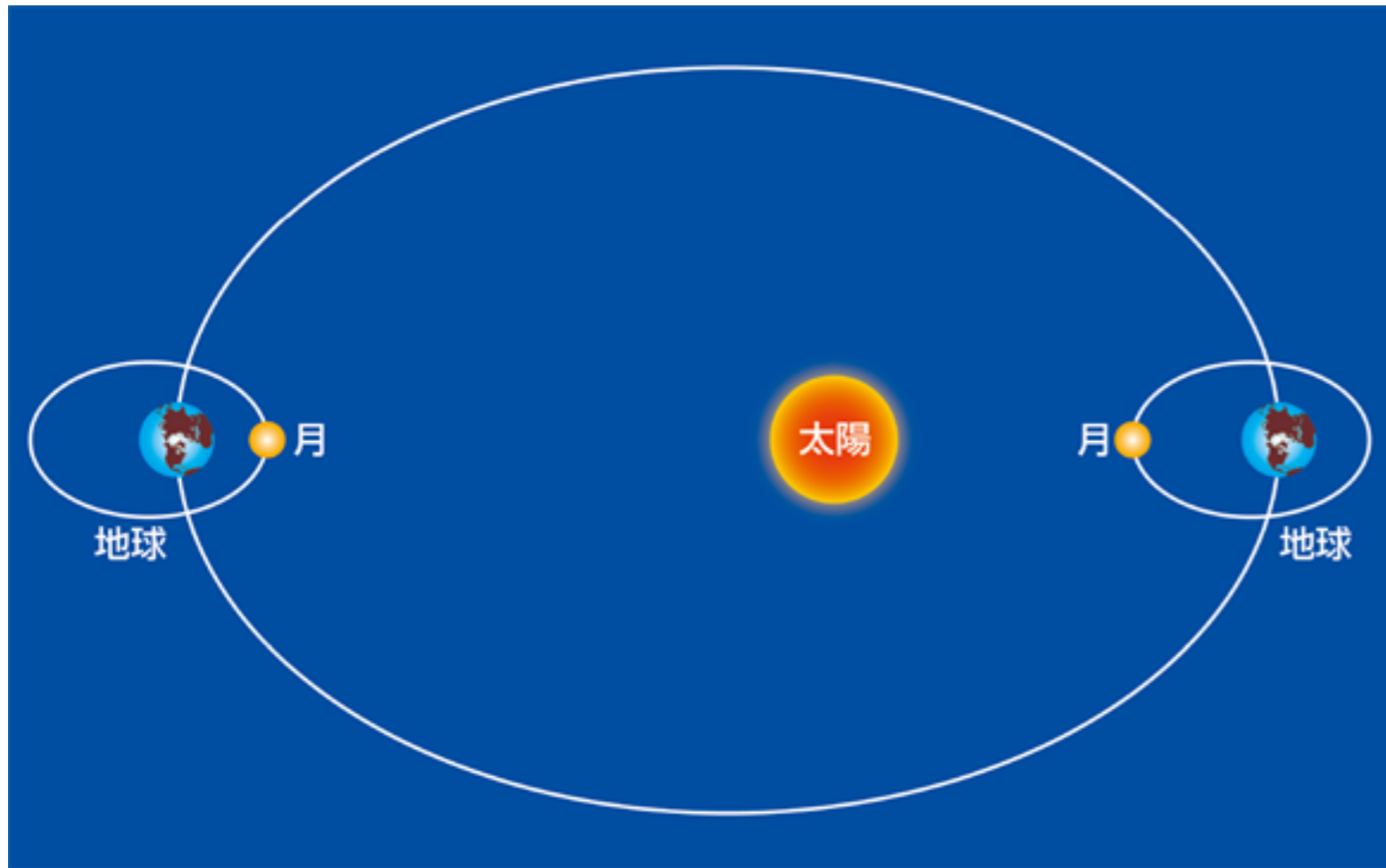
Physics at near speed of light

「Time is relative」

Newton's Mechanics

$$F = ma$$

Gravity is universal



Newton

Universal Force

= Every objects have attractive force

重力の正体は？



<http://hikingartist.com/>

「万有引力があるからだ」 (ニュートン, 1687)

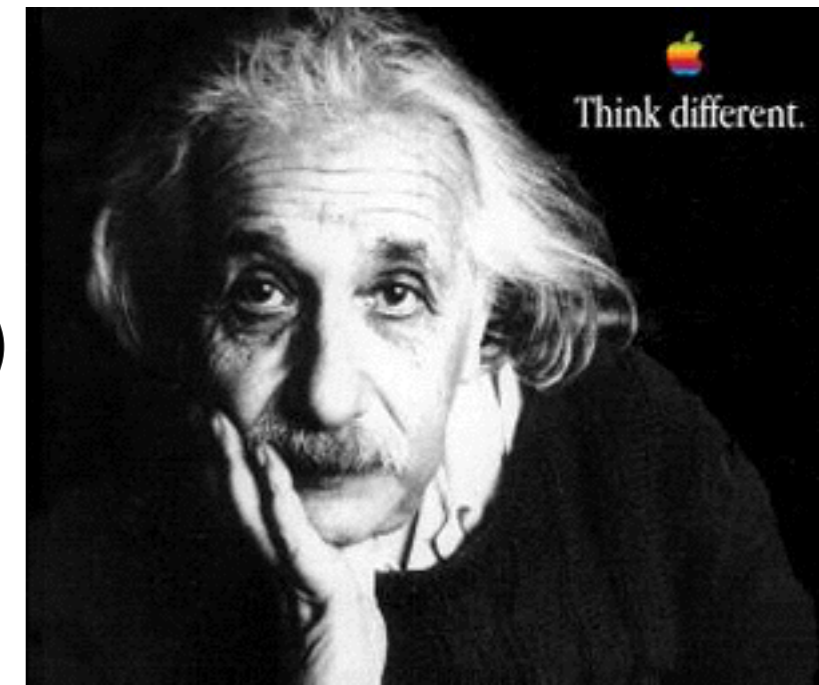
$$F = G \frac{Mm}{r^2}$$

$$m \frac{d^2 x}{dt^2} = F$$

「時空のゆがみだ」

(アインシュタイン, 1915)

一般相対性理論

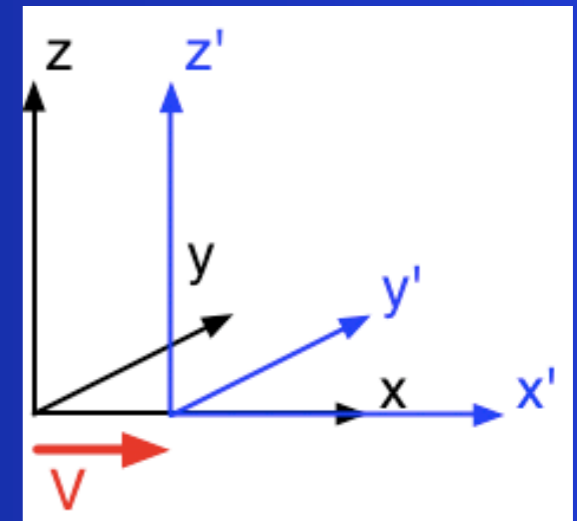


$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

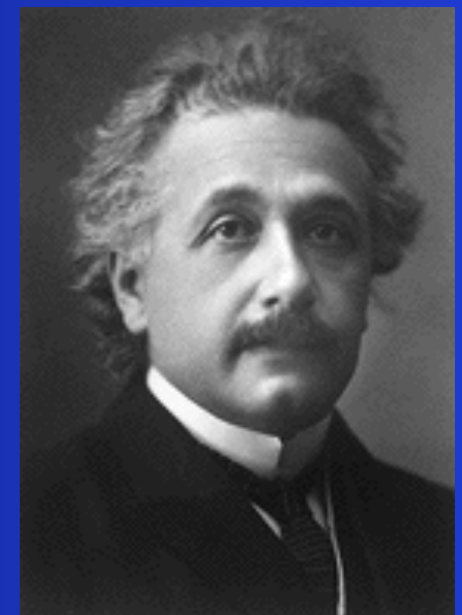
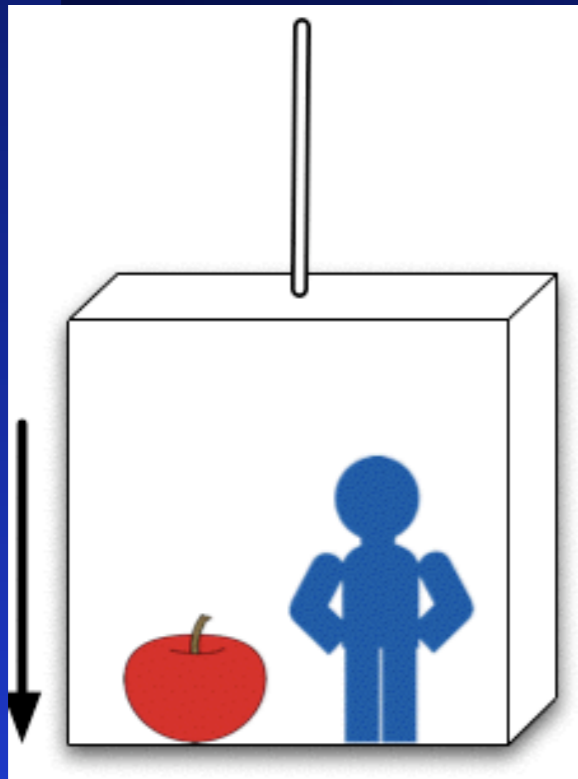
$$\frac{d^2 \xi^\mu}{d\tau^2} = R^\mu{}_{\nu\rho\sigma} \frac{d\xi^\nu}{d\tau} \frac{d\xi^\rho}{d\tau} \xi^\sigma$$

What is “acceleration”?

- ★ In Special Relativity, only constant velocity observer can be discussed.
- ★ What is the acceleration?

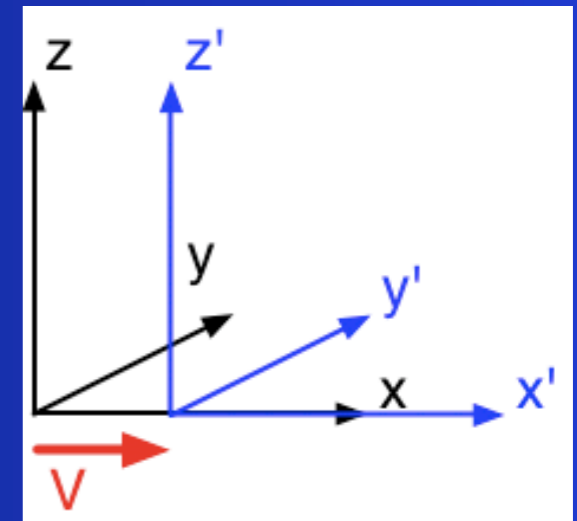


Thought Experiment of Elevator



What is “acceleration”?

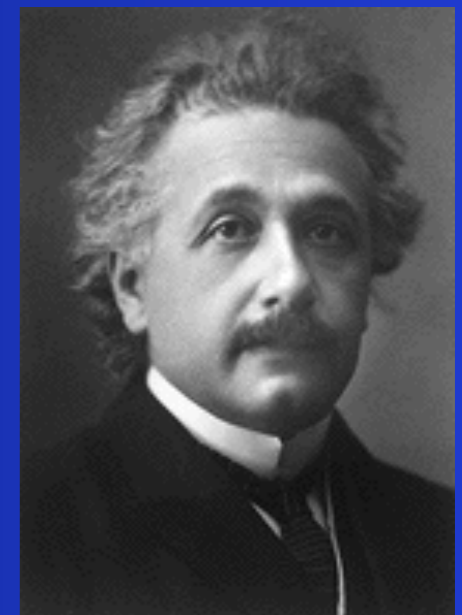
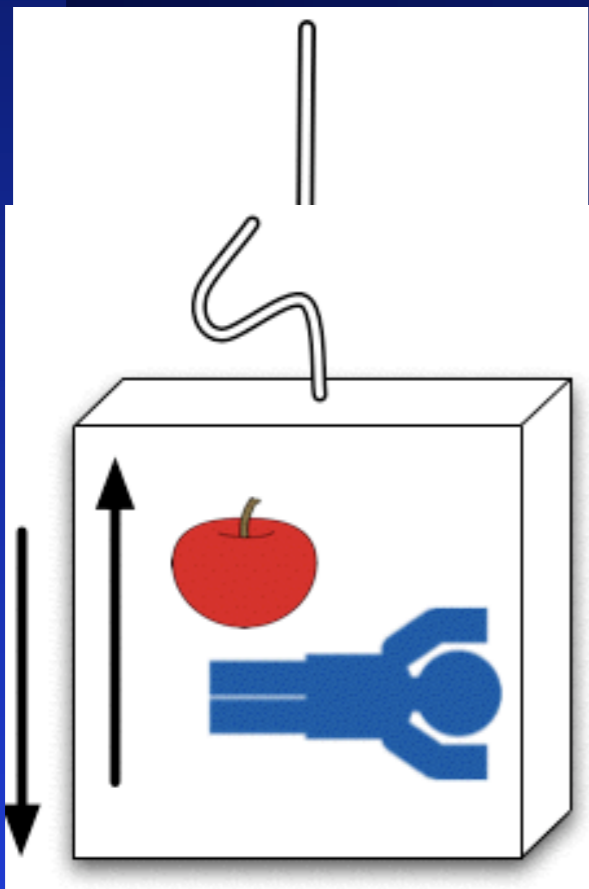
- ★ In Special Relativity, only constant velocity observer can be discussed.
- ★ What is the acceleration?



Thought Experiment of Elevator

「Gravitational acceleration can be compensated.」
= acceleration can be removed **locally**. (Equivalence Principle)

「Greatest idea in my life」



Zero Gravity by Free-Falling Airplane



無重力体験 ～アメリカ～



ボーイング727による宇宙体験・無重力体験飛行がアメリカ・ラスベガス、ケネディースペースセンターなどで楽しめます。

天井や壁を歩いてみたり、スーパーマンのように宙を飛んだり、水球になった水を飲んだり(?)... などなど。楽しみ方は、自由! 無重力状態でどんな事を試してみますか?

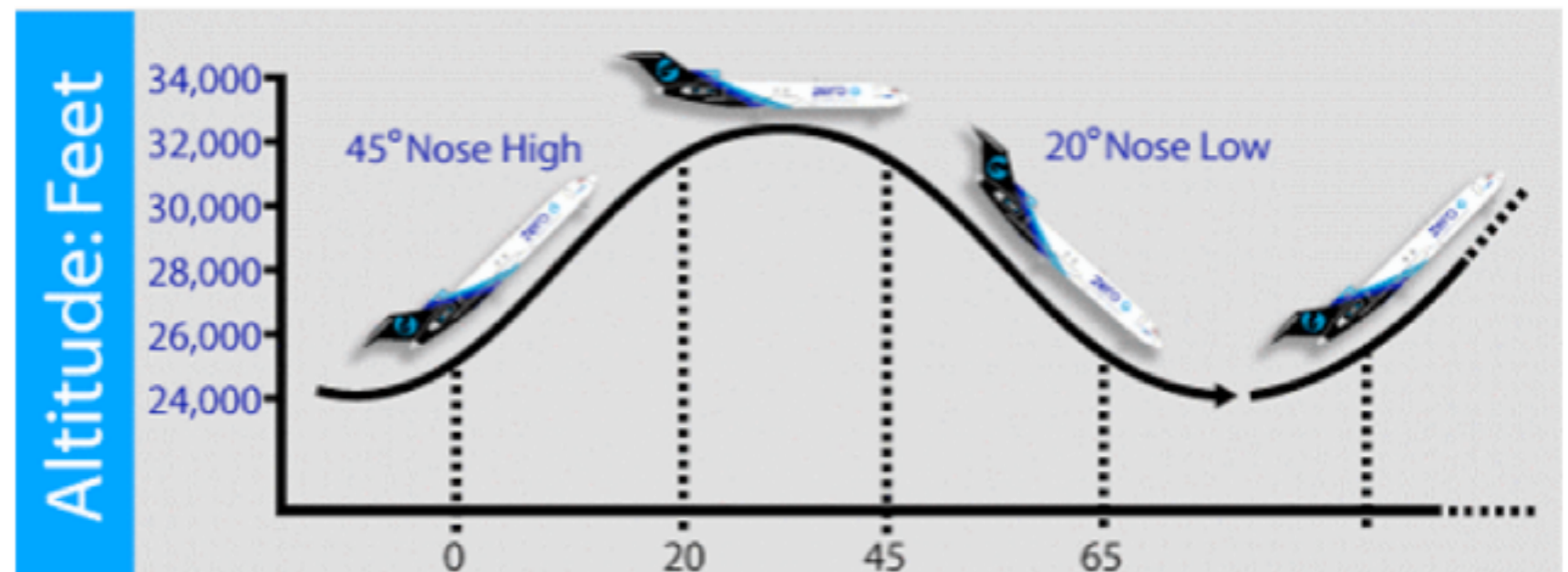
このプログラムでは、無重力体験のみでなく火星や月の重力も体験できま

無重力体験・概要

高度24,000ft (約7,315m) から45度の角度で34,000ft (約10,360m) まで地上の1.8倍 (1.8G) の重力を感じながら、一気に上昇し、その後下降します (パラボラテック飛行)。この時に無重力を25-30秒程度お楽しみいただけます。

