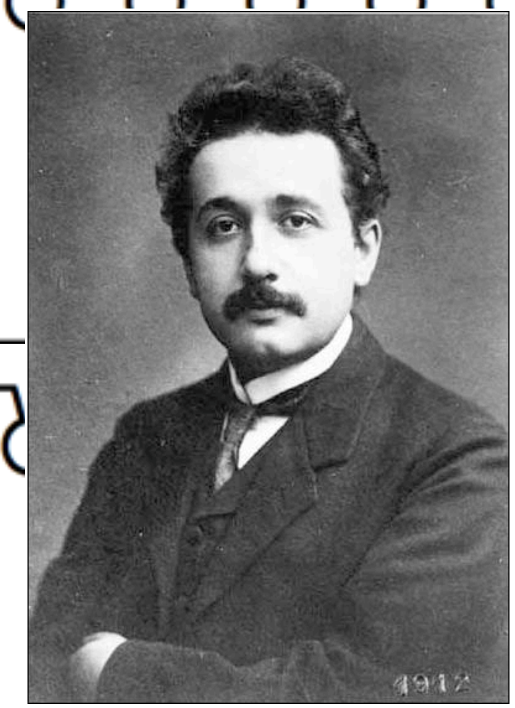
Why & How we know there are black-holes?
Introduction to Einstein's Relativity