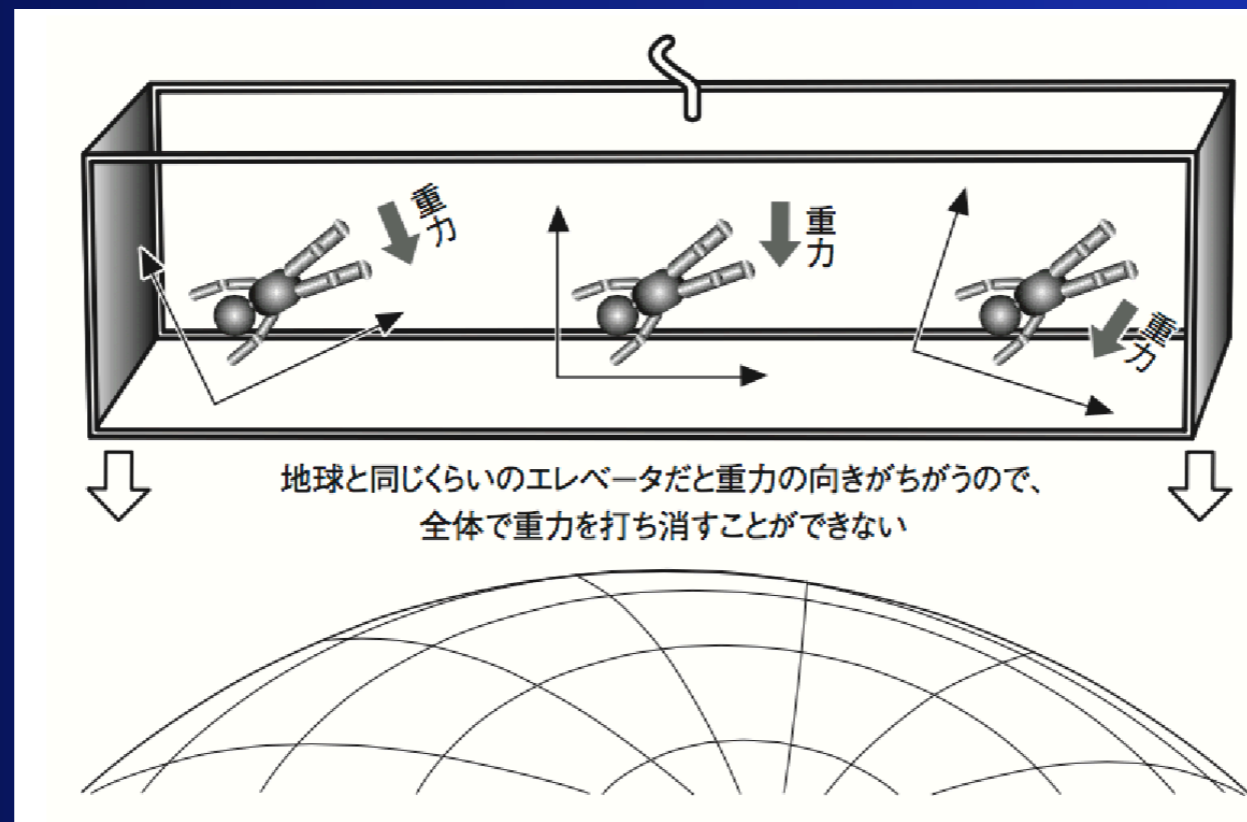
30秒という時間は短いようですが、通常スカイダイビングでのフリーホールの時間 (バンジージャンプの5回分より長い) 時間とほぼ同じです。無重力の間は、スーパーマンになったように空を飛ぶなど思い思いにお楽しみ下さい。

また、このプログラムでは、10回の無重力が予定されていますが、それ以外に3回の月 (地球の1/6) や2回の火星 (地球の1/3) の重力も同じフライトでお楽しみいただく予定です。



What is “gravity”?

「Gravitational acceleration can be compensated.」
=acceleration can be removed **locally**. (Equivalence Principle)



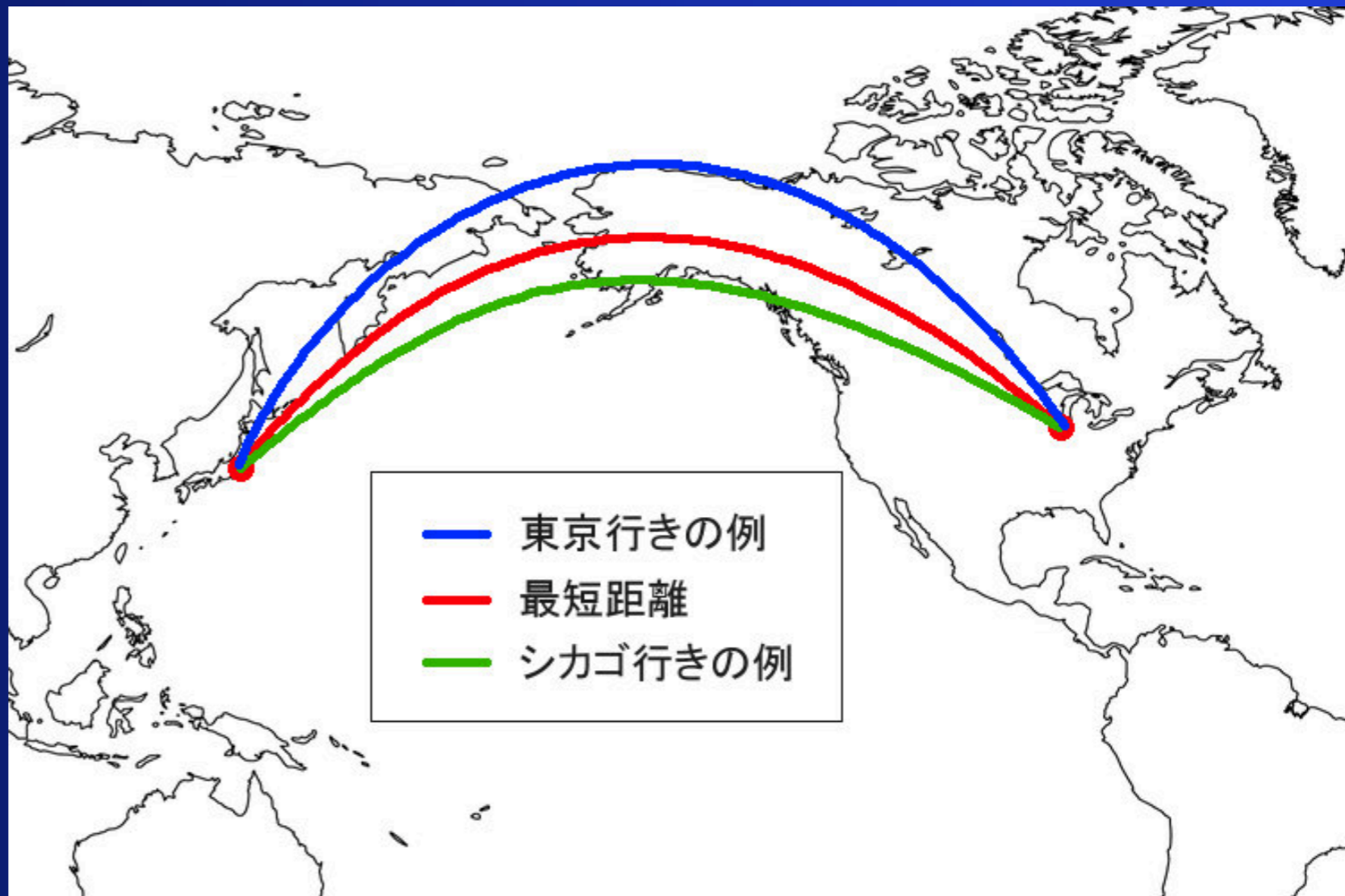
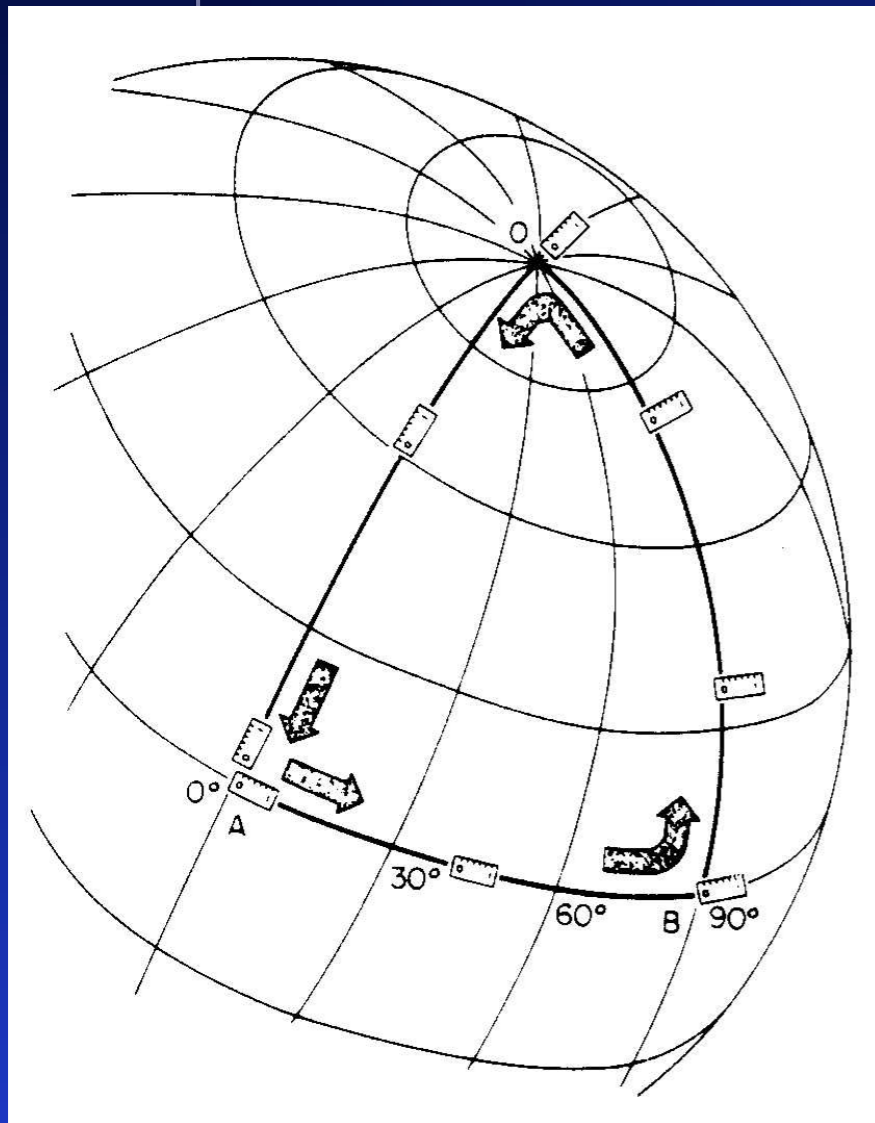
=gravity cannot be removed **globally**.

=gravity is the result of space-time bending.

Geometry of Curved Space = 「General Relativity」

General Relativity [metric]

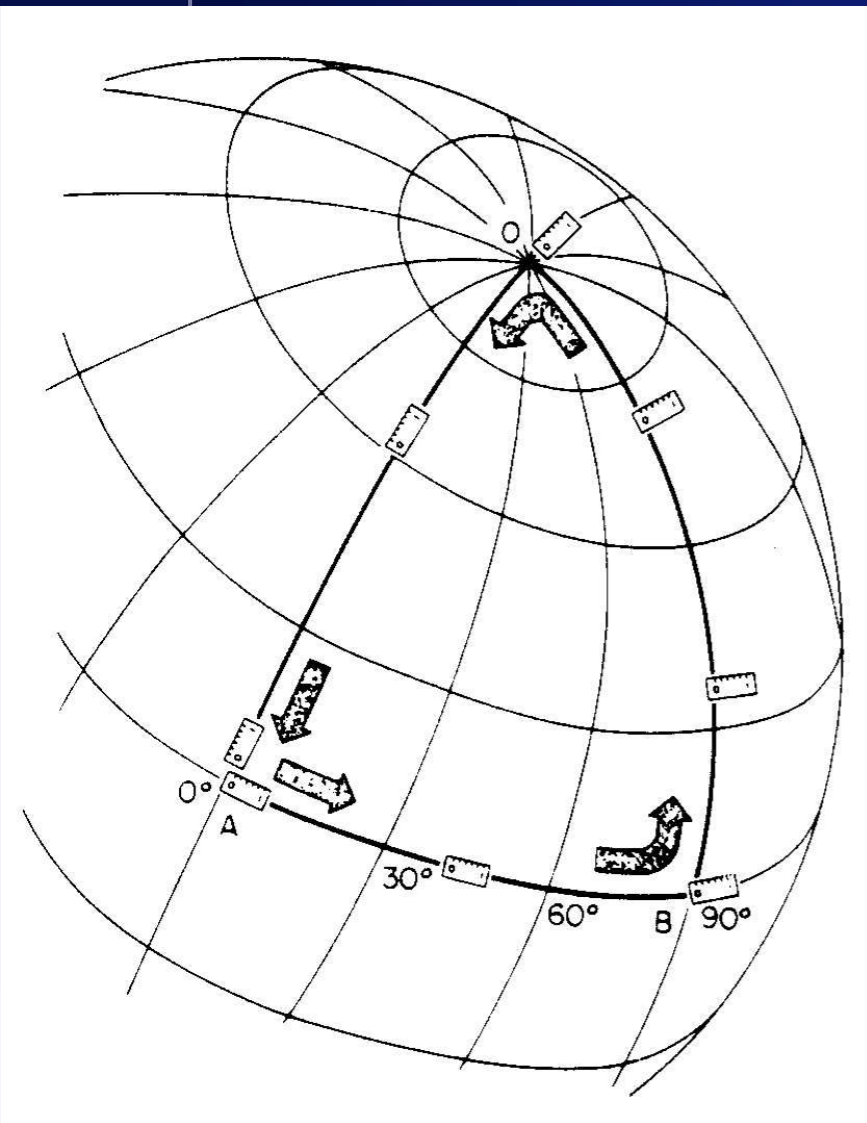
Geometry of Curved Space = Riemannian Geometry



General Relativity [metric]

Geometry of Curved Space = Riemannian Geometry

Generalized distance 「metric」 .



$$ds^2 = \sum_{\mu=0}^3 \sum_{\nu=0}^3 g_{\mu\nu}(x) dx^\mu dx^\nu = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ & g_{xx} & g_{xy} & g_{xz} \\ & & g_{yy} & g_{yz} \\ sym. & & & g_{zz} \end{pmatrix}$$

In flat spacetime,

$$\begin{aligned} ds^2 &= -c^2 dt^2 + dx^2 + dy^2 + dz^2 \\ &= -c^2 dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \end{aligned}$$

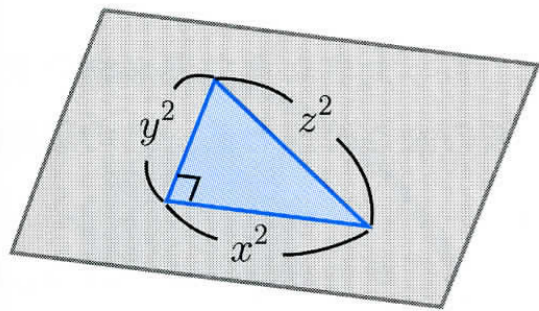
General Relativity [metric]

Geometry of Curved Space = Riemannian Geometry

Generalized distance 「metric」 .