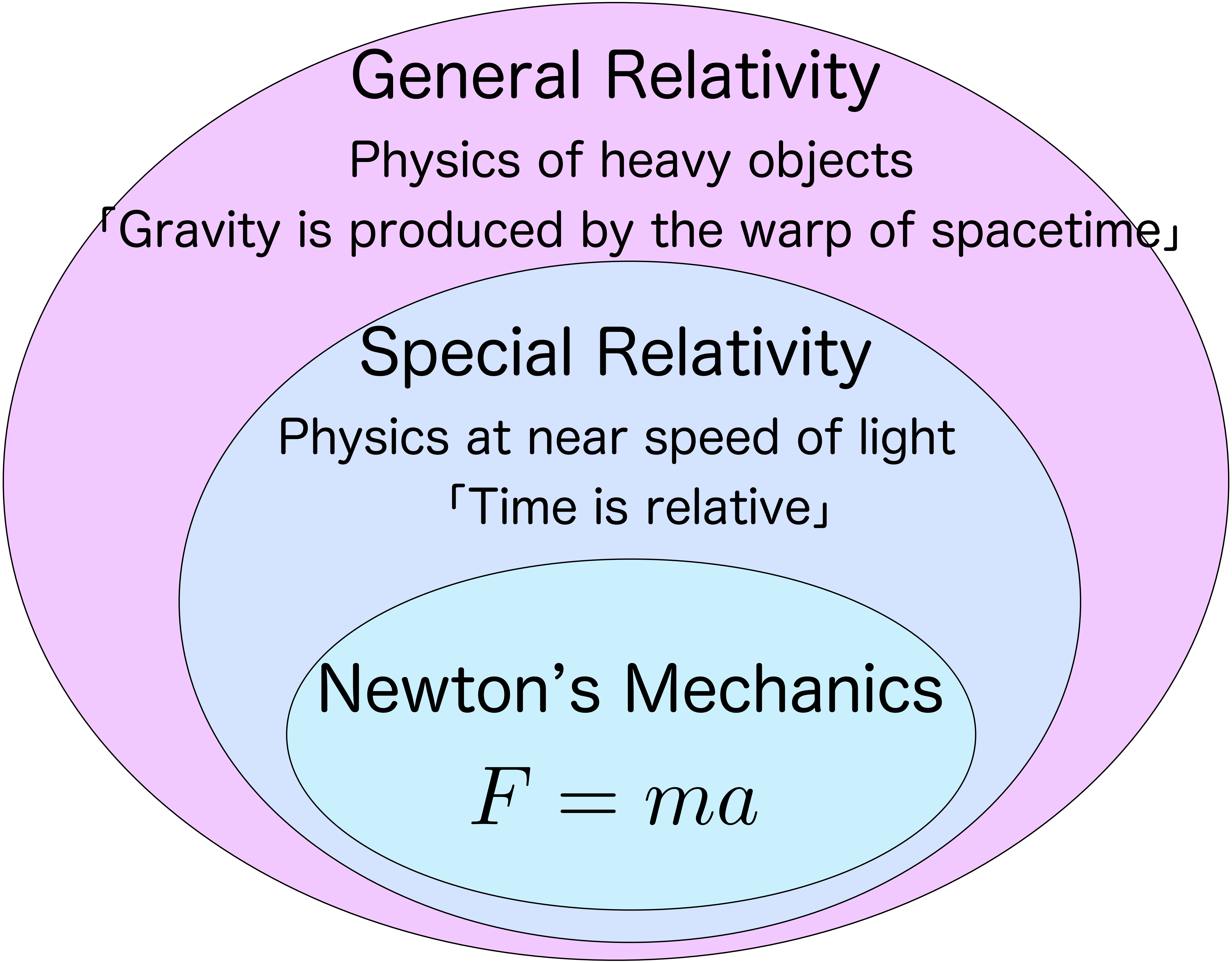
Hisaaki Shinkai



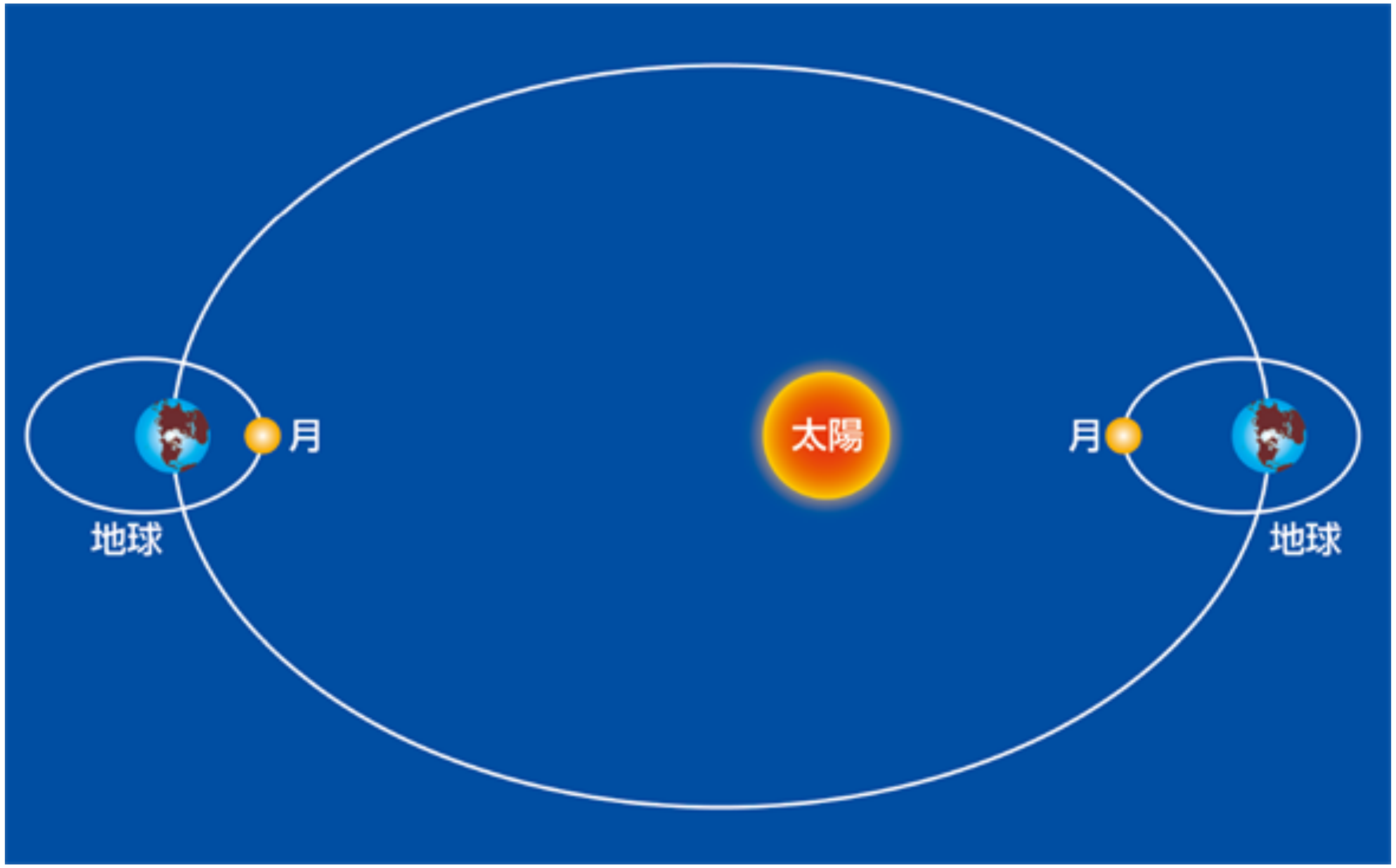
1 Special Relativity: Theory of time

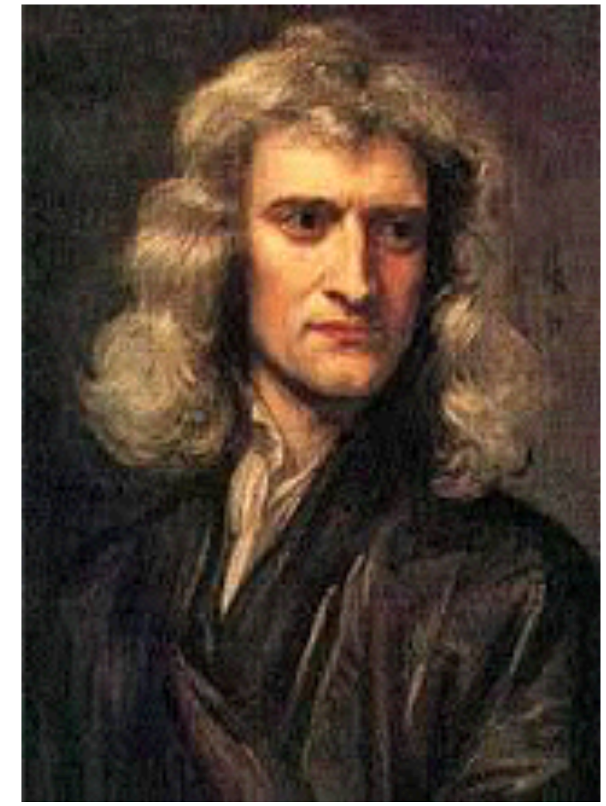
2 General Relativity: Theory of gravity

真貝　寿明
Hisaaki Shinkai
http://www.oit.ac.jp/is/~shinkai/


General Relativity
Physics of heavy objects
「Gravity is produced by the warp of spacetime」
Special Relativity
Physics at near speed of light
「Time is relative」
Newton's Mechanics
$F = ma$

Gravity is universal





Newton

Universal Force

=Every objects have attractive force

重力の正体は？

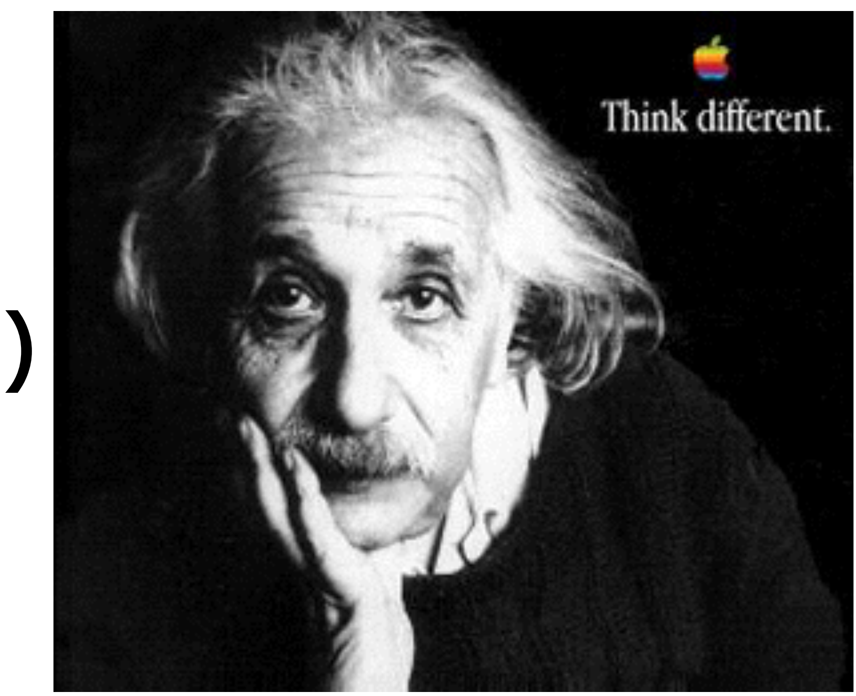
http://hikingartist.com/

「万有引力があるからだ」（ニュートン，1687）

$$F = G\frac{Mm}{r^2} \qquad m\frac{d^2x}{dt^2} = F$$

「時空のゆがみだ」

（アインシュタイン，1915）



一般相対性理論

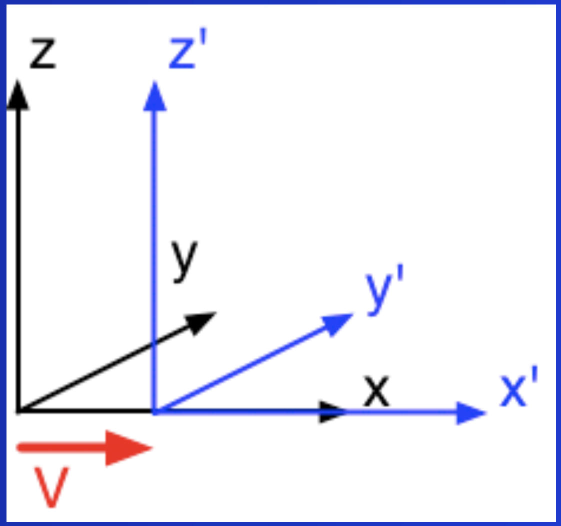
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$\frac{d^2\xi^\mu}{d\tau^2} = R^\mu{}_{\nu\rho\sigma}\frac{d\xi^\nu}{d\tau}\frac{d\xi^\rho}{d\tau}\xi^\sigma$$

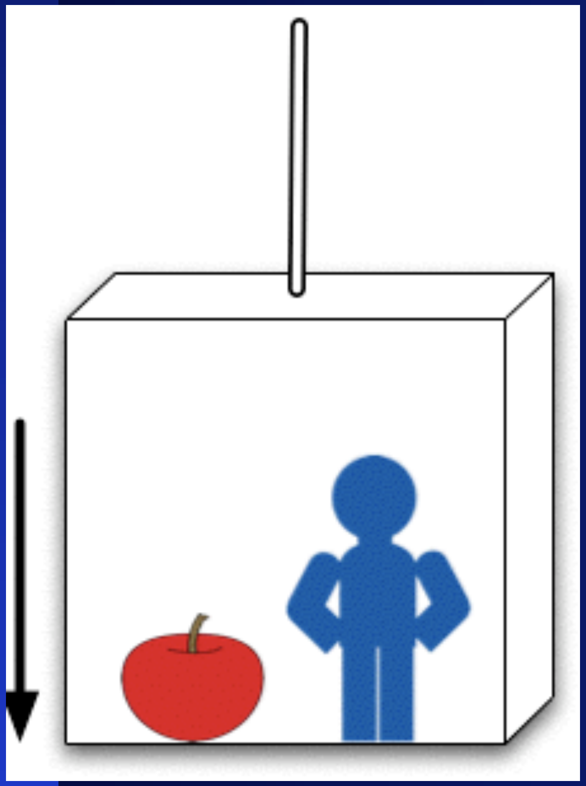
What is “acceleration”?