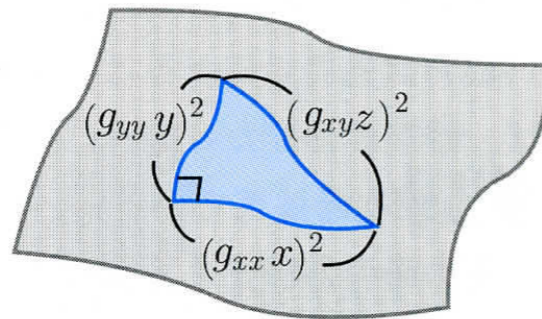
時空の曲がりを表す計量 $g_{\mu\nu}$

平らな面ではピタゴラスの定理が成り立つ。



$$x^2 + y^2 - z^2 = 0$$

曲がった空間でも成立するために計量関数で調整する。



$$(g_{xx}x)^2 + (g_{yy}y)^2 - (g_{xyz})^2 = 0$$

このように、曲がり具合は関数 $g_{\mu\nu}$ に押し付けて表すことができる。
 アインシュタイン方程式は、3次元空間+時間の4次元の曲がり $g_{\mu\nu}$ を解く方程式である。



Marcel Grossmann

$$ds^2 = \sum_{\mu=0}^3 \sum_{\nu=0}^3 g_{\mu\nu}(x) dx^\mu dx^\nu = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ & g_{xx} & g_{xy} & g_{xz} \\ & & g_{yy} & g_{yz} \\ sym. & & & g_{zz} \end{pmatrix}$$

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General Relativity

【The Einstein equation】

Field equation (1915)

how spacetime is curved \Leftrightarrow how matters are distributed

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

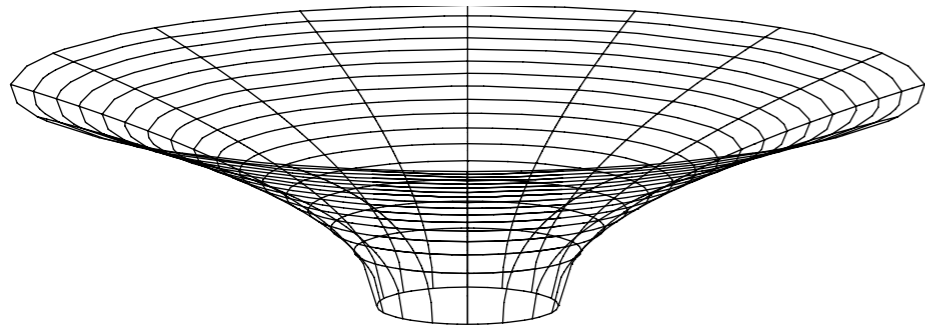
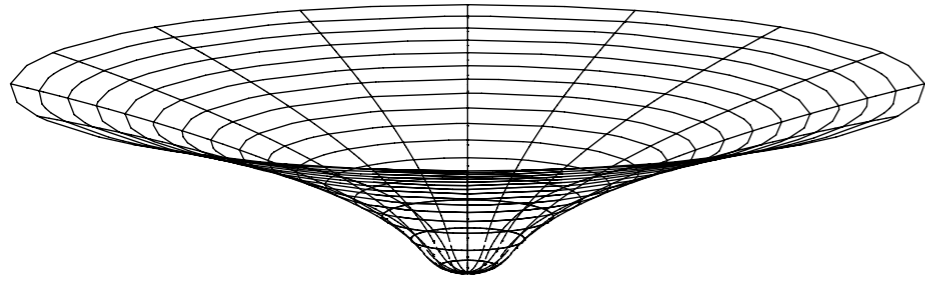
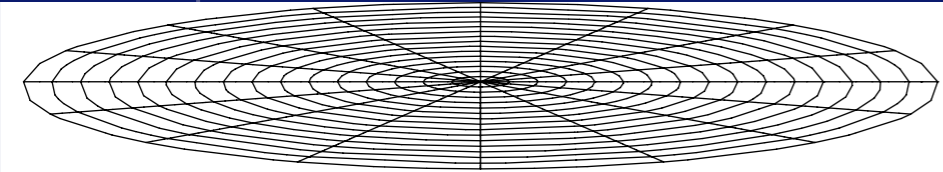
Riemann curvature

Ricci tensor, Einstein tensor

Energy-Momentum tensor

$$\begin{aligned}\Gamma_{\mu\nu}^{\alpha} &\equiv \frac{1}{2}g^{\alpha\beta}(\partial_{\nu}g_{\beta\mu} + \partial_{\mu}g_{\beta\nu} - \partial_{\beta}g_{\mu\nu}) \\ R^{\mu}_{\nu\alpha\beta} &\equiv \partial_{\alpha}\Gamma_{\nu\beta}^{\mu} - \partial_{\beta}\Gamma_{\nu\alpha}^{\mu} + \Gamma_{\sigma\alpha}^{\mu}\Gamma_{\nu\beta}^{\sigma} - \Gamma_{\sigma\beta}^{\mu}\Gamma_{\nu\alpha}^{\sigma} \\ R_{ab} \equiv R^{\mu}_{a\mu b} &\equiv \partial_{\mu}\Gamma_{ab}^{\mu} - \partial_b\Gamma_{a\mu}^{\mu} + \Gamma_{\nu\mu}^{\mu}\Gamma_{ab}^{\nu} - \Gamma_{\nu b}^{\mu}\Gamma_{a\mu}^{\nu} \\ R &= g^{ab}R_{ab}\end{aligned}$$

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}$$



IMAGINE THAT SPACE IS A GIANT SHEET OF RUBBER...

THINGS THAT HAVE MASS CAUSE THAT RUBBER SHEET TO BEND, LIKE A BOWLING BALL ON A TRAMPOLINE.

THE MORE MASS, THE MORE THAT SPACE GETS BENT AND DISTORTED BY GRAVITY.

FOR EXAMPLE, THE REASON THE EARTH GOES AROUND THE SUN IS THAT THE SUN IS VERY MASSIVE, CAUSING A BIG DISTORTION OF THE SPACE AROUND IT.

IF YOU JUST TRY TO MOVE IN A STRAIGHT LINE AROUND SUCH A BIG DISTORTION, YOU WILL FIND YOURSELF ACTUALLY MOVING IN A CIRCLE.

THAT'S HOW ORBITS WORK: THERE'S NO ACTUAL FORCE PULLING THE PLANETS AROUND, JUST A BENDING OF THE SPACE.

GRAVITATIONAL WAVES ARE PRODUCED WHENEVER MASSES ACCELERATE, CHANGING THE DISTORTION OF SPACE.

EVERYTHING WITH MASS AND/OR ENERGY CAN MAKE GRAVITATIONAL WAVES.

IF YOU AND I STARTED TO

The field equation (The Einstein equation)

The origin of the gravity is the curvature of space-time. The relation is expressed as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (2.18)$$

- The left-hand side expresses how space-time is curved using Riemannian geometry.
- The right-hand side expresses how the matter distributes in space-time. ($T_{\mu\nu}$ is called the energy-momentum tensor, and $T_{\mu\nu} = 0$ if vacuum.)
- The indice μ, ν indicate the coordinate (t, x, y, z) , so that (2.18) consists of 10 equations.

This equation tells us that if the matter exists then the surrounding space-time curves. If space-time curves, then the matter moves along the curve (geodesics).

General Relativity (1916)

- **Motivation :**

Relativity (1905) only treats observers with constant speed.

What is the acceleration?

- **Breakthrough :**

No gravity effects in free-falling elevator !

But we cannot erase the gravity in an Earth-sized elevator (globally).

- **Einstein's conclusion :**

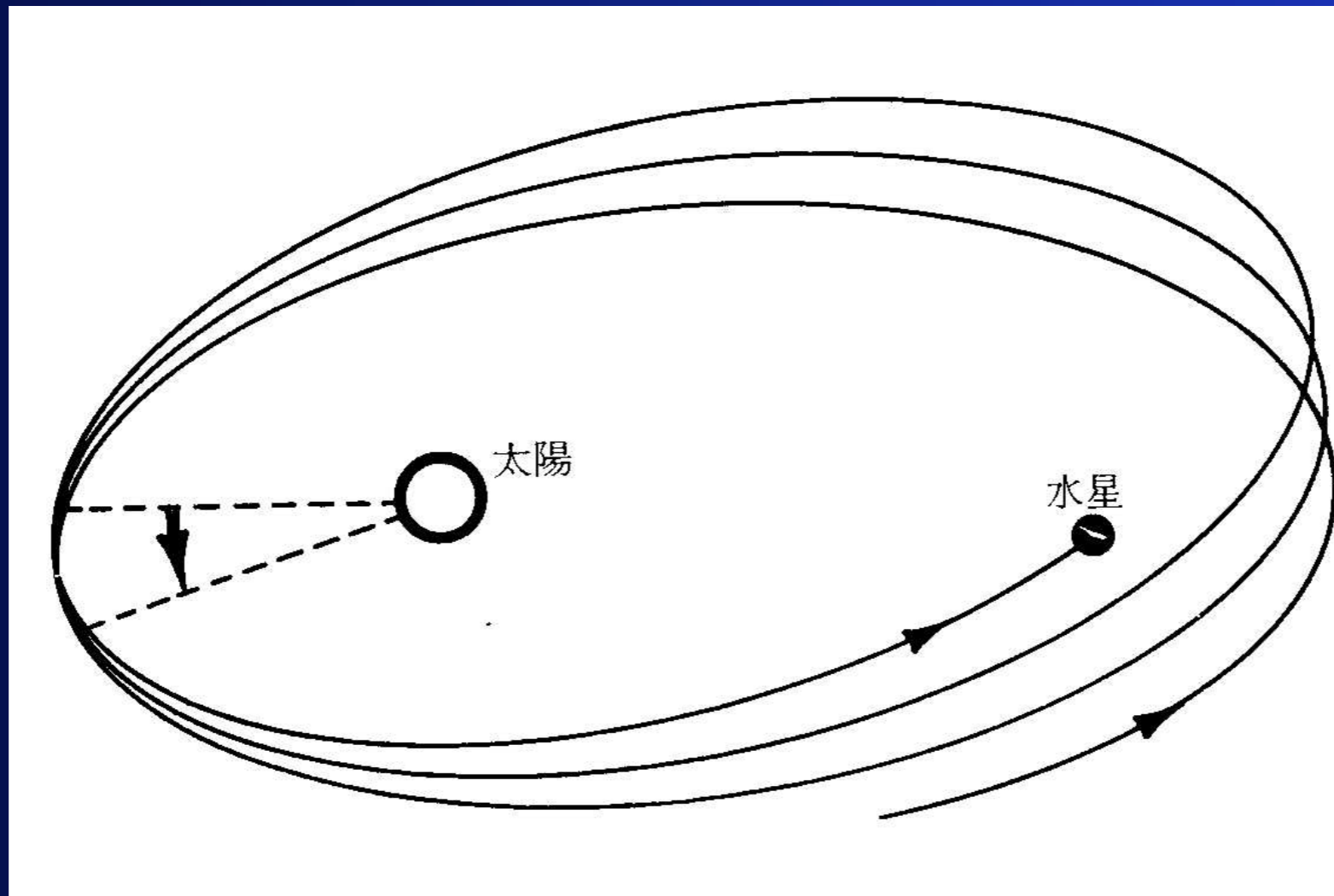
Gravity is space-time curvature.

Prediction of General Relativity

— Orbit of Mercury : rotation of perihelion

近日点移動

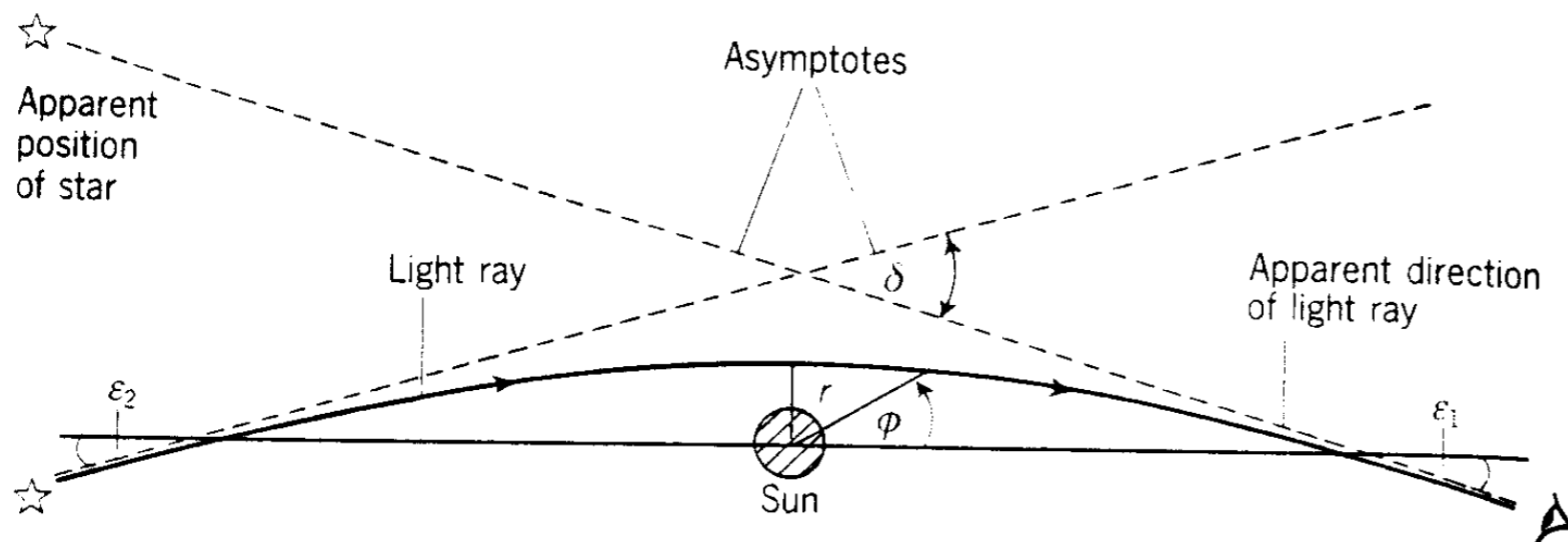
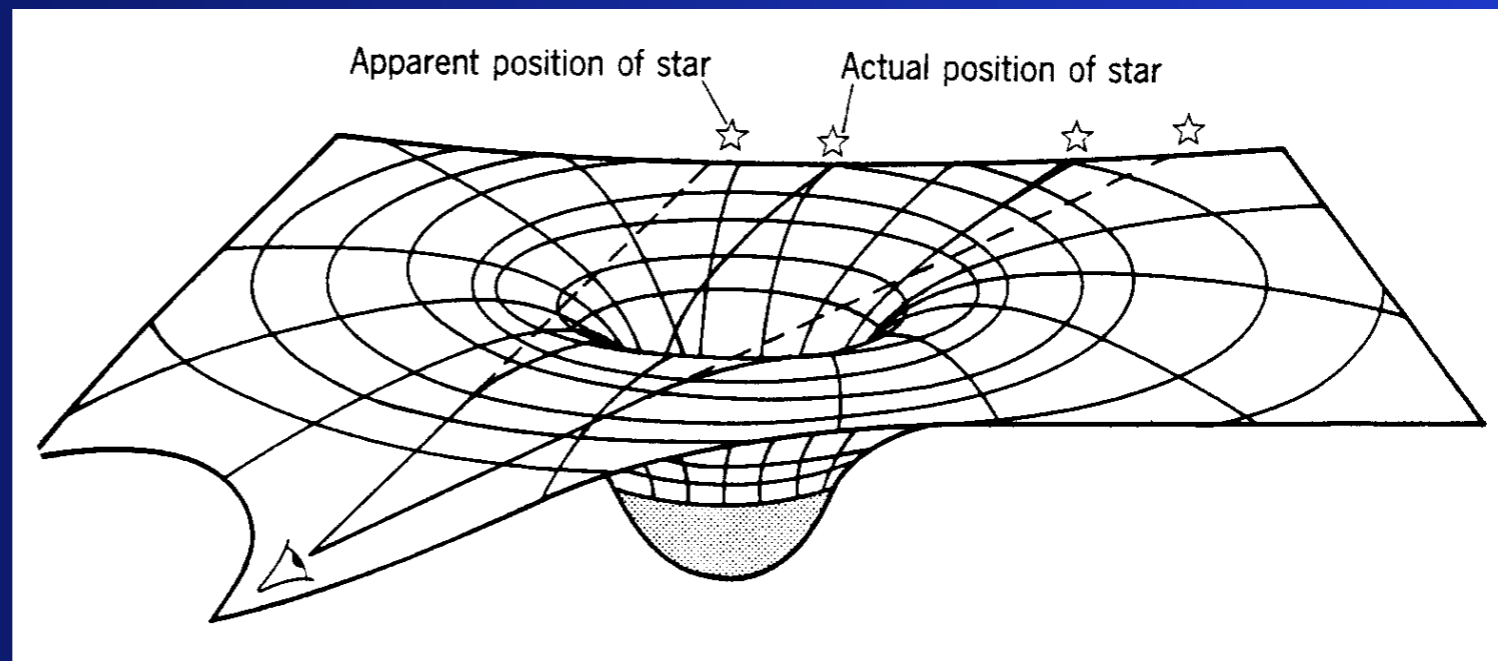
Unexplained orbital change of Mercury (Le Verrier 1854)



Einstein's theory explained this effect. (1915)

Prediction of General Relativity

—Bending of Light: gravitational lens—



Taken from the 22 November 1919 edition of the Illustrated London News.

Coverage in the (more excitable) New York Times.

LIGHTS ALL ASKEW IN THE HEAVENS

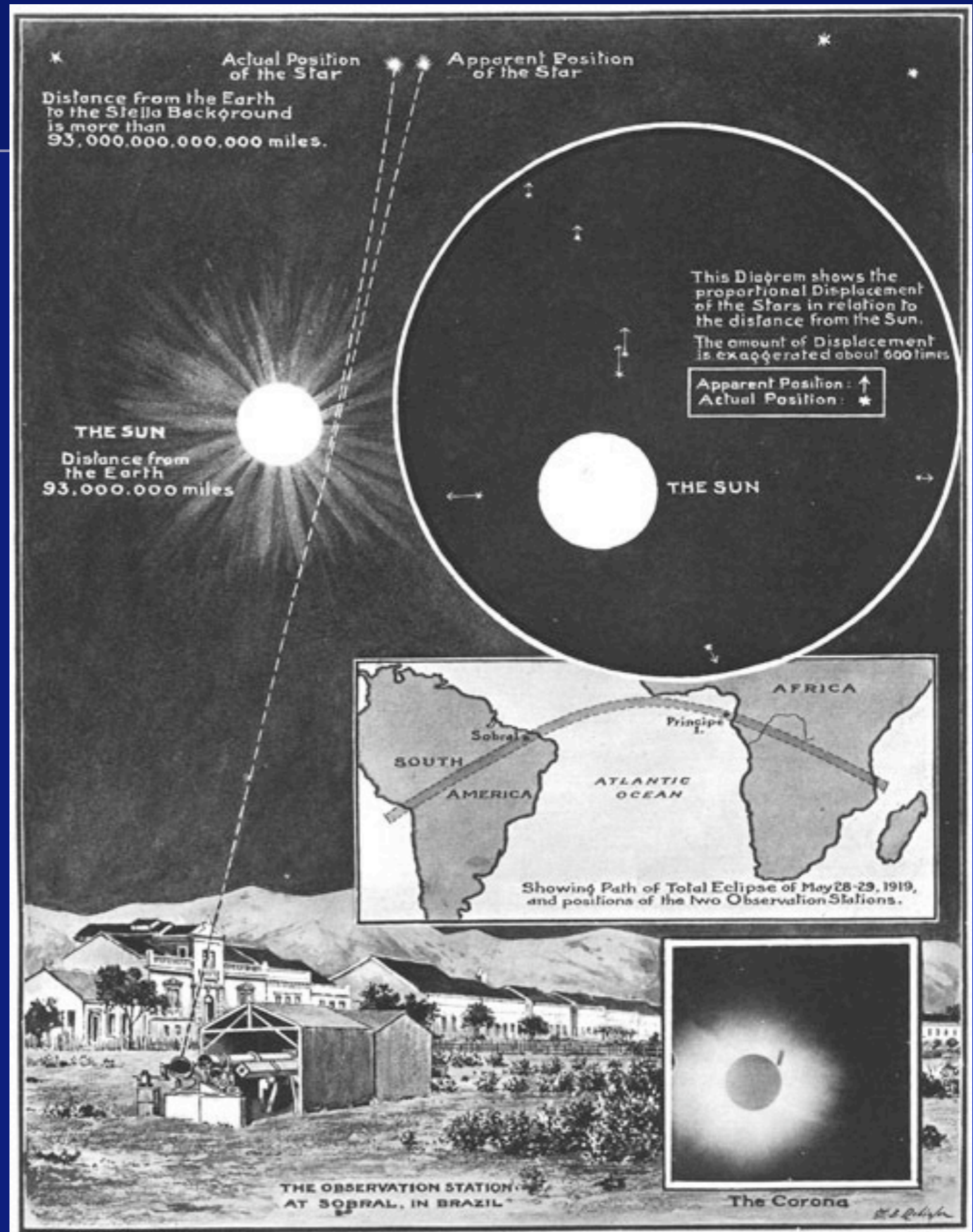
Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could
Comprehend It, Said Einstein When
His Daring Publishers Accepted It.



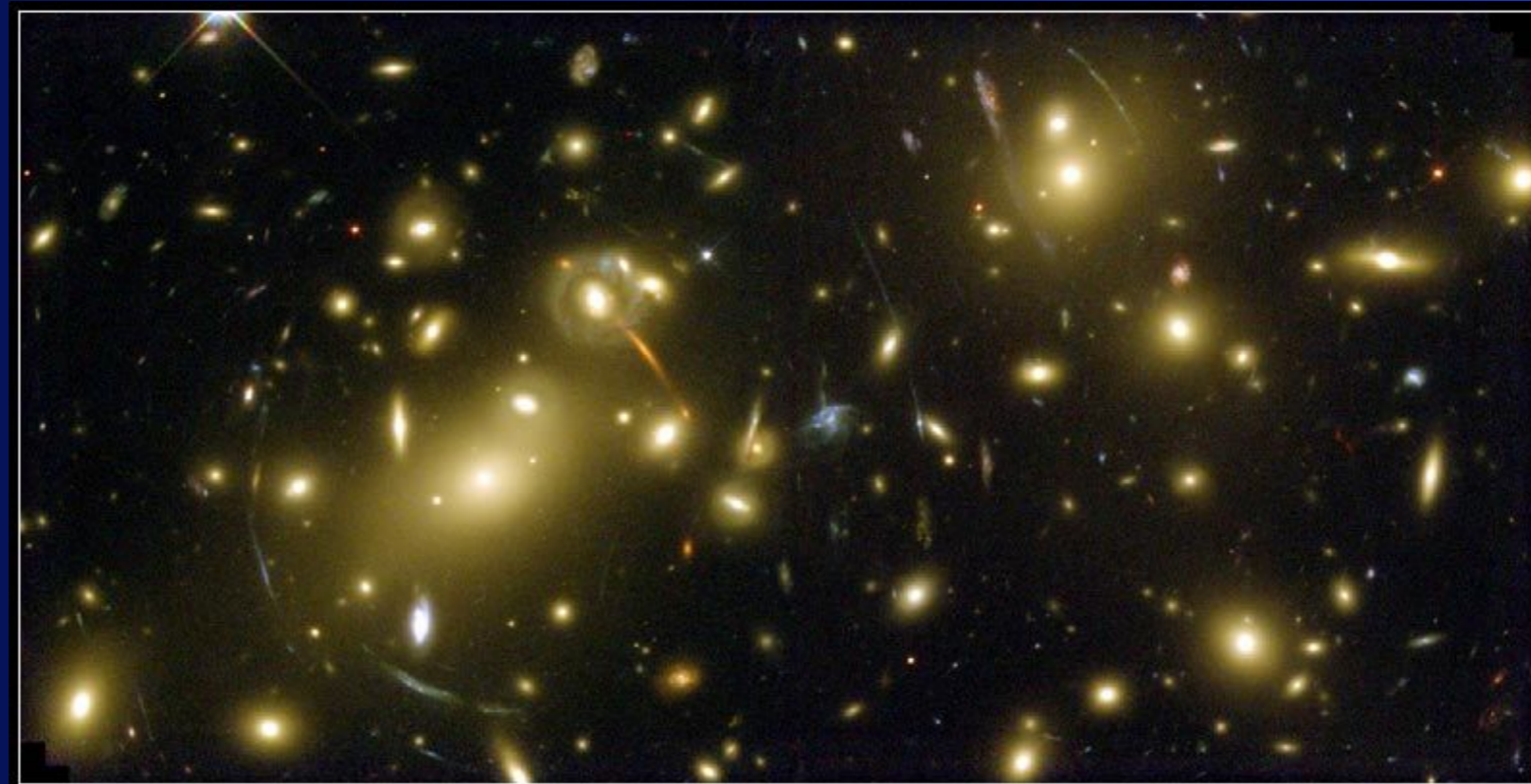


Arthur Stanley Eddington
(1882–1944)

He was an early advocate of Einstein's General Relativity, and an interesting anecdote well illustrates his humour and personal intellectual investment: Ludwig Silberstein, a physicist who thought of himself as an expert on relativity, approached Eddington at the Royal Society's (6 November) 1919 meeting where he had defended Einstein's Relativity with his Brazil-Principe Solar Eclipse calculations with some degree of scepticism and ruefully charged Arthur as one who claimed to be one of three men who actually understood the theory (Silberstein, of course, was including himself and Einstein as the other two). When Eddington refrained from replying, he insisted Arthur not be "so shy", whereupon Eddington replied, **"Oh, no! I was wondering who the third one might be!"**

Prediction of General Relativity

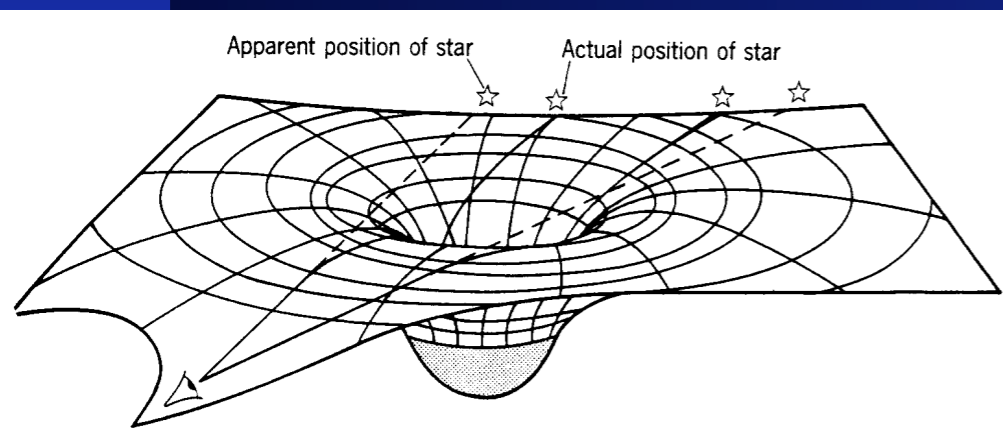
—Bending of Light: gravitational lens—

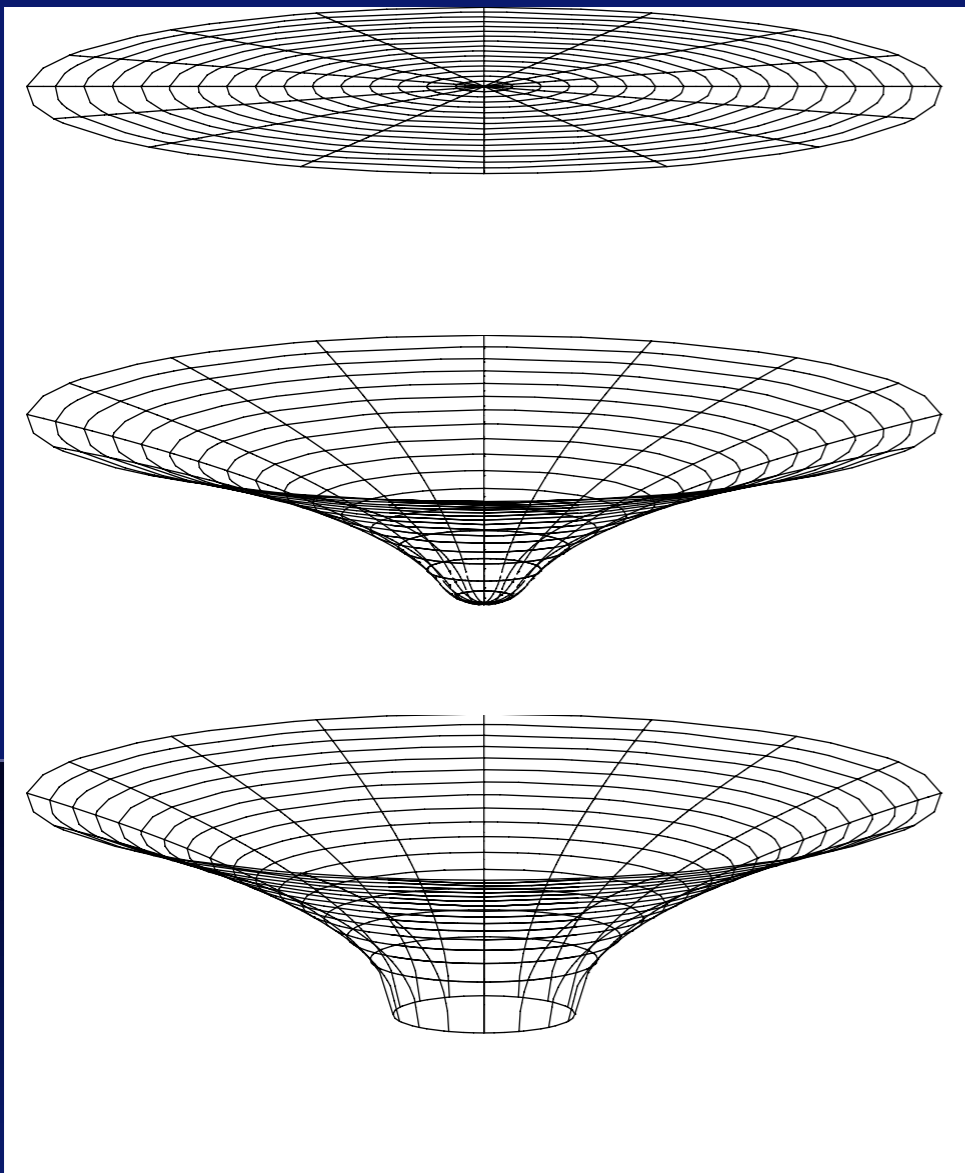


Galaxy Cluster Abell 2218

HST • WFPC2

NASA, A. Fruchter and the ERO Team (STScI, ST-ECF) • STScI-PRC00-08

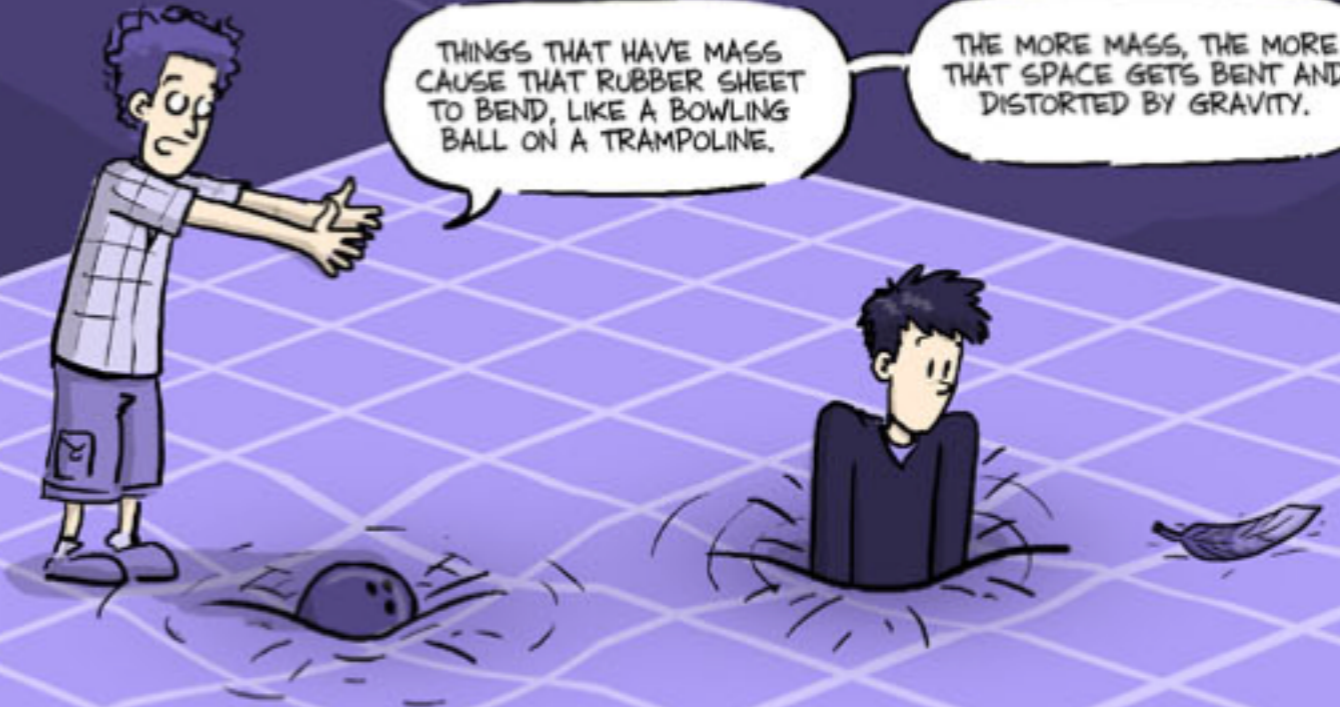




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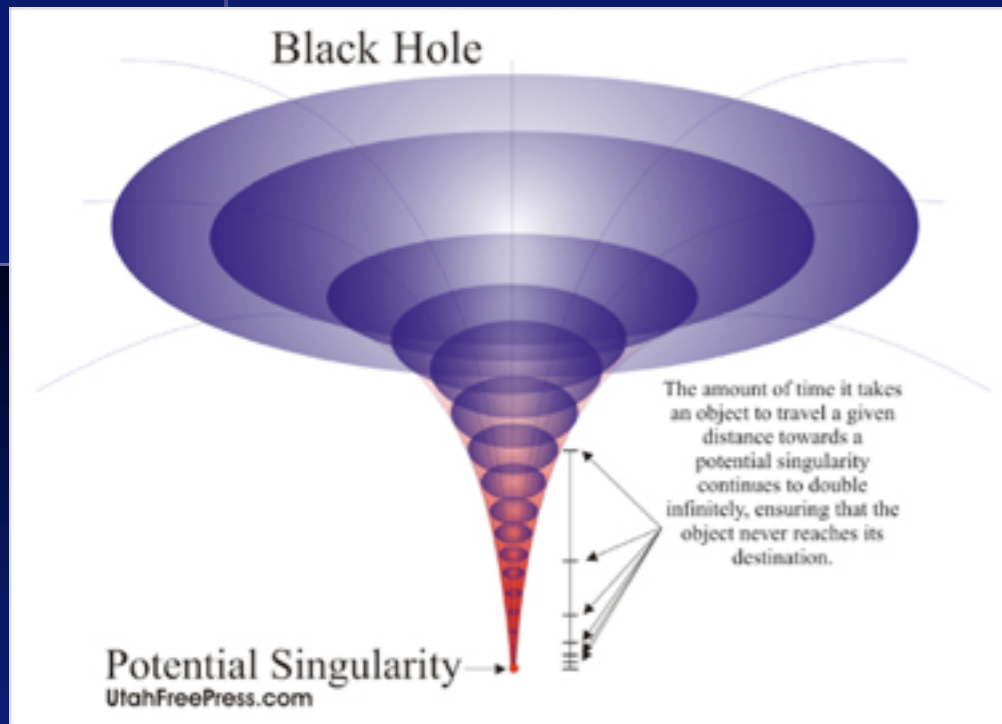
EVERYTHING WITH MASS AND/OR ENERGY CAN MAKE GRAVITATIONAL WAVES.

IF YOU AND I STARTED TO DANCE AROUND EACH OTHER.