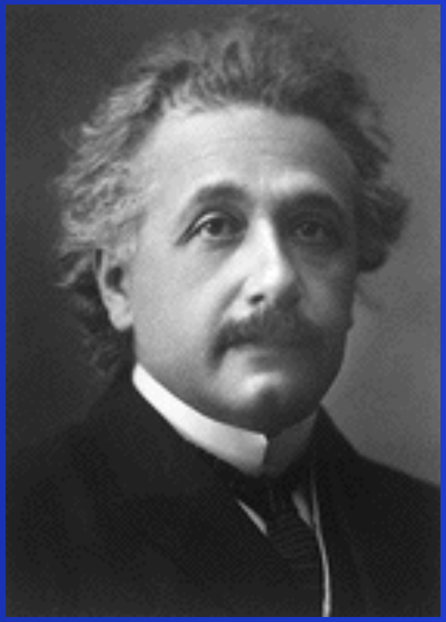
★ In Special Relativity, only constant velocity observer can be discussed.

★ What is the acceleration ?



Thought Experiment of Elevator





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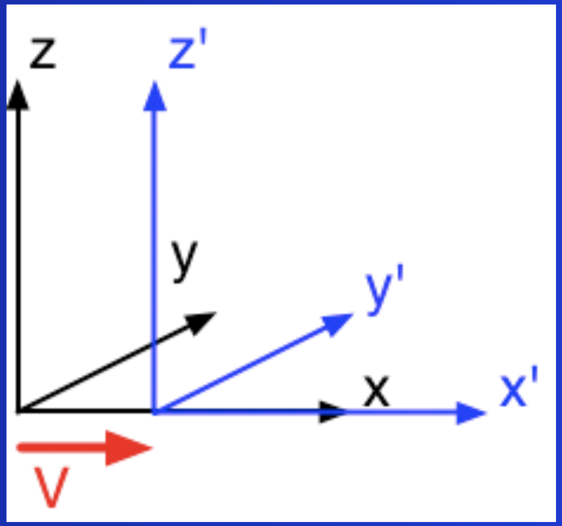
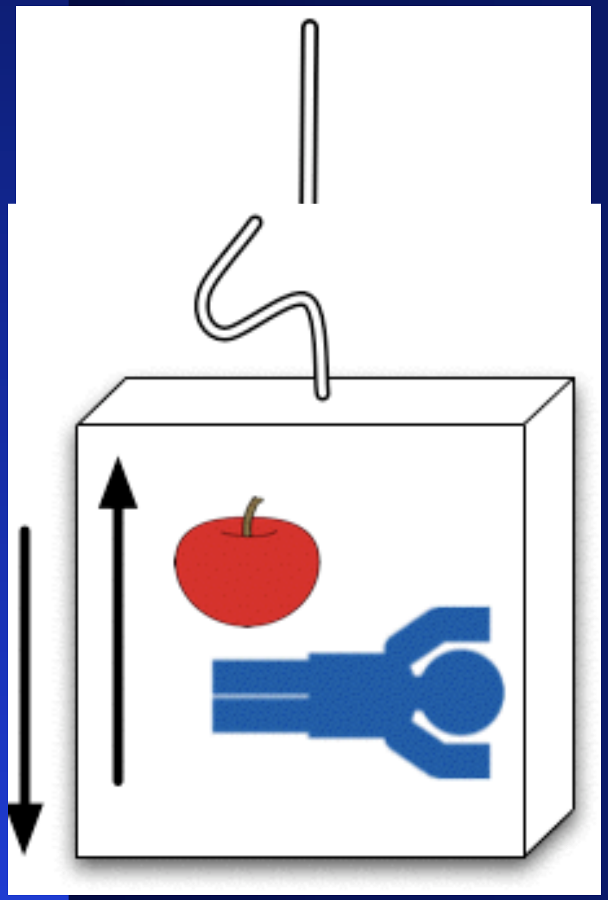


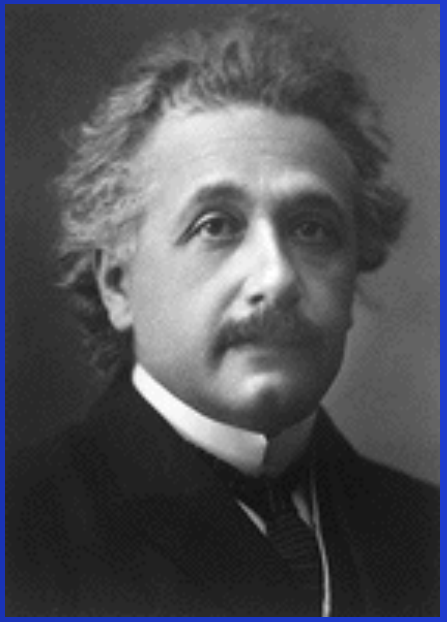
Thought Experiment of Elevator



「Gravitational acceleration can be compensated. 」

＝acceleration can be removed locally. (Equivalence Principle)

「Greatest idea in my life」



Zero Gravity by Free-Falling Airplane

Zero Gravity　　**無重力体験　～アメリカ～**

ボーイング727による**宇宙体験・無重力体験飛行**がアメリカ・ラスベガス、ケネディースペースセンターなどで楽しめます。

天井や壁を歩いてみたり、スーパーマンのように宙を飛んだり、水球になった水を飲んだり（？）．．などなど。楽しみ方は、自由！無重力状態でどんな事を試してみますか？

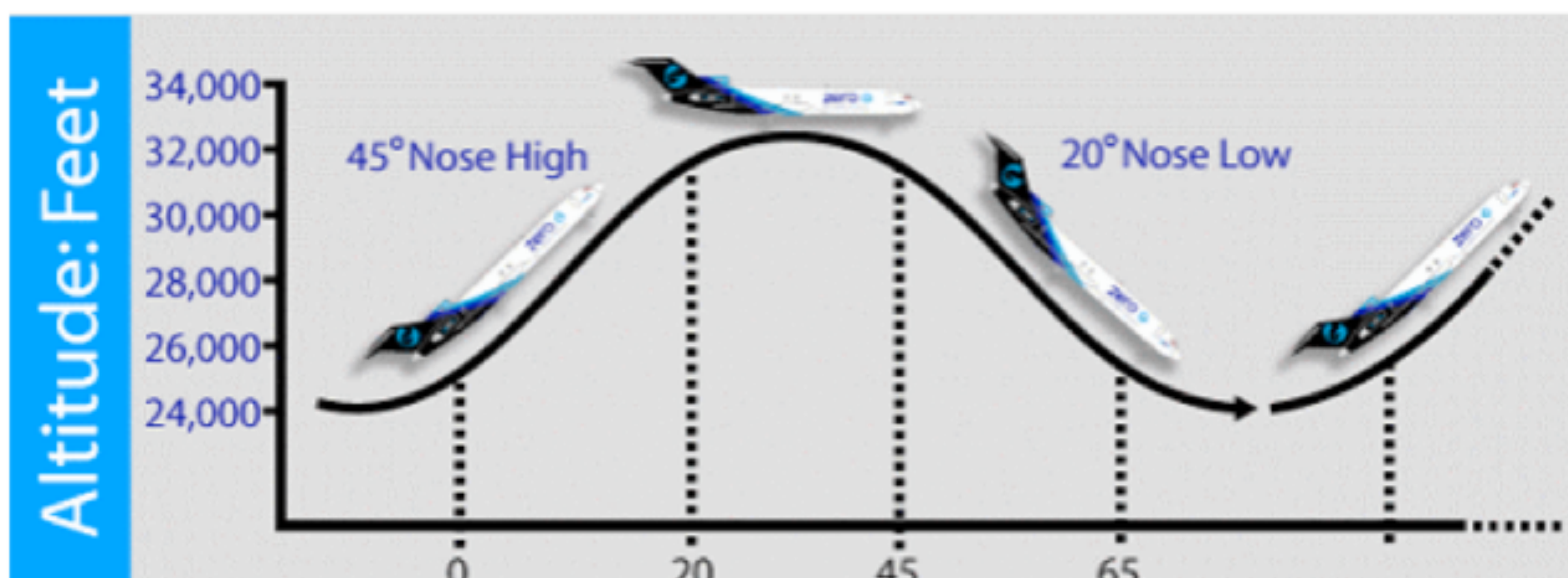
このプログラムでは、無重力体験のみでなく火星や月の重力も体験できま

無重力体験・概要

高度24,000ft（約7,315m）から45度の角度で34,000ft（約10360m）まで地上の1.8倍（1.8G）の重力を感じながら、一気に上昇し、その後下降します（パラボラテック飛行）。この時に無重力を25-30秒程度お楽しみいただけます。