FailsShot

Black-hole



2.5 How and Why we know there is a black hole

Schwarzschild solution (black-hole solution)

The solution of the Einstein equation (2.18) of spherically symmetric, static, and vacuum space-time.

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (2.19)$$

where G and c are gravitational constant and speed of light, M is the mass at the center, r is radial coordinate.

Exercise 3 The radius of black-hole, R_{BH} can be given by

$$R_{BH} = \frac{2GM}{c^2}, \quad (2.20)$$

where M is the mass of black-hole, $G = 6.67 \times 10^{-11} [\text{m}^3/\text{kg}/\text{s}^2]$ is gravitational constant, and $c = 3.0 \times 10^8 [\text{m}/\text{s}]$ is the speed of light. Calculate the size of

- an Earth-mass black hole ($M = 6.0 \times 10^{24} [\text{kg}]$)
- a Sun-mass black hole ($M_{\odot} = 2.0 \times 10^{30} [\text{kg}]$)
- a black-hole of the center of our galaxy ($M = 4.2 \times 10^6 M_{\odot}$)



Figure 26: Karl Schwarzschild (1873–1916)

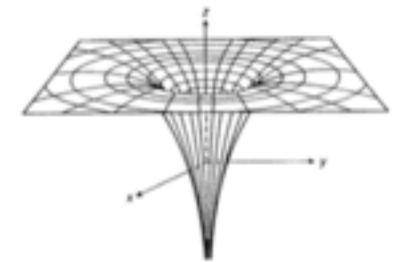
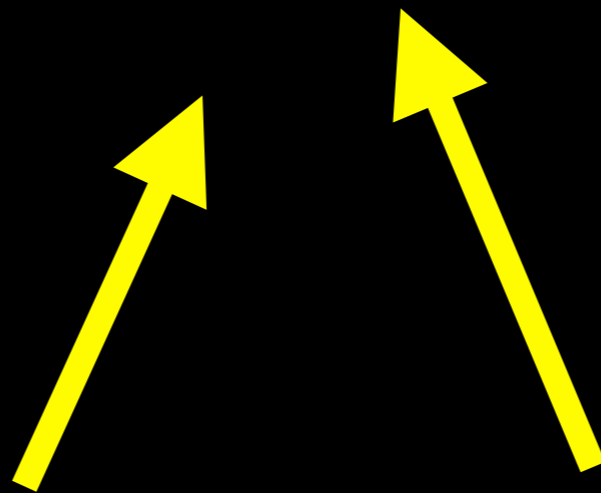
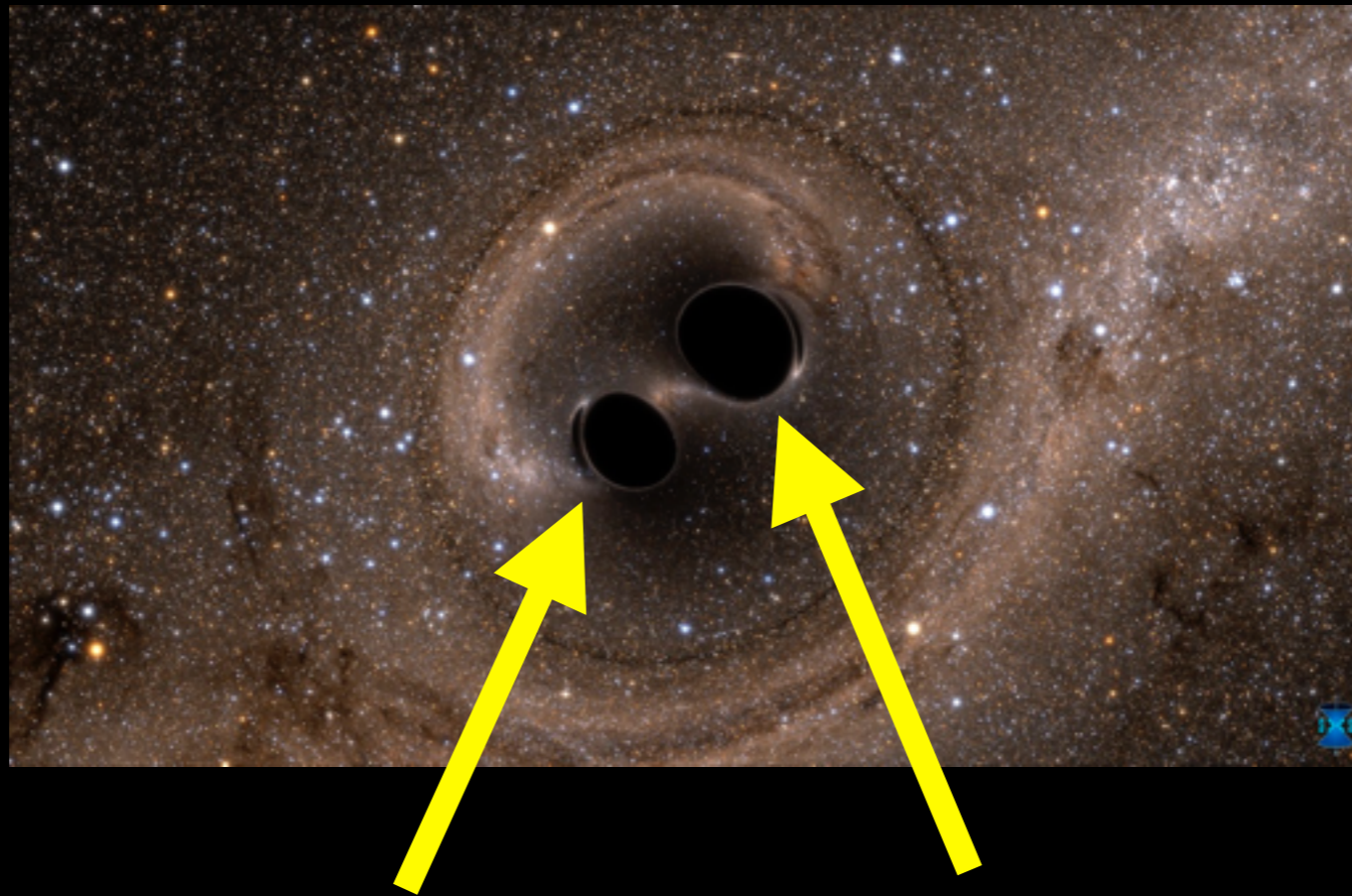


Figure 27: Black-hole is the bended trampoline to infinity.

Black hole = Too heavy object
Even the Light cannot escape

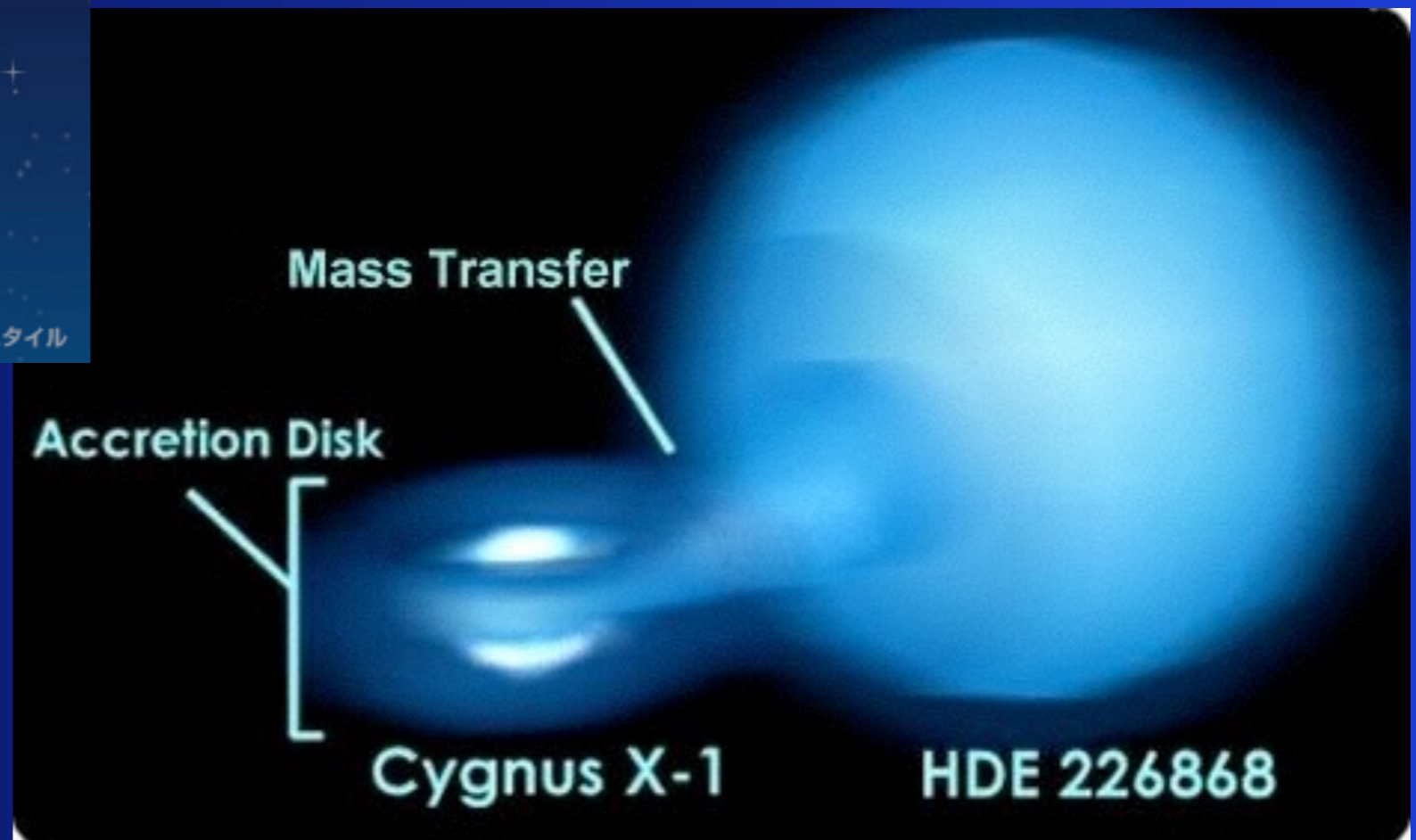
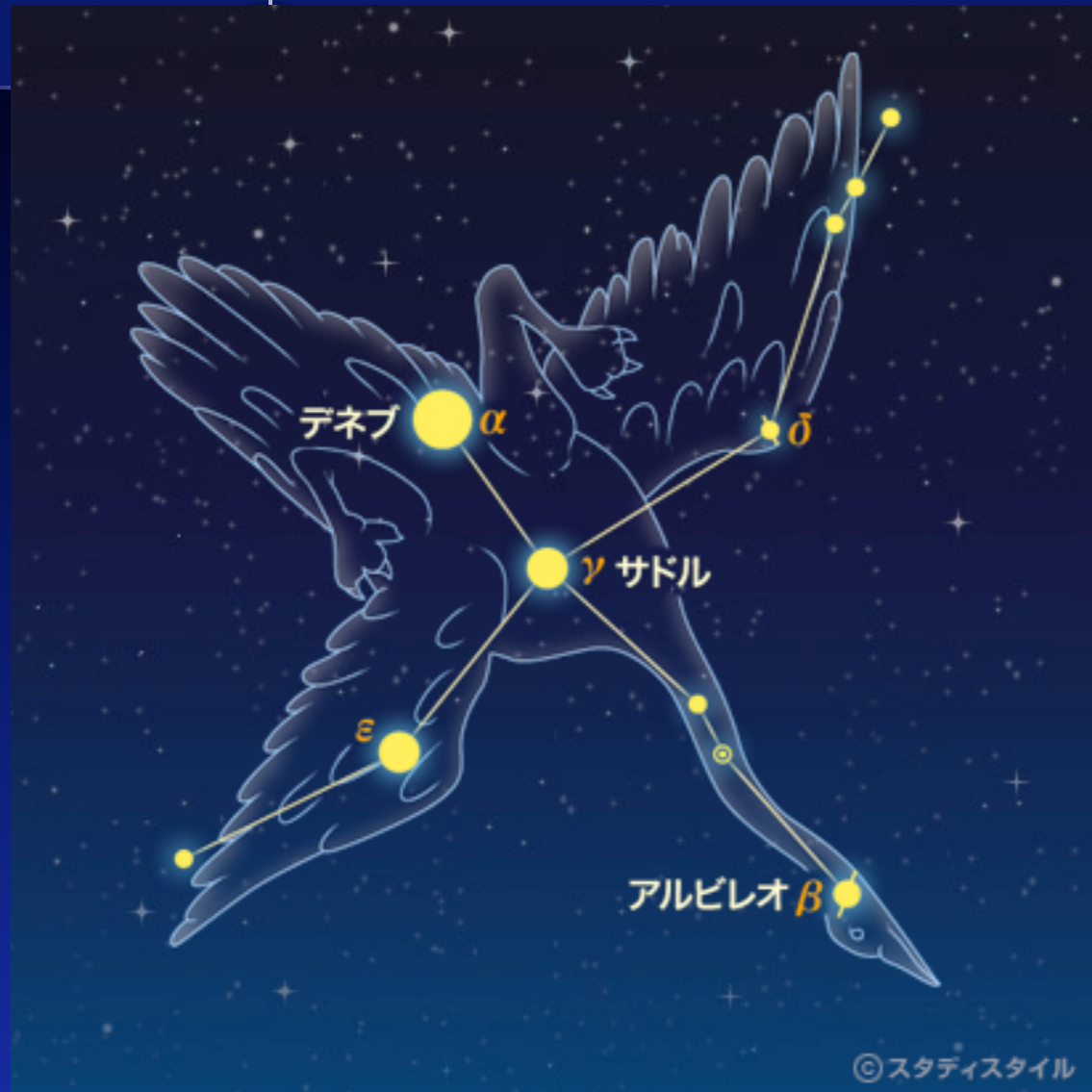


Black hole = Too heavy body
Even the Light cannot escape



Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

BH candidate: Cygnus X-1



Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Gravitational Lensing

MATTHEW
McCONAUGHEY

ANNE
HATHAWAY

JESSICA
CHASTAIN

MICHAEL
AND
CAINE




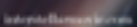




GO FURTHER.

FROM THE DIRECTOR OF **THE DARK KNIGHT TRILOGY** AND **INCEPTION**

INTERSTELLAR

IN THEATRES AND **IMAX**
EVERYWHERE
NOVEMBER 7

A black and white movie title card for the film 'Interstellar'. The title 'INTERSTELLAR' is centered in a serif font against a dark, starry background. The stars are scattered across the field, with some appearing as bright points and others as faint, diffuse clouds.

INTERSTELLAR

Making of **Interstellar (2014)**

0'57"

キップ・ソーン
Kip Thorne



Interstellar (2014)



Executive Producer: Kip Thorne

<https://www.youtube.com/watch?v=qZZ9jRan9eo>



Interstellar (2014)

Executive Producer: Kip Thorne

<https://www.youtube.com/watch?v=qZZ9jRan9eo>

Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Kepler motion

Supermassive BH in the center of Galaxy

THE MILKY WAY

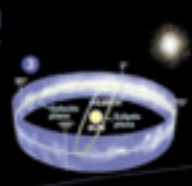


How galaxy of Earth, the Milky Way is a spiral-shaped system of a few hundred billion stars. Bright regions of recently formed stars highlight its arms, while older stars replete or edge their outer layers as beautiful planetary nebulas, then fade away and die. A thick swarm of orange and red stars marks the galactic bulge, encapsulating the star-packed galactic center. At its core may be a black hole, a region so dense that not even light can escape its gravitational pull. All objects in the Milky Way orbit the galactic center, much like planets in Earth's solar system revolve around the sun. But the scale is staggering: Light from a star at one edge of the galaxy takes about 100,000 years to reach the opposite side.



GUIDE TO THE GALAXY

- For beyond the galactic disk, yet orbit by its gravity, lone stars and globular clusters swirl in the galaxy's halo. Regions of dark matter—stream but felt through its gravitational effects—surround the central core.
- Star clouds of interstellar dust block much of our sight.
- Our view of the Milky Way, which from our position in the flat galactic disk appears as a fuzzy band of light, is obscured in some places by the dust that surrounds the galaxy's core.
- Earth's orbit around the sun lies at a severe angle to the galactic plane.



A TURBULENT HEART

A graph based on a radio survey reveals the whirlwind structure of molecular gas in the inner part of our galaxy. Gas moving away from Earth (top half) toward Earth (bottom half). This diagram (top) shows what the gas looks like. The gas is shown in blue and red, indicating different temperatures and densities. The bottom half shows a similar view but with different data points.

This computer-generated image of the Milky Way—our perspective of a 3-D model newly compiled for National Geographic—incorporates the actual positions of hundreds of thousands of stars and nebulas.

- Stellar star cluster
- Interstellar gas and dust
- Galactic
- Star-forming region
- Stellar cluster
- Galactic bulge or core
- Galactic halo

Reference positions for galaxies, nebulas, and star clusters

NGC Star General Catalog

Dark matter system-related

PLANETARY NEBULA NGC 8

A small, hot core, which will cool and fade over time to form, its stellar wind, sweeps of charged particles, creates an oval of opposite directions, but without from back-to-back an engine. This expansion, created by the Helix Space Nebula, is common among interstellar nebulas. Ultraviolet light from the star heats the gas and causes it to glow. The Helix Nebula is a classic example of this process.

LAGOON NEBULA

Light from the hot star is absorbed by and scatters the dust, creating a glow. As the star evolves, it sheds its outer layers, creating a cloud of gas and dust. This cloud is then ionized by the star's ultraviolet light, creating the colorful emission lines we see. The Lagoon Nebula is a classic example of this process.