30秒という時間は短いようですが、通常スカイダイビングでのフリーホールの時間（バンジージャンプの5回分より長い）時間とほぼ同じです。無重力の間は、スーパーマンになったように空を飛ぶなど思い思いにお楽しみ下さい。

また、このプログラムでは、10回の無重力が予定されていますが、それ以外に3回の月（地球の1/6）や2回の火星（地球の1/3）の重力も同じフライトでお楽しみいただく予定です。

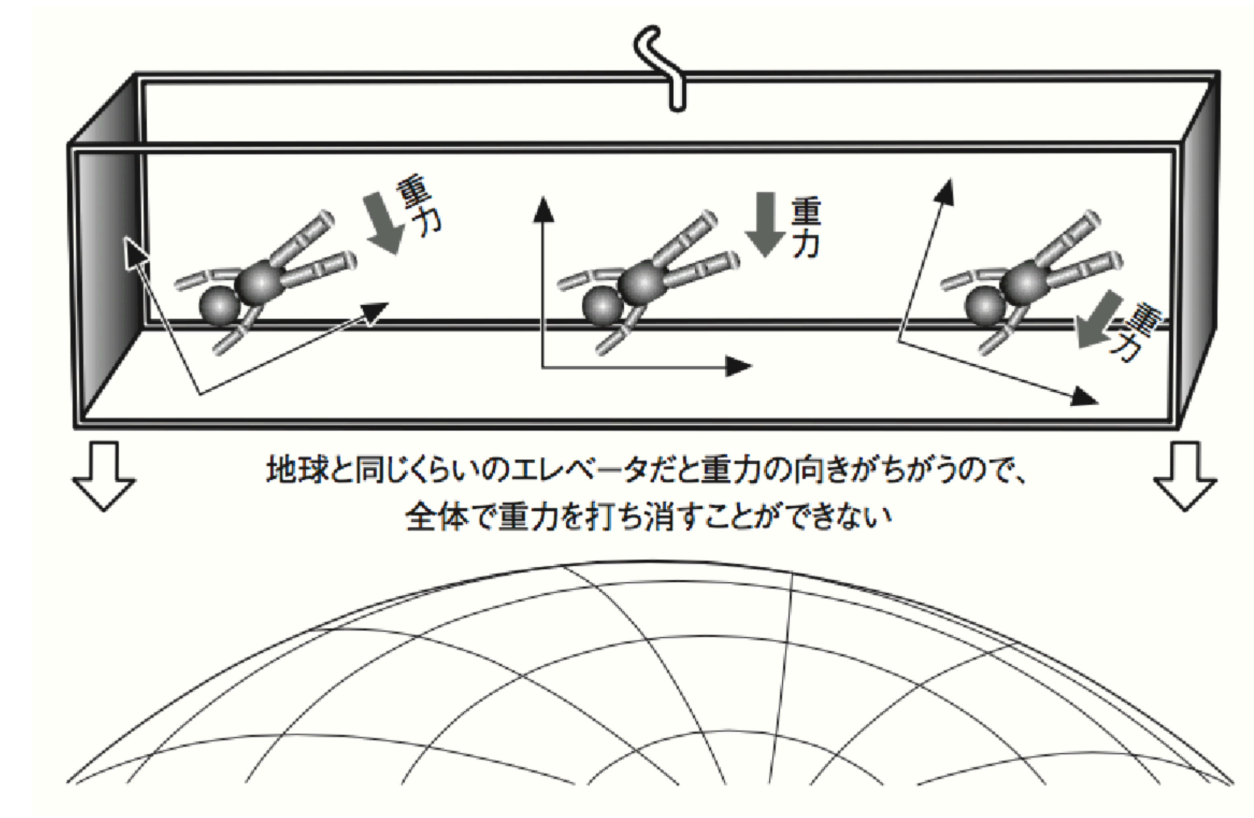


http://www.ctn-japan.com/tracon/space/zerog_us.html

What is “gravity”?

「Gravitational acceleration can be compensated.」

＝acceleration can be removed locally.（Equivalence Principle)



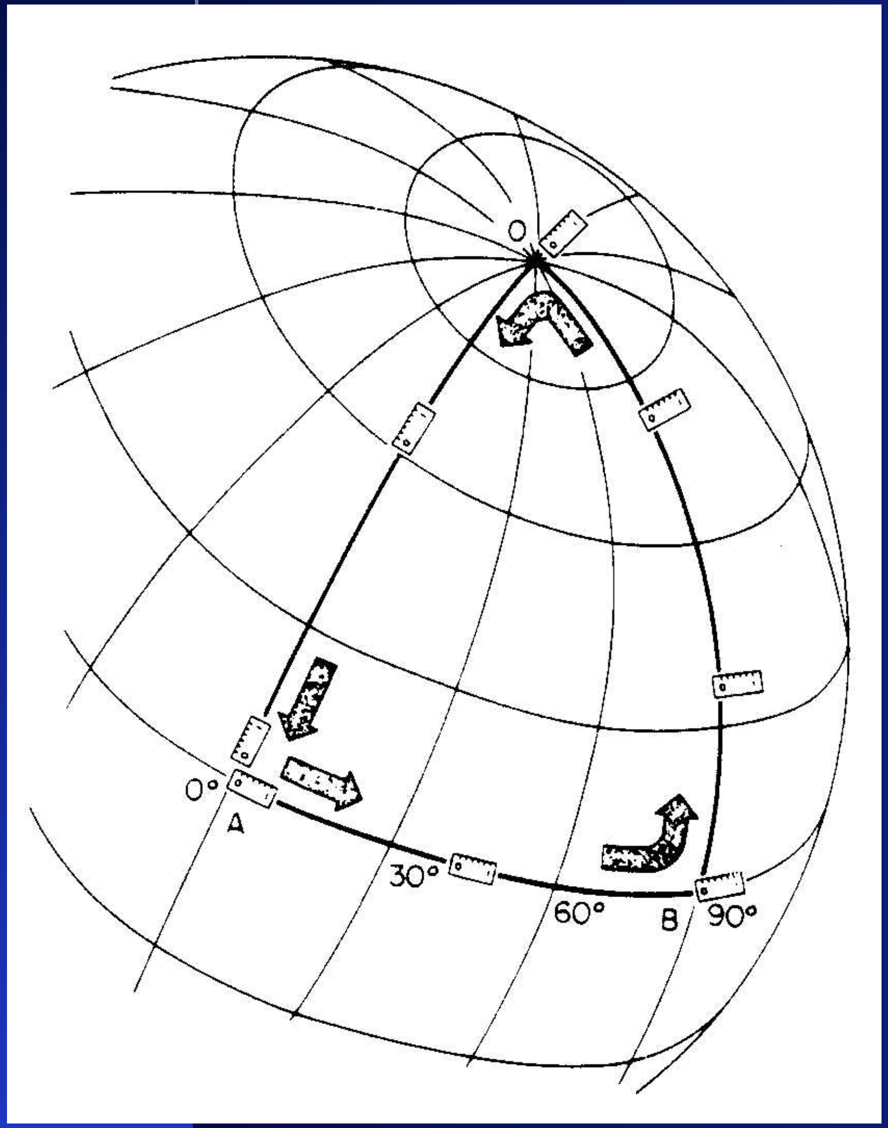
＝gravity cannot be removed globally.

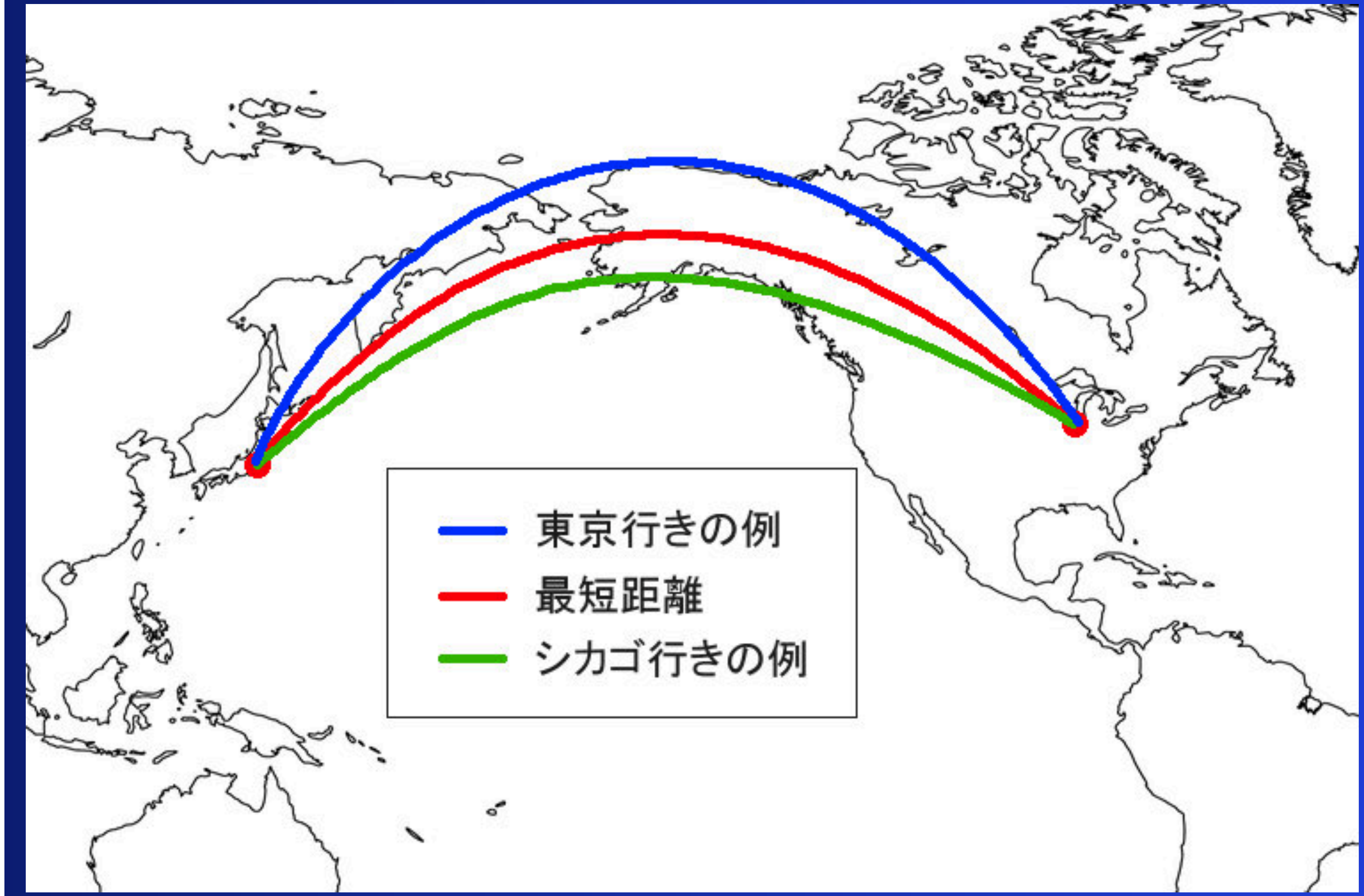
＝gravity is the result of space-time bending.

Geometry of Curved Space ＝「General Relativity」

General Relativity【metric】

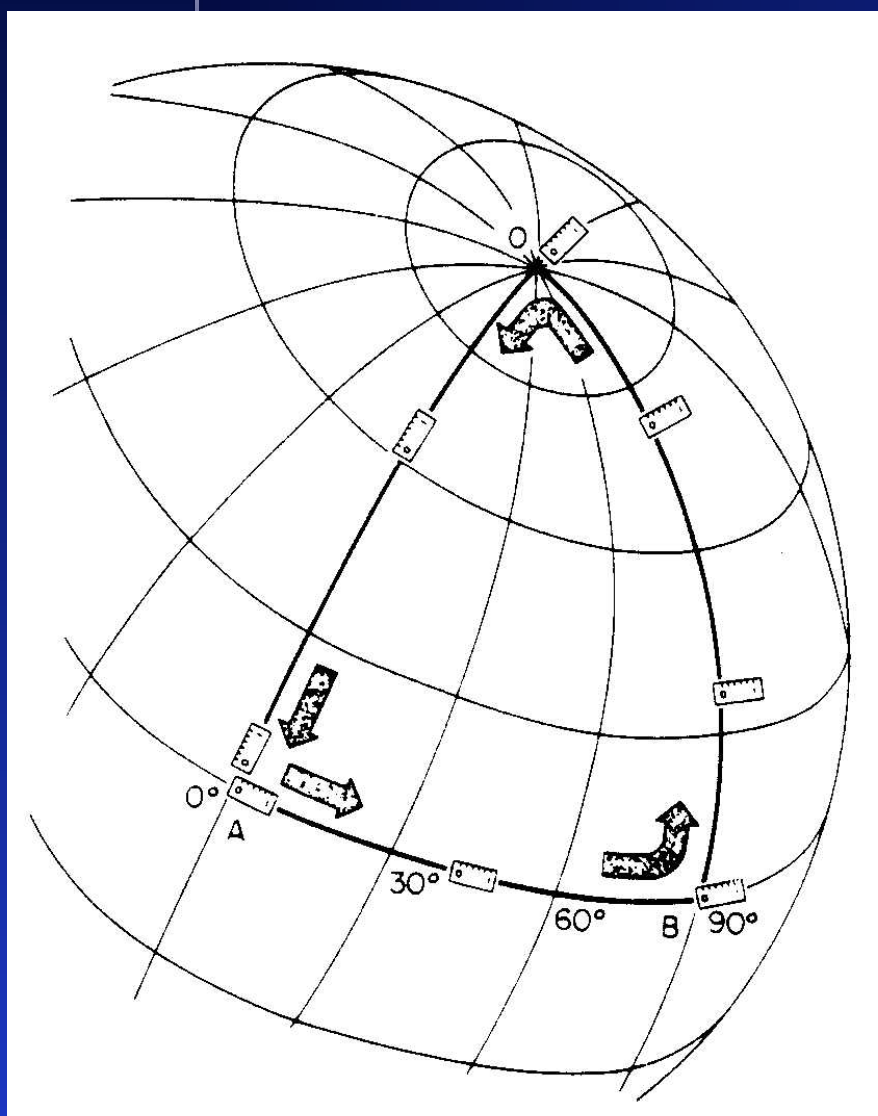
Geometry of Curved Space＝Riemannian Geometry





General Relativity【metric】

Geometry of Curved Space＝Riemannian Geometry



Generalized distance「metric」.

$$ds^2 = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu}(x)dx^\mu dx^\nu = g_{\mu\nu}dx^\mu dx^\nu$$

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ & g_{xx} & g_{xy} & g_{xz} \\ & & g_{yy} & g_{zz} \\ sym. & & & g_{zz} \end{pmatrix}$$

In flat spacetime,

$$\begin{aligned} ds^2 &= -c^2dt^2 + dx^2 + dy^2 + dz^2 \\ &= -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \end{aligned}$$