WE ARE HERE

Our location is shown in the Orion arm, approximately 26,000 light-years from the galactic center. The diagram shows the spiral arms and the central bulge, with Earth's position marked in the Orion arm.

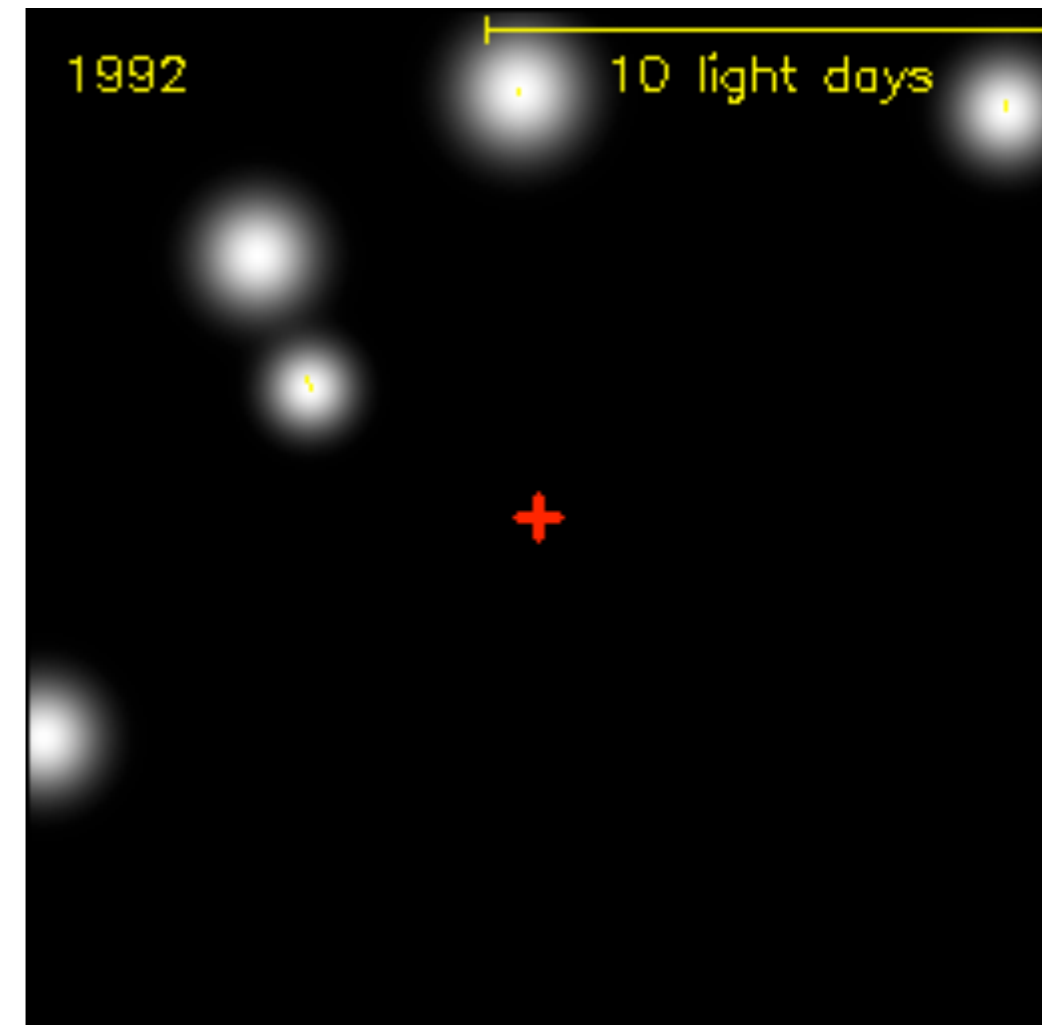
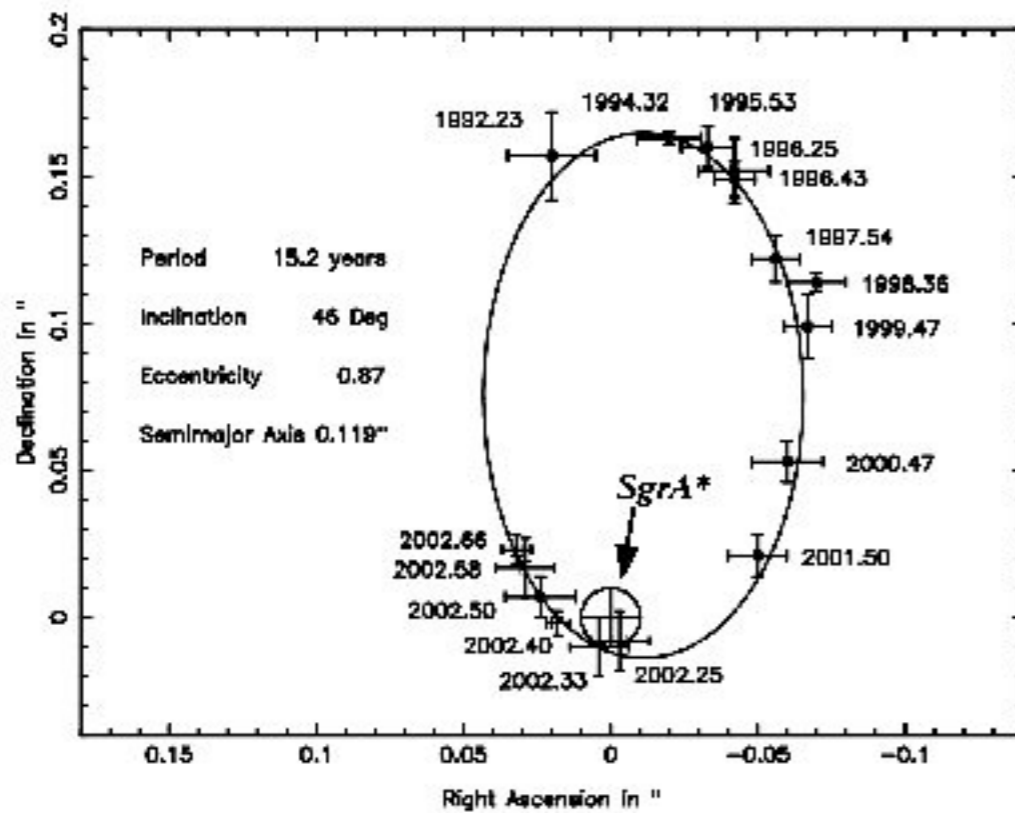
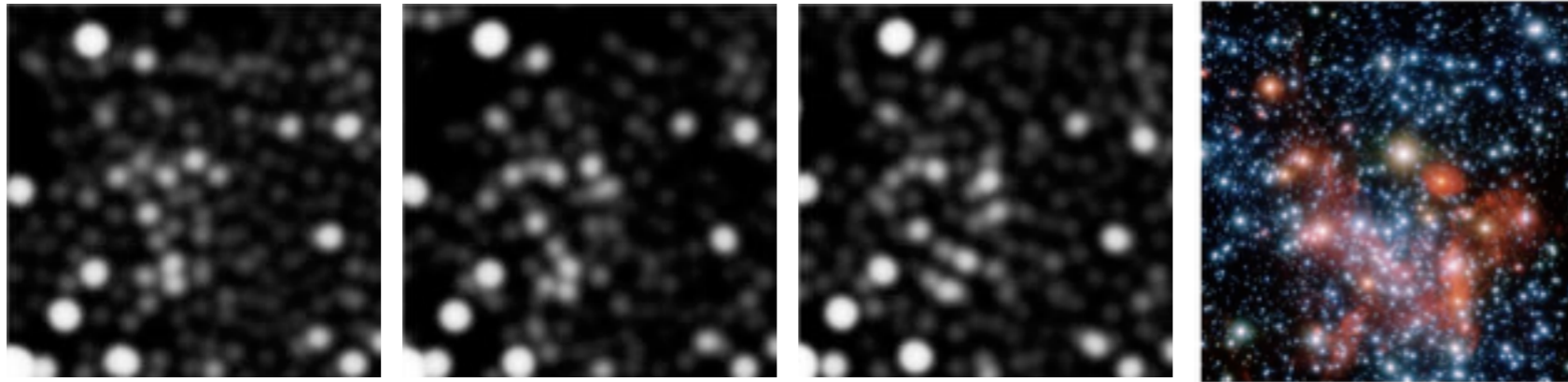
LAGOON NEBULA

The Lagoon Nebula is a classic example of a planetary nebula. It is a cloud of gas and dust that has been ejected by a star in the final stages of its life. The nebula is ionized by the star's ultraviolet light, creating the colorful emission lines we see. The Lagoon Nebula is a classic example of this process.

Zooming in the center of the Milky Way



Supermassive BH in the center of Galaxy

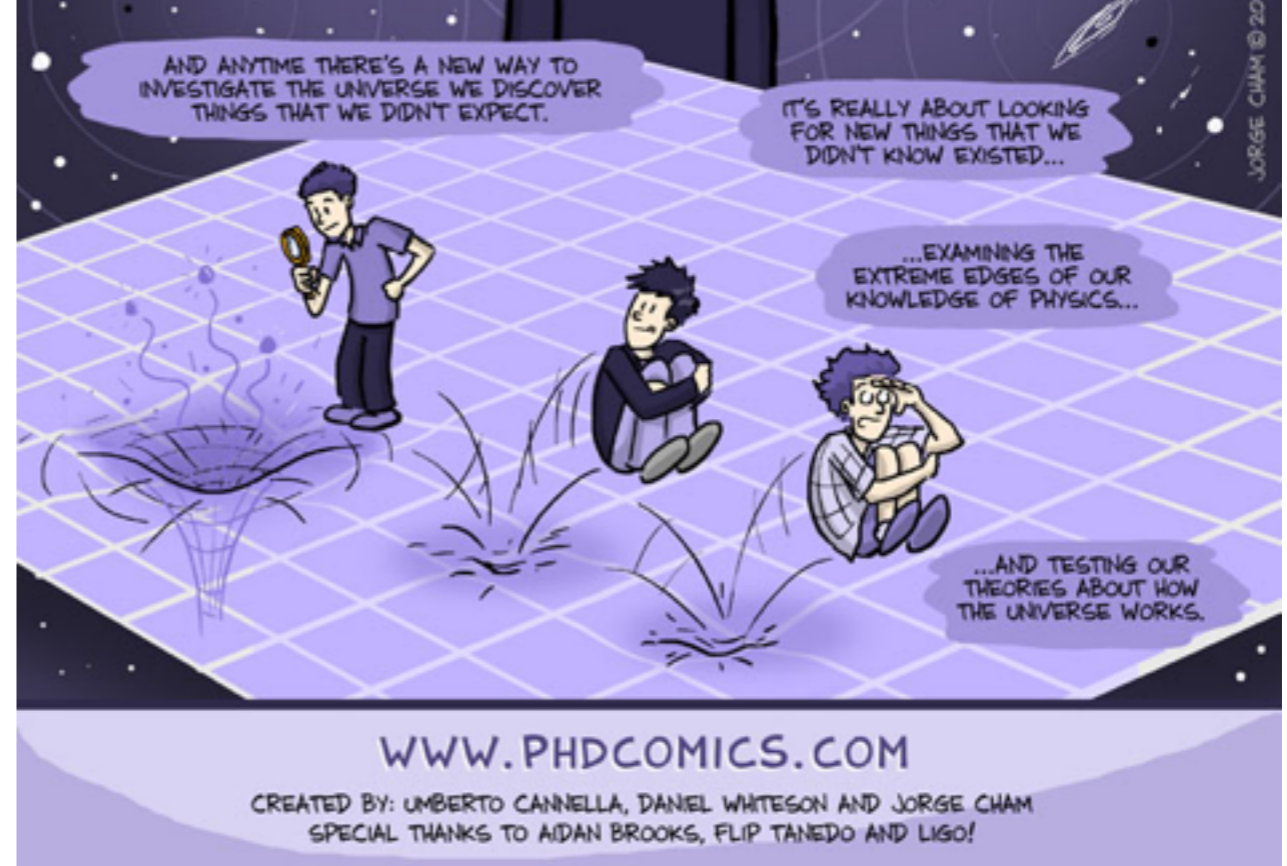
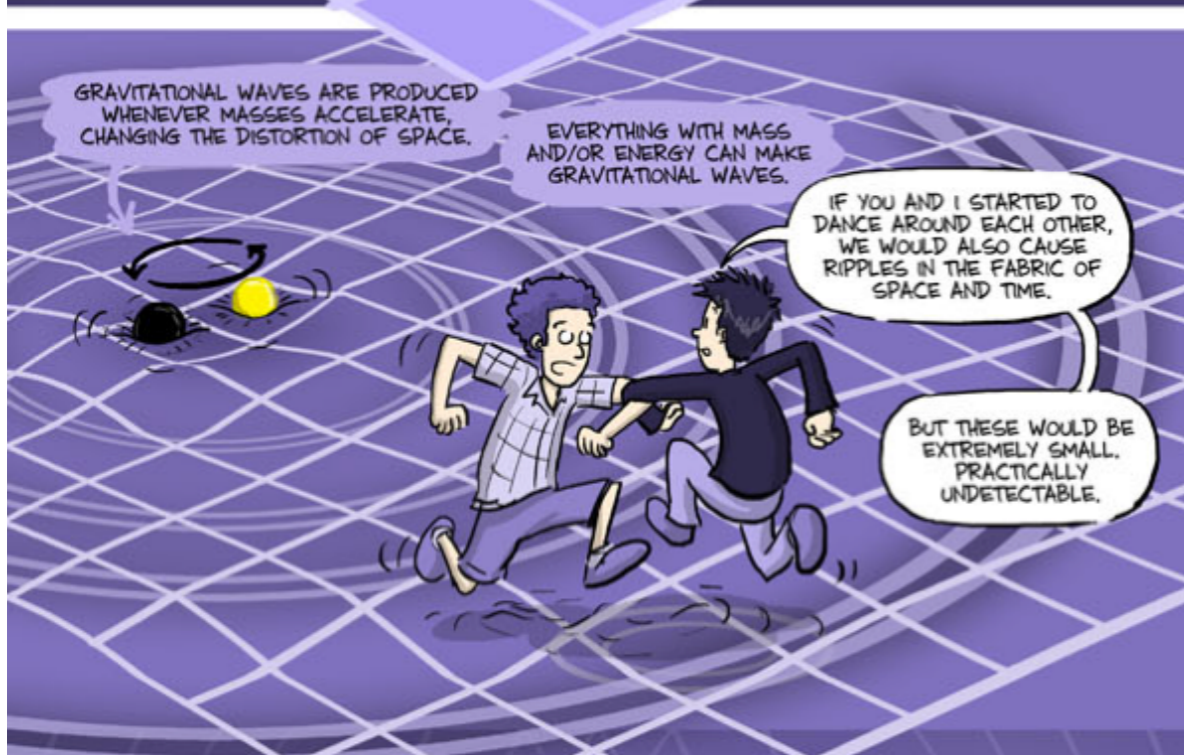
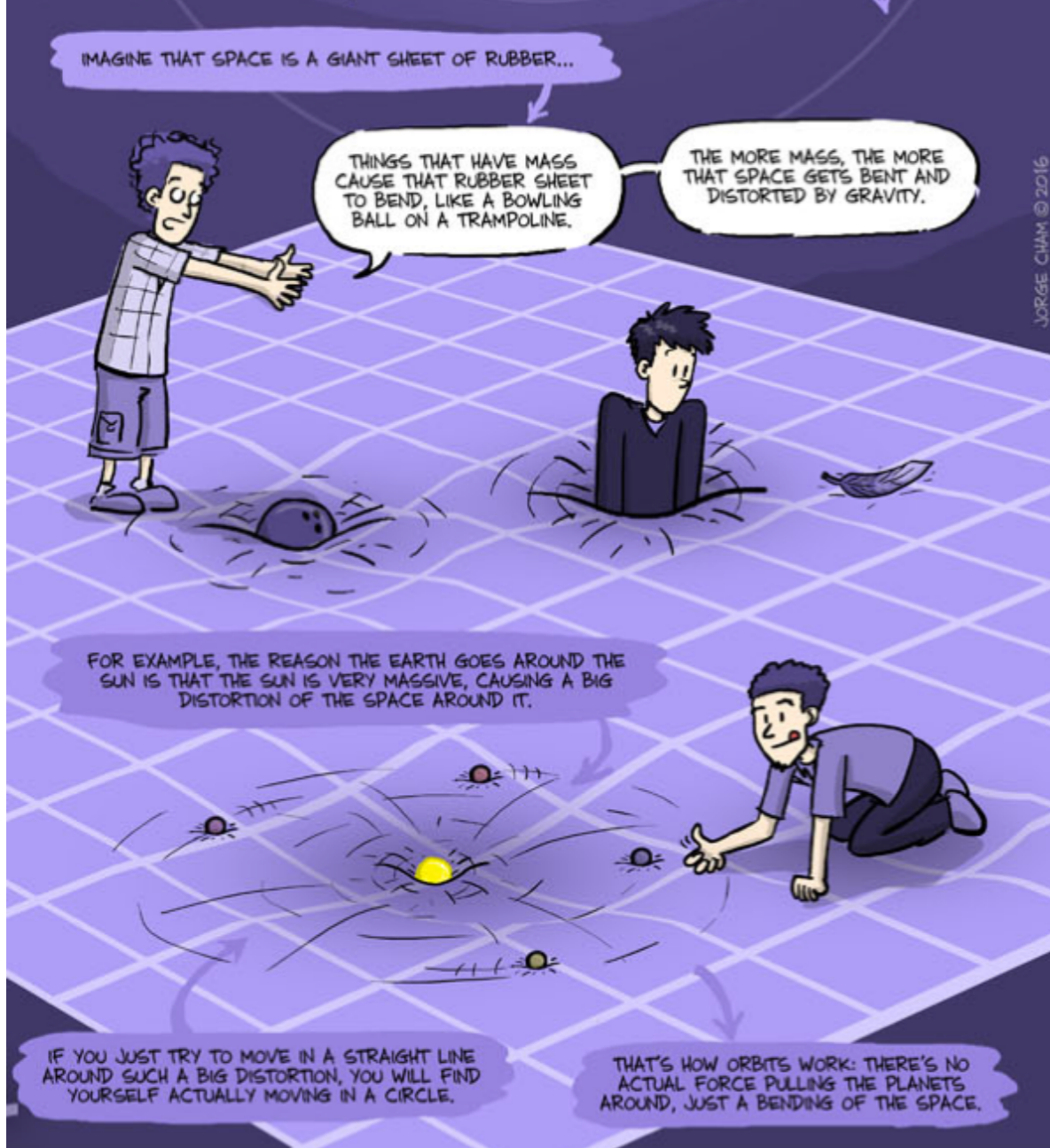