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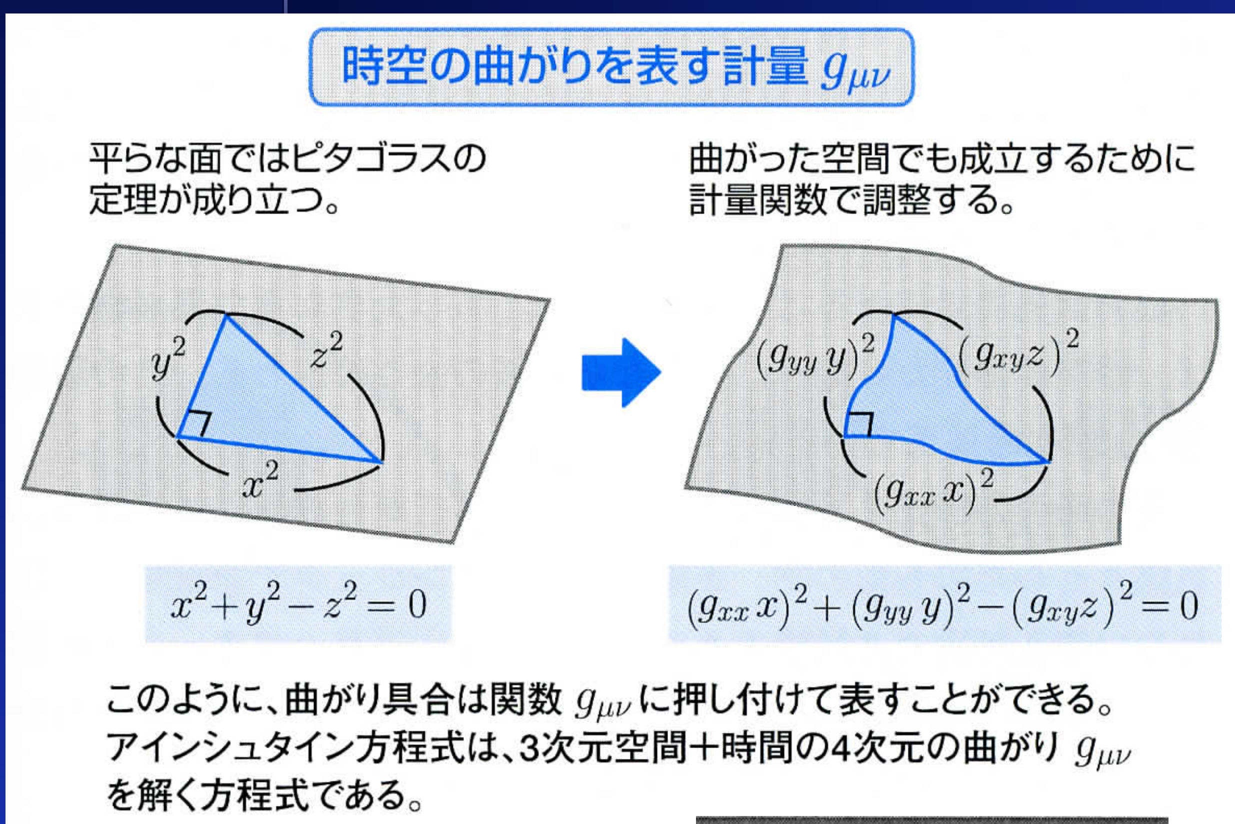

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Marcel Grossmann

General Relativity 【The Einstein equation】

Field equation (1915)