<http://www.extinctionsift.com/SignificantFindings08.htm>

<http://www.brighthub.com/science/space/articles/13435.aspx#>

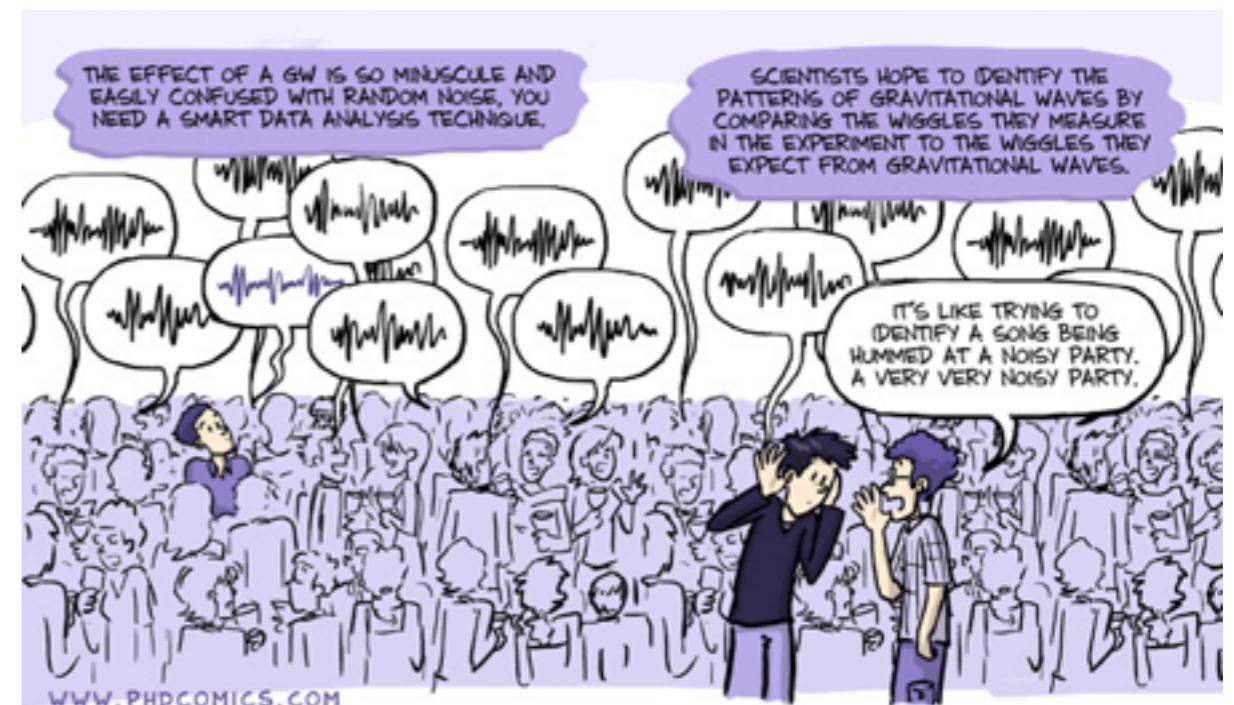
Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Gravitational Wave

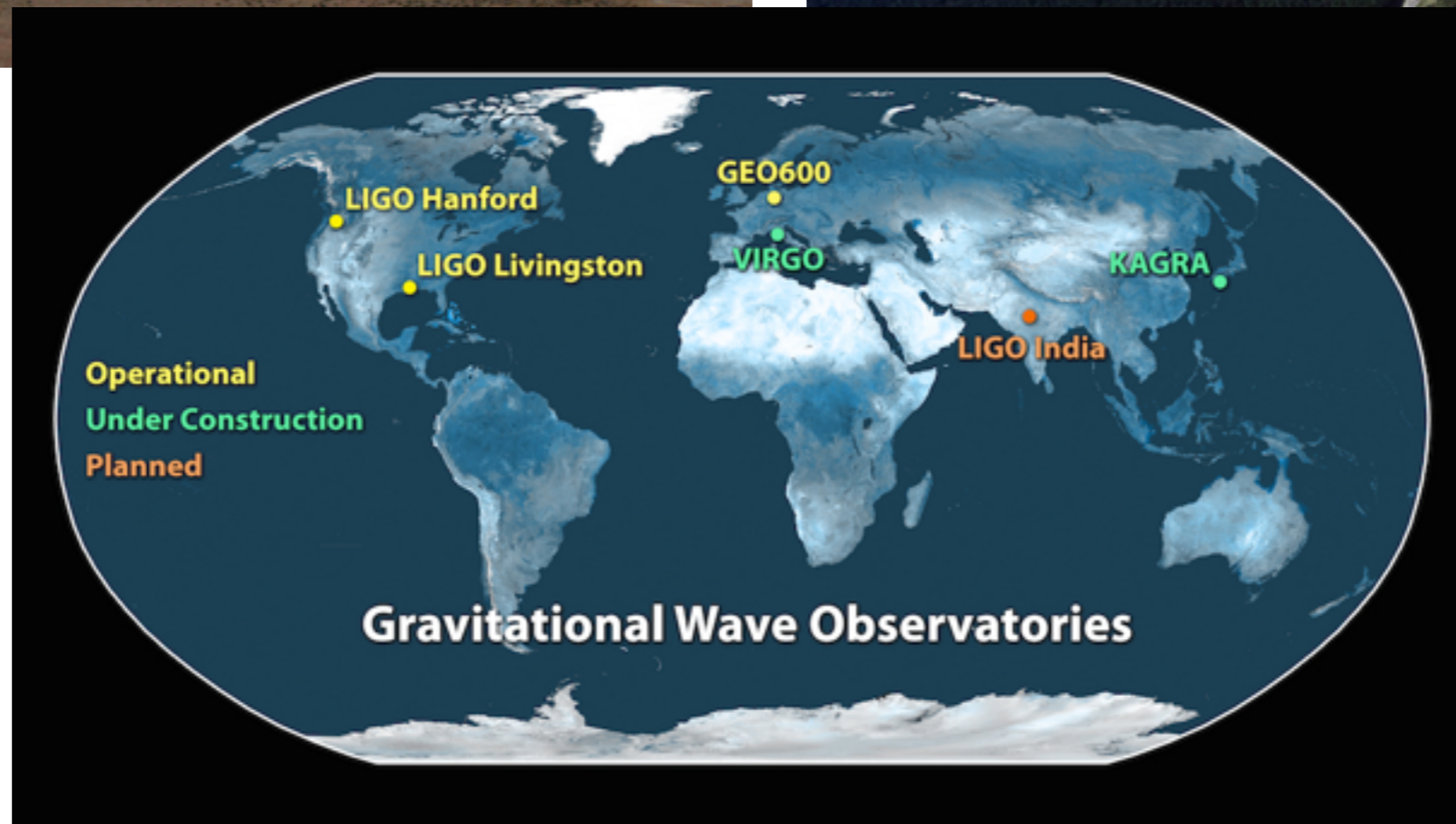
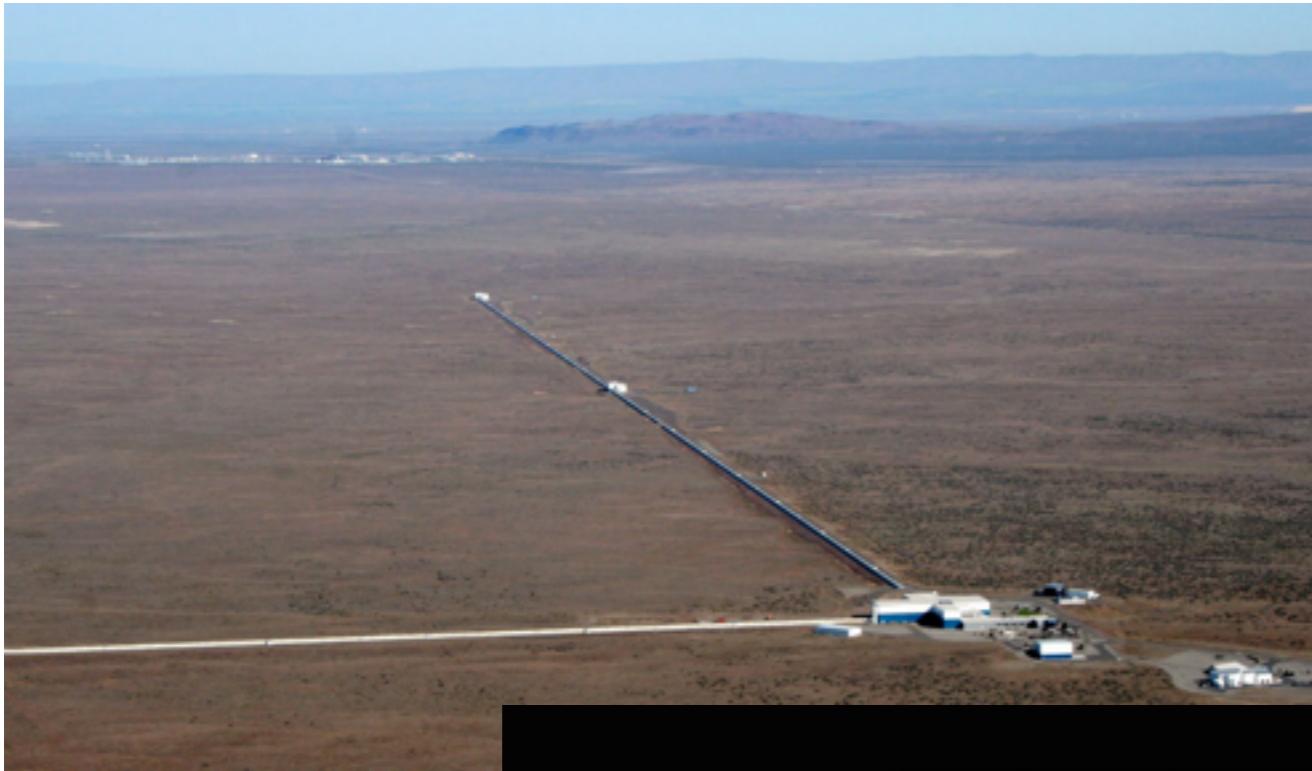


www.phdcomics.com

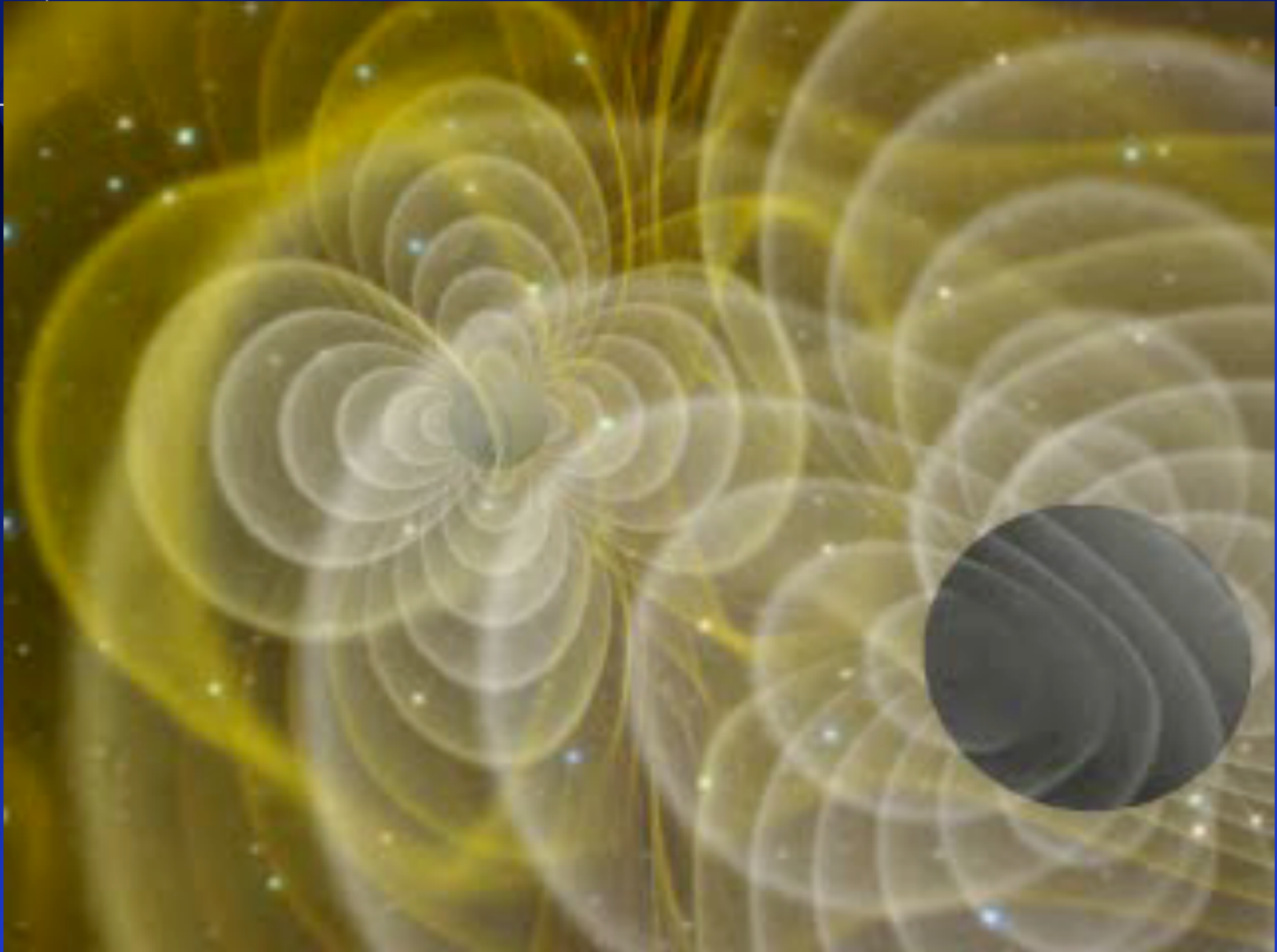
“gravitational waves explained”



LIGO (Laser Interferometer Gravitational-Wave Observatory)



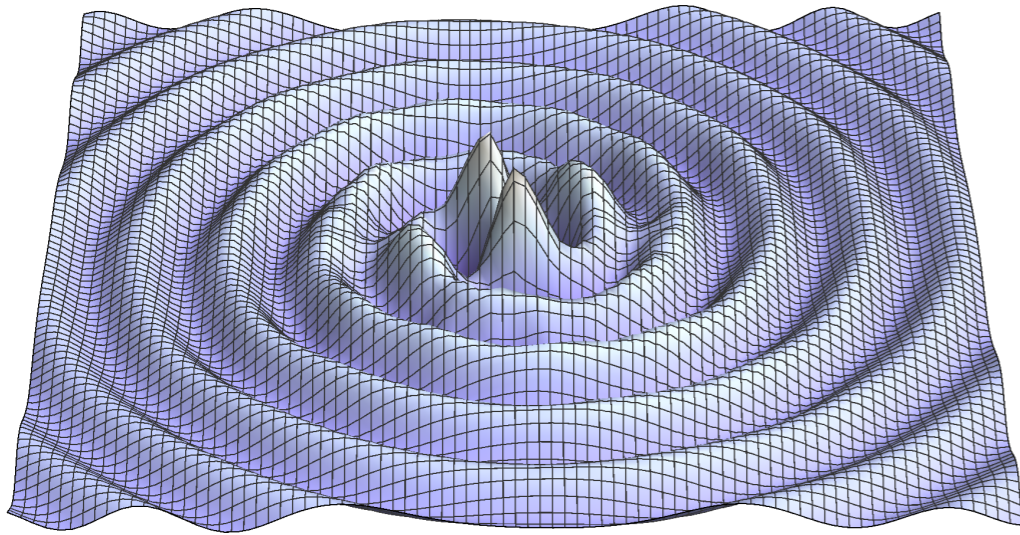
Collision of BH-BH binary



NCSA-AEI group (1998)