how spacetime is curved ⇔ how matters are distributed

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

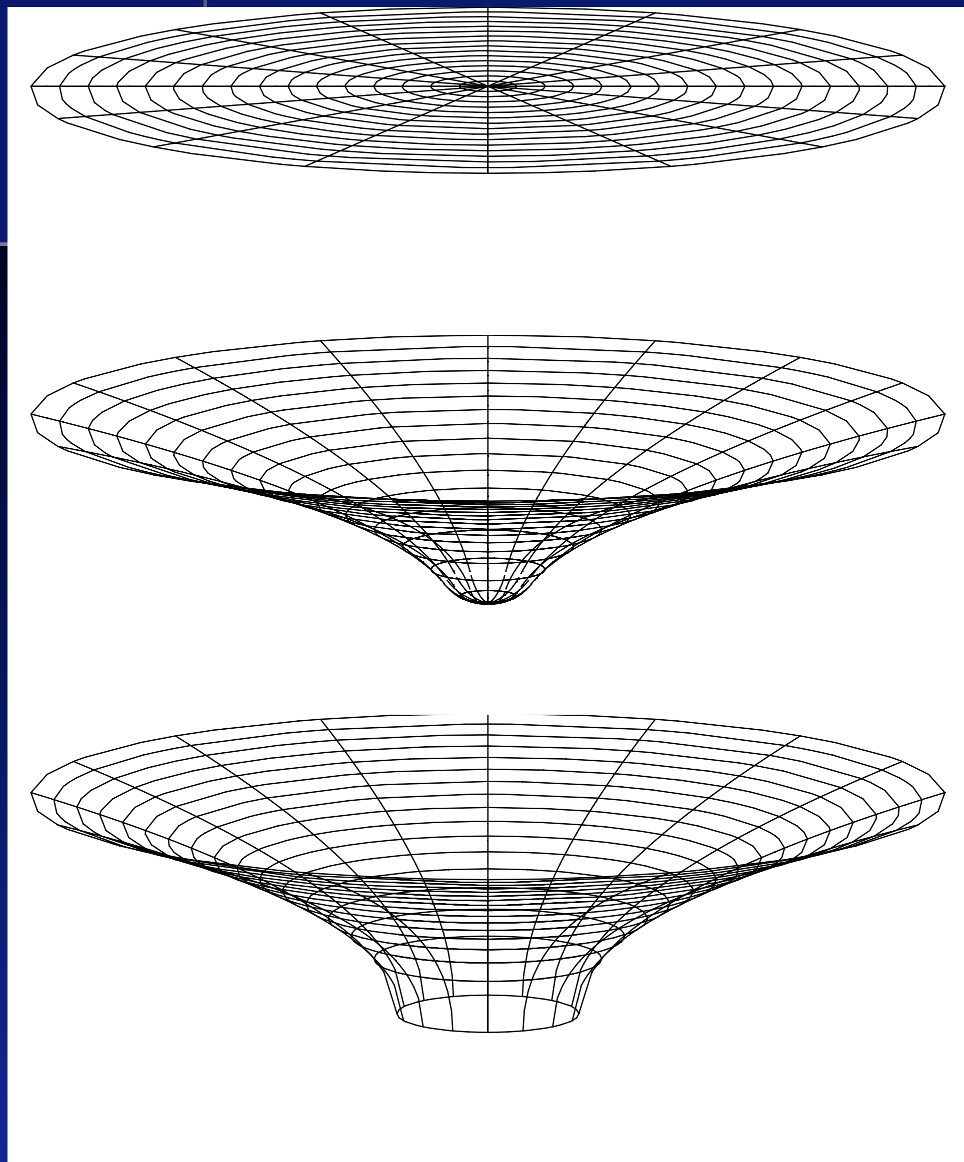
Riemann curvature

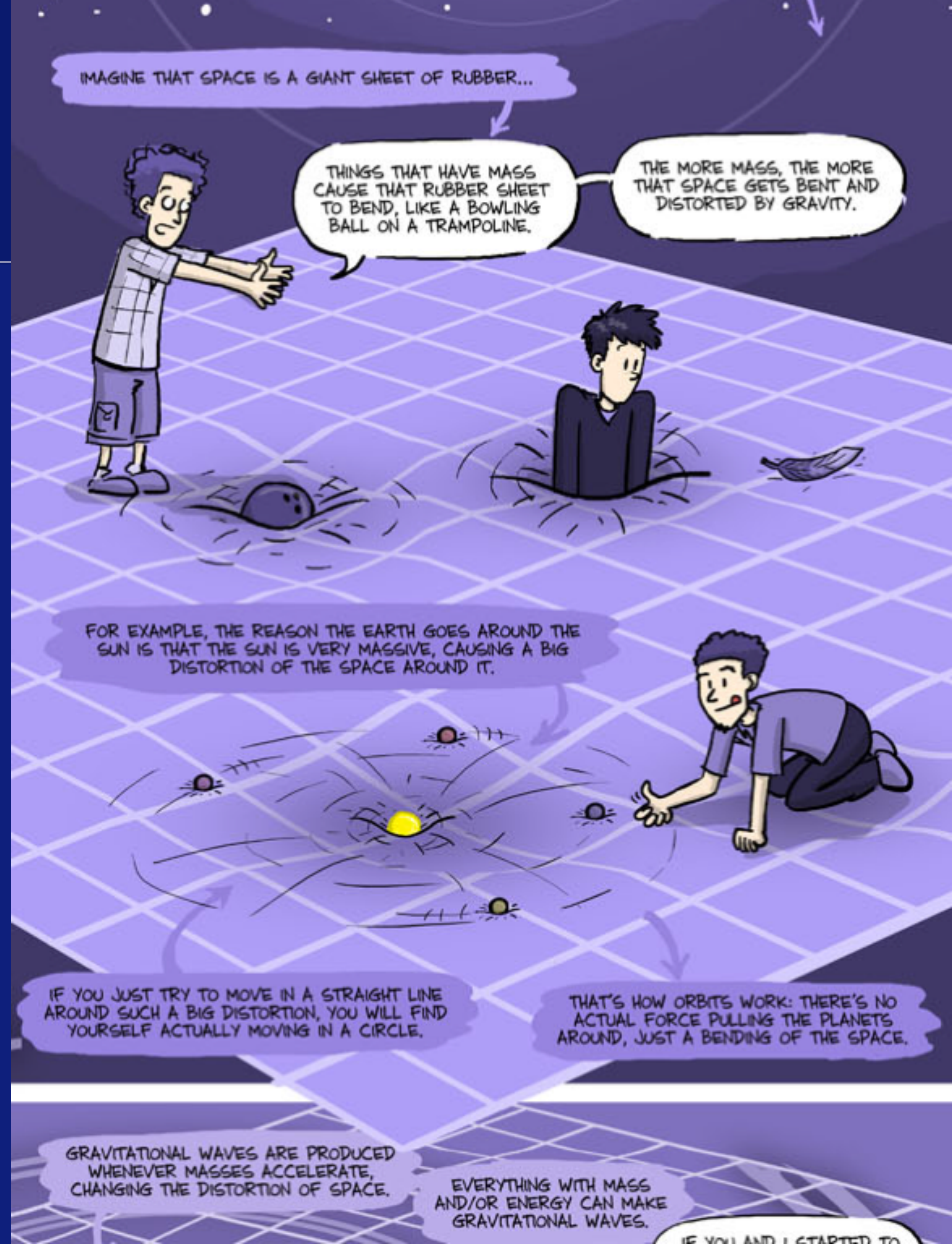
Ricci tensor, Einstein tensor

$$\begin{aligned}
\Gamma^{\alpha}_{\mu\nu} &\equiv \frac{1}{2} g^{\alpha\beta}(\partial_\nu g_{\beta\mu} + \partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu}) \\
R^{\mu}{}_{\nu\alpha\beta} &\equiv \partial_\alpha \Gamma^{\mu}_{\nu\beta} - \partial_\beta \Gamma^{\mu}_{\nu\alpha} + \Gamma^{\mu}{}_{\sigma\alpha}\Gamma^{\sigma}{}_{\nu\beta} - \Gamma^{\mu}{}_{\sigma\beta}\Gamma^{\sigma}{}_{\nu\alpha} \\
R_{ab} \equiv R^{\mu}{}_{a\mu b} &\equiv \partial_\mu \Gamma^{\mu}_{ab} - \partial_b \Gamma^{\mu}_{a\mu} + \Gamma^{\mu}{}_{\nu\mu}\Gamma^{\nu}{}_{ab} - \Gamma^{\mu}{}_{\nu b}\Gamma^{\nu}{}_{a\mu} \\
R &= g^{ab} R_{ab}
\end{aligned}$$

Energy-Momentum tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$




IMAGINE THAT SPACE IS A GIANT SHEET OF RUBBER...
THINGS THAT HAVE MASS CAUSE THAT RUBBER SHEET TO BEND, LIKE A BOWLING BALL ON A TRAMPOLINE.
THE MORE MASS, THE MORE THAT SPACE GETS BENT AND DISTORTED BY GRAVITY.
FOR EXAMPLE, THE REASON THE EARTH GOES AROUND THE SUN IS THAT THE SUN IS VERY MASSIVE, CAUSING A BIG DISTORTION OF THE SPACE AROUND IT.
IF YOU JUST TRY TO MOVE IN A STRAIGHT LINE AROUND SUCH A BIG DISTORTION, YOU WILL FIND YOURSELF ACTUALLY MOVING IN A CIRCLE.
THAT'S HOW ORBITS WORK: THERE'S NO ACTUAL FORCE PULLING THE PLANETS AROUND, JUST A BENDING OF THE SPACE.
GRAVITATIONAL WAVES ARE PRODUCED WHENEVER MASSES ACCELERATE, CHANGING THE DISTORTION OF SPACE.
EVERYTHING WITH MASS AND/OR ENERGY CAN MAKE GRAVITATIONAL WAVES.

The field equation (The Einstein equation)

The origin of the gravity is the curvature of space-time. The relation is expressed as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{2.18}$$

- The left-hand side expresses how space-time is curved using Riemannian geometry.
- The right-hand side expresses how the matter distributes in space-time. ($T_{\mu\nu}$ is called the energy-momentum tensor, and $T_{\mu\nu} = 0$ if vacuum.)
- The indice μ, ν indicate the coordinate (t, x, y, z), so that (2.18) consists of 10 equations.

This equation tells us that if the matter exists then the surrounding space-time curves. If space-time curves, then the matter moves along the curve (geodesics).

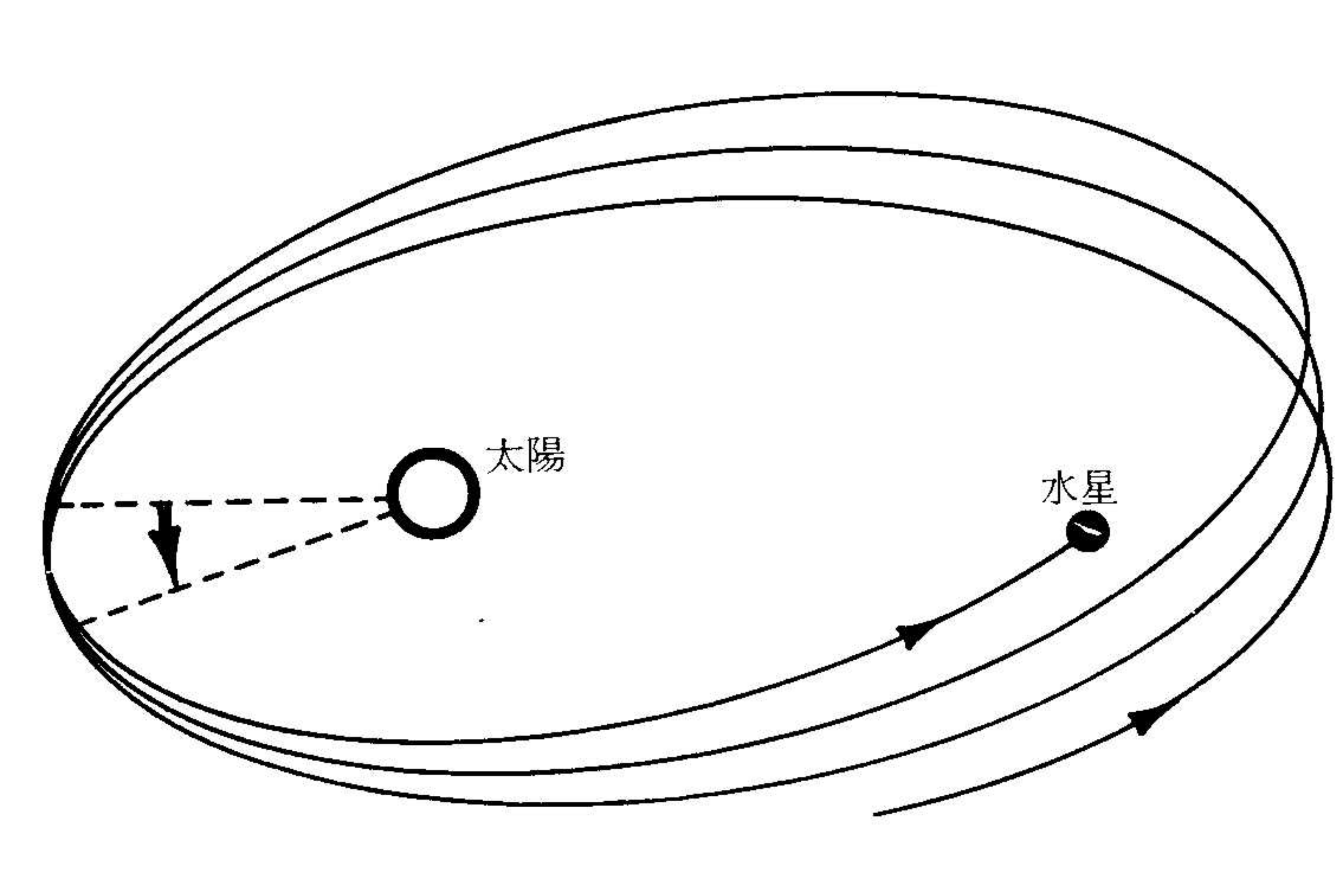
General Relativity (1916)