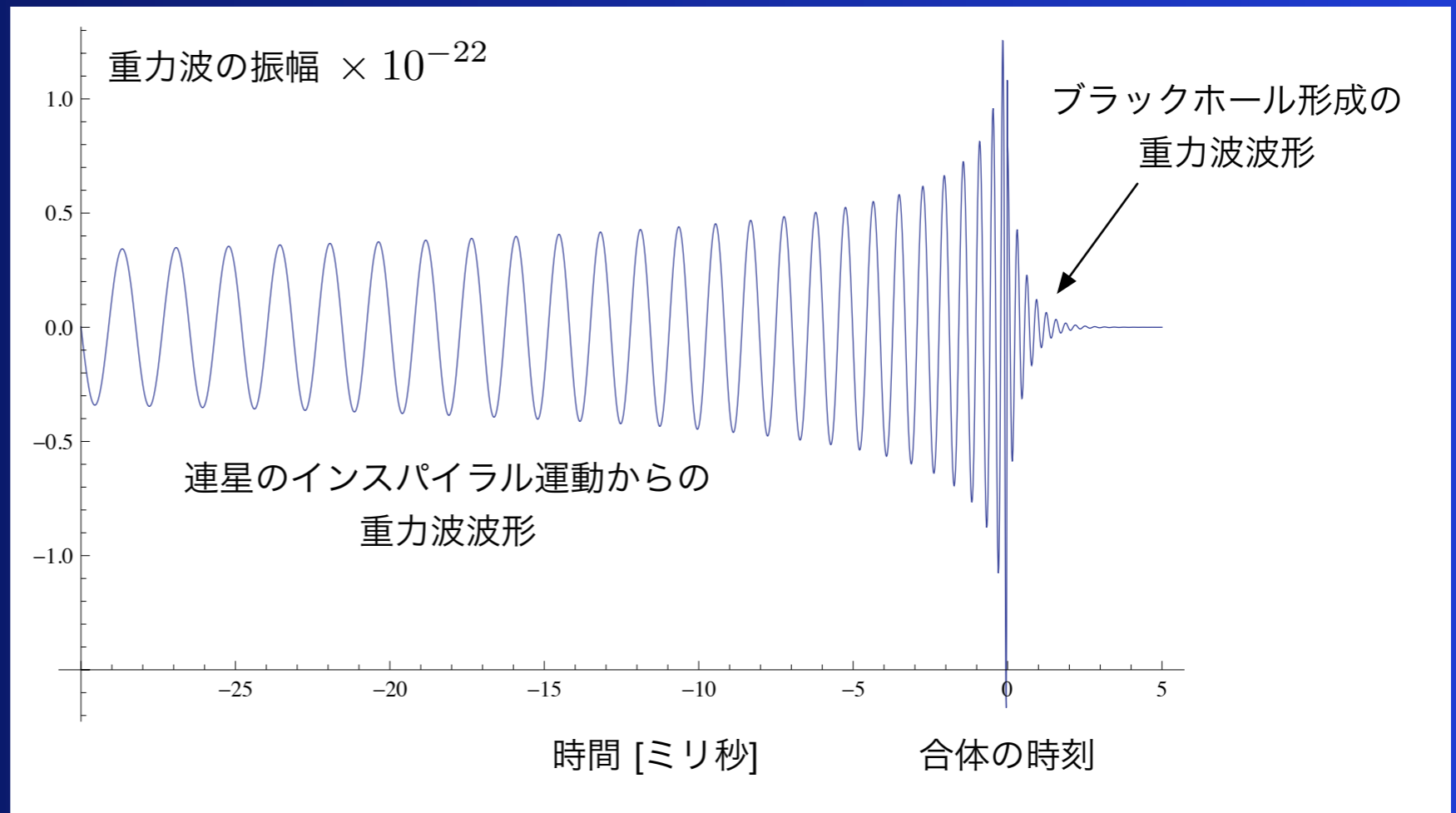
重力波の存在が間接的に確かめられた。

重力波の直接観測をしたい！



連星中性子星

連星ブラックホール



David Reitze, LIGO Director

Feb 11, 2016

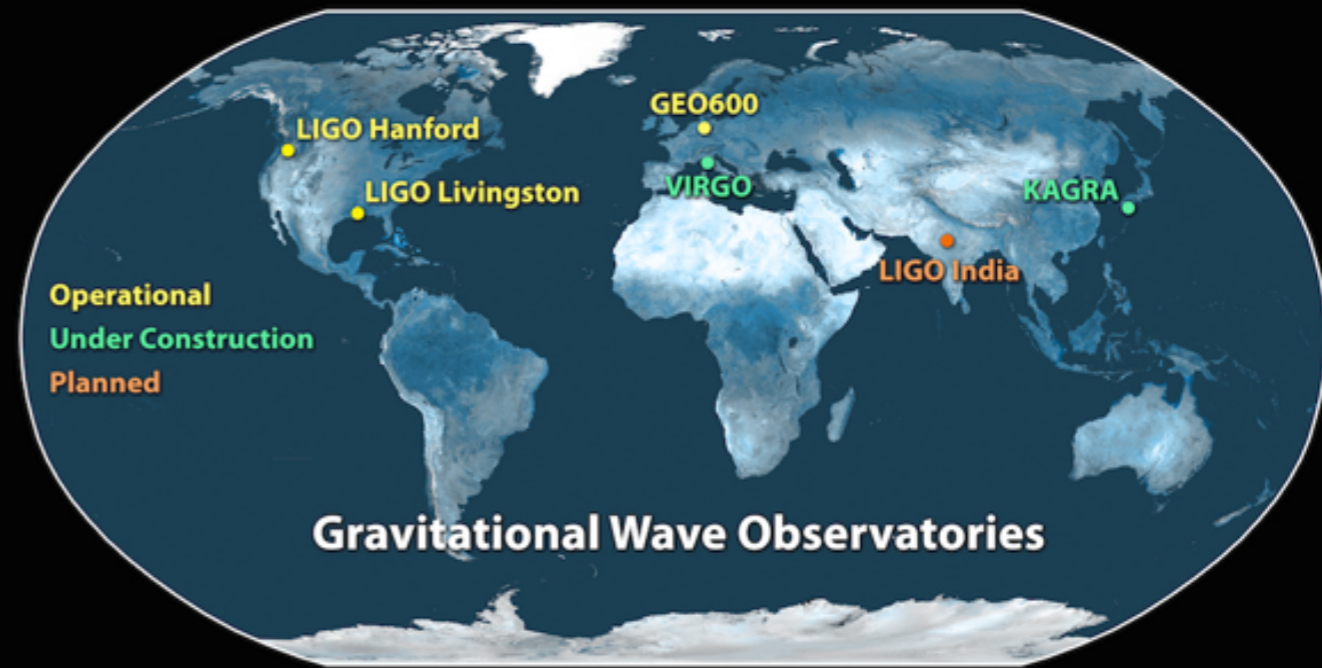
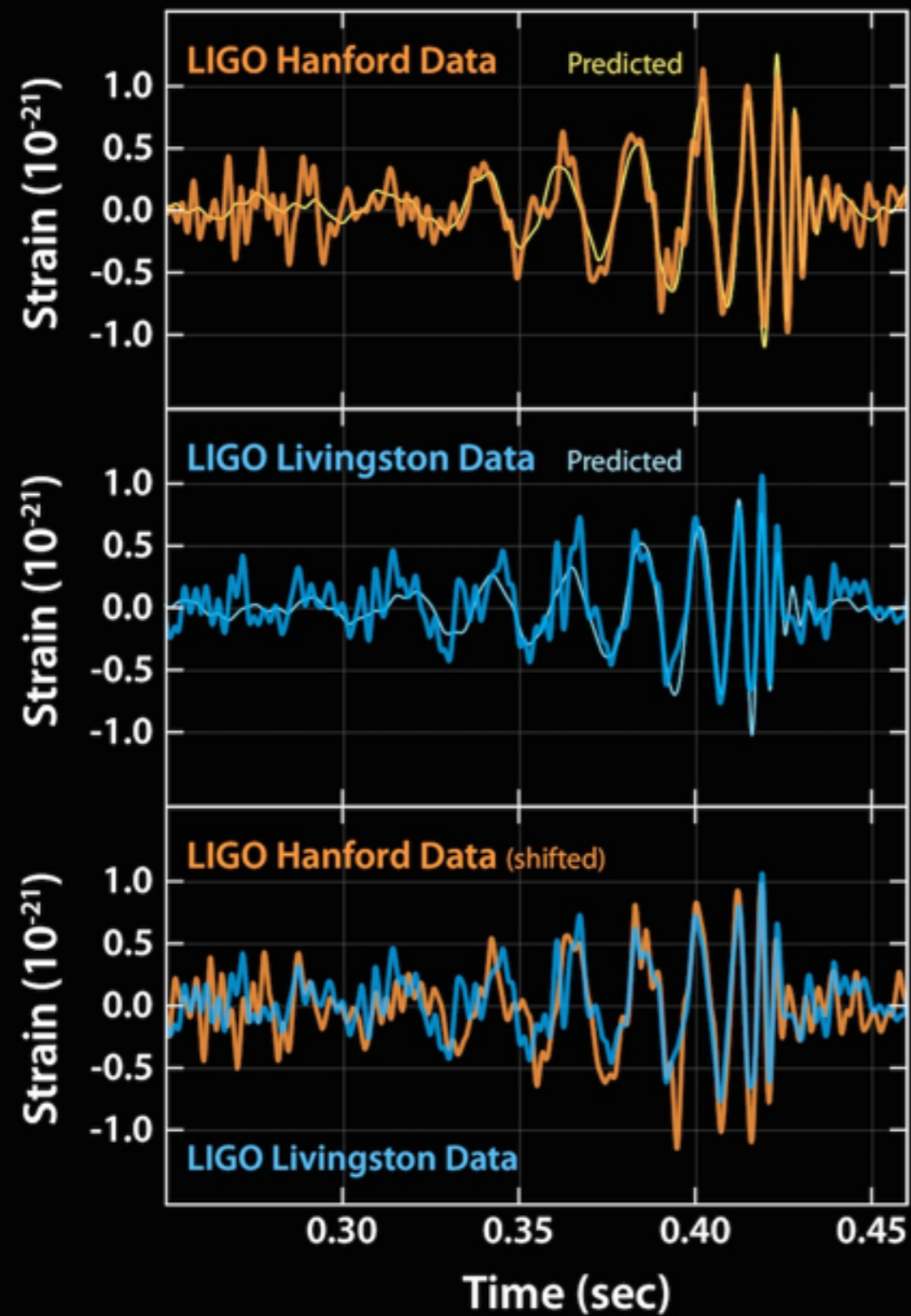


“We had detected gravitational waves. We did it.”

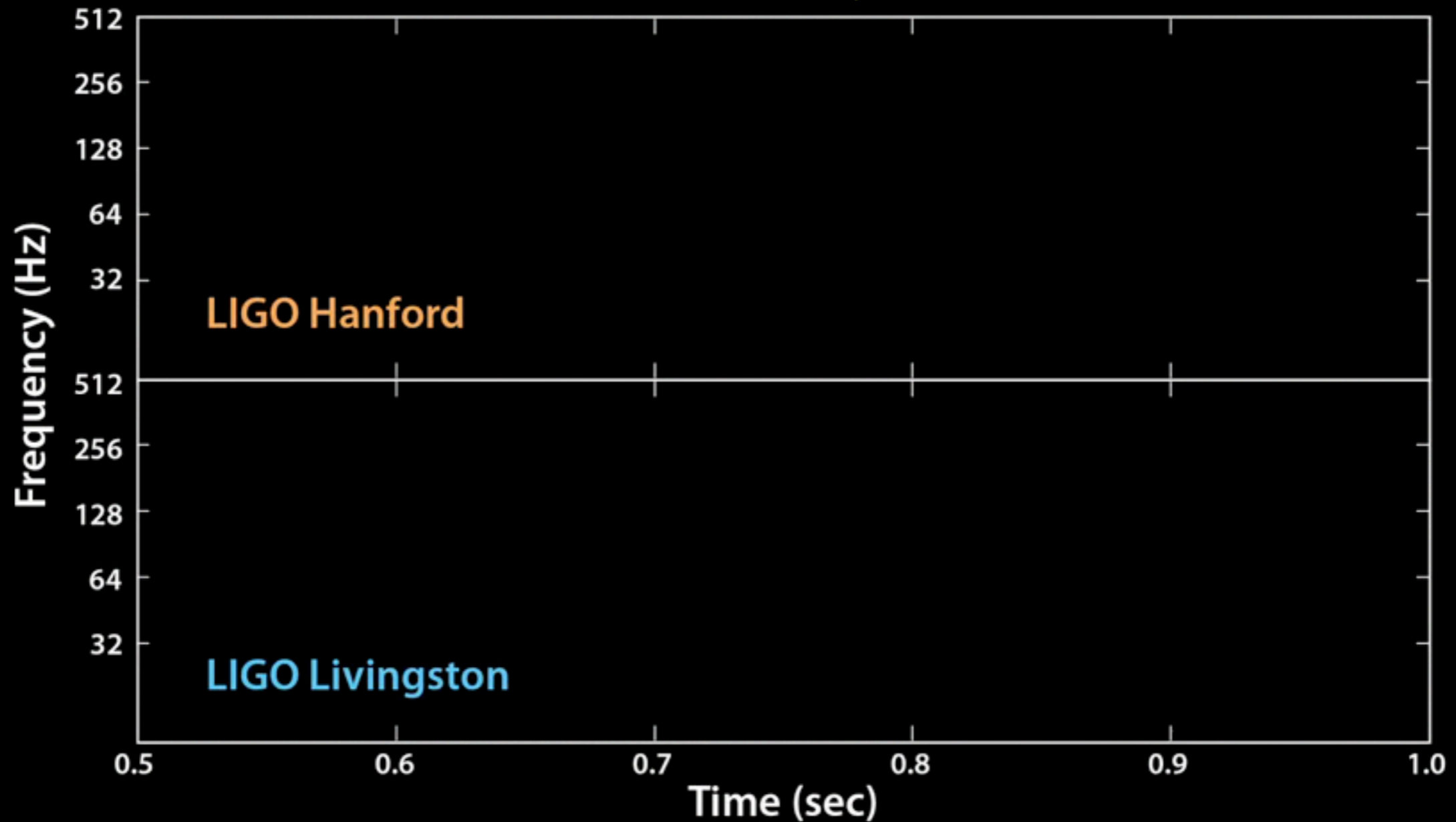
“我々は、重力波を検出した。やり遂げたのだ。”

<https://www.youtube.com/watch?v=aEPlwEJmZyE>

Sep 14, 2015



Hear the sound of BH-BH collision, 12 billions yrs ago



始め2回は実周波数, 後の2回は聞きやすいように+400Hz

<https://mediaassets.caltech.edu/gwave>

きけ距て すのそす変る伸、波 則らフG形を

ブラックホール 解明に期待

る精度を目指す。実現すれば七億光年の範囲にある連星からの重力波を捉えられる。一年で十回ほどキヤッチできる計算だという。



重力波の大きな特徴はブラックホールからも放出されること。連星が合体してブラックホールが生まれる瞬間を観測できると期待される。時間と空間をねじ曲げてすべてをのみ込むブラックホールは、光も電波も出さないため直接には観測

神田教授は、岐阜県飛騨市の大型低温重力波望遠鏡「KAGRA」のプロジェクトでもデータ管理グループのリーダーを務める。説明会では観測されたデータの見方などを解説し「我々にとっても勇気づけられるものだった」と語った。発表を受けて、研究室の学生

が締め切り間際の論文を慌てて書き換えた工ピソードを披露すると、会場は笑いに包まれた。

同大大学院理学研究科2年の和知慎吾さん(23)は「重力波だけでなく、ブラックホールも直接観測したことになる」と分かったため「速でささ波のように宇

重力波 初の直接観測

米国を中心とした国際研究チームが「重力波」を初めて直接観測したとの発表を受け、重力波の研究が専門の神田展行・大阪市立大学大学院教授(51)は12日、発表内容についての説明会を大阪市住吉区の同大杉本キャンパスで開いた。成果を詳しく理解してもらおう狙い。学生ら約1000人が参加し、真剣な表情で聴き入った。

【畠山哲郎】

「研究者勇気づけた」

大阪市大院・神田教授 学生らに解説



重力波観測について解説する大阪市立大学大学院理学研究科の神田展行教授—大阪市住吉区で、川平愛撮影

宙空間に伝わる現象。物理学者のアインシュタインが「一般相対性理論」で存在を予言し、世界中の研究者が観測に挑戦していた。

大阪工大「予想通りで驚いた」

大阪工業大情報科学部の真貝寿明教授(一般相対性理論)は「祝・重力波の直接検出」と題して、研究室のウェブページに

一般向けの緊急解説記事を掲載した。昨年には一般向けの解説書「ブラックホール・膨張宇宙・重力波 一般相対性理論の

100年と展開」を出版している。「こんなにも予想通りのものが見つかるのかと驚いた。素晴らしい発見だ」と感想を語った。今後の研究については「日本でもKAGRAを

使い、改めて重力波を確認したり、海外のチームと協力して重力波がどこから来たものなのかを調べたりしていくことが重要だ」と話した。

【畠山哲郎】

Tokyo Shimbun 2016/2/12

Mainichi Shimbun 2016/2/13

GW exists !
GW was detected !
BHs exist !
BH binaries exist !
GR is right !

Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Gravitational Lensing

Kepler motion

Gravitational Wave

**After 100 years, we are finally in the era of
Astronomy with General Relativity.**