- **Motivation :**
  Relativity (1905) only treats observers with constant speed.
  What is the acceleration?
- **Breakthrough :**
  No gravity effects in free-falling elevator !
  But we cannot erase the gravity in an Earth-sized elevator (globally).
- **Einstein's conclusion :**
  Gravity is space-time curvature.

Prediction of General Relativity
— Orbit of Mercury : rotation of perihelion

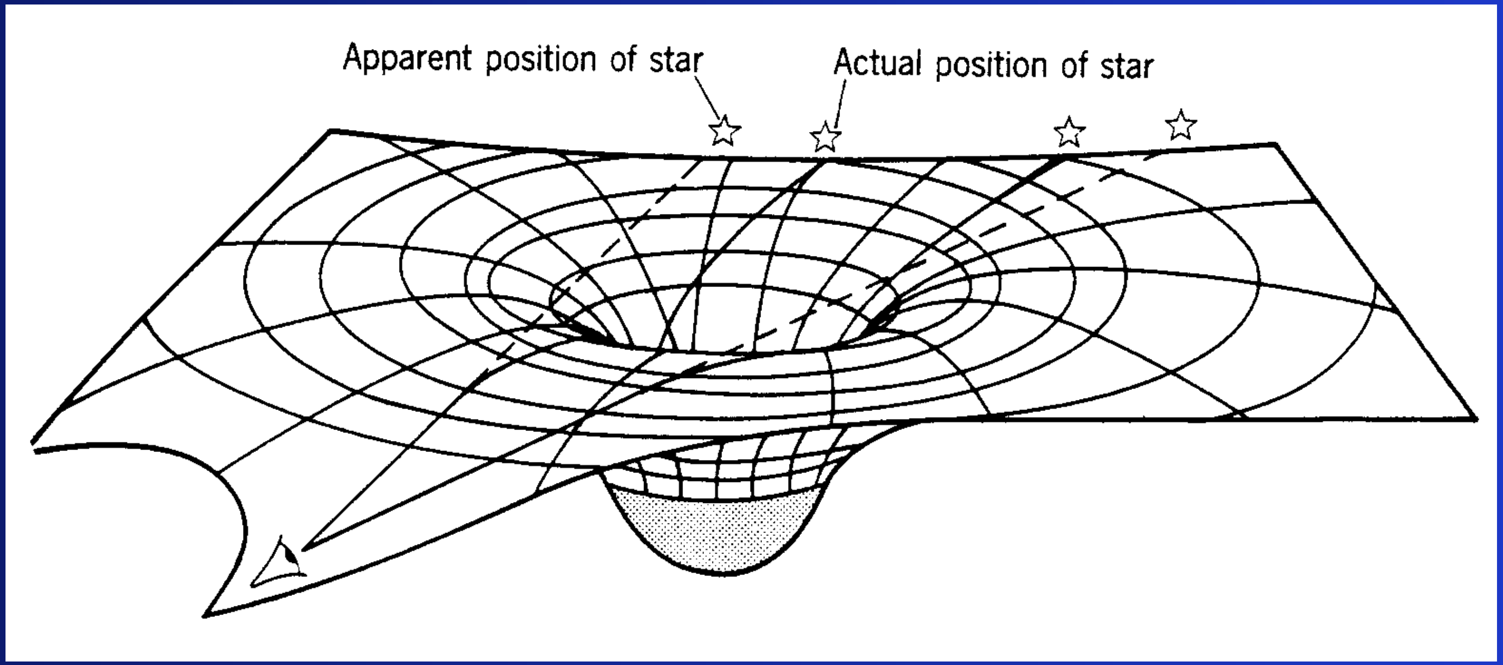
近日点移動

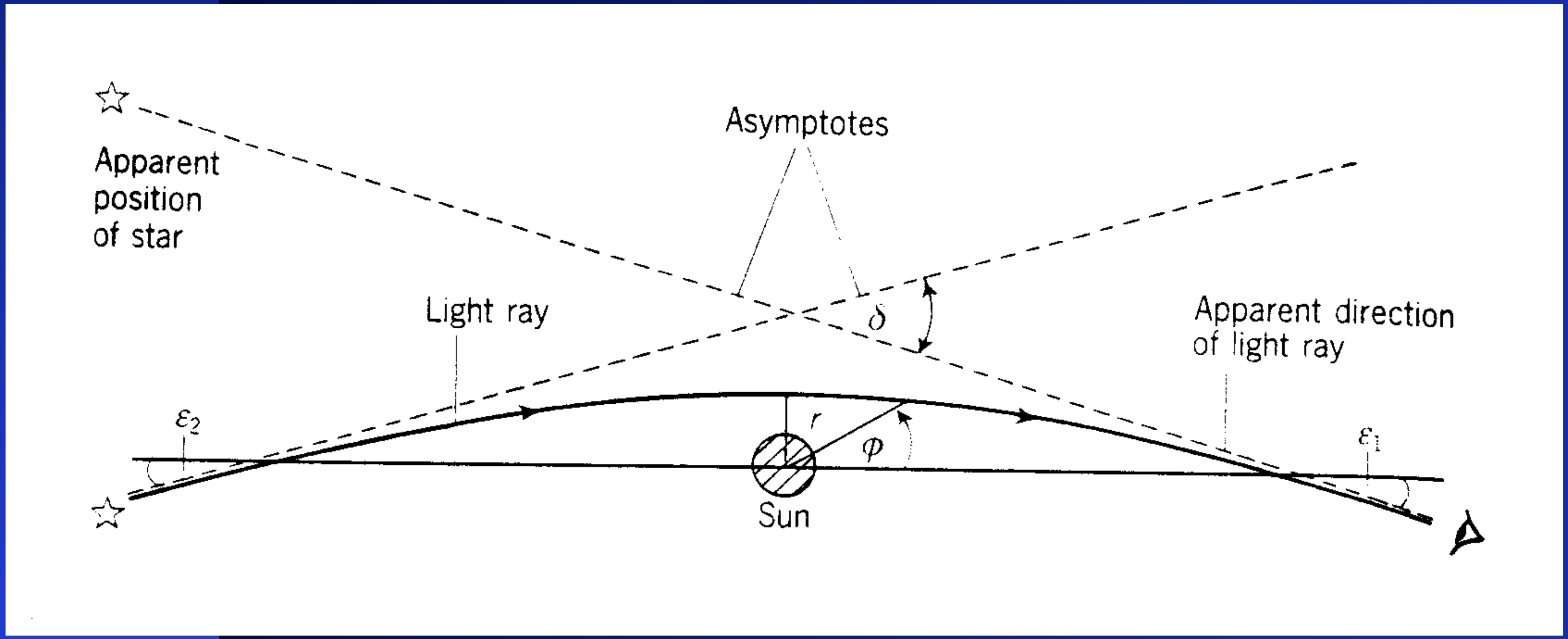
Unexplained orbital change of Mercury (Le Verrier 1854)



Einstein’s theory explained this effect. (1915)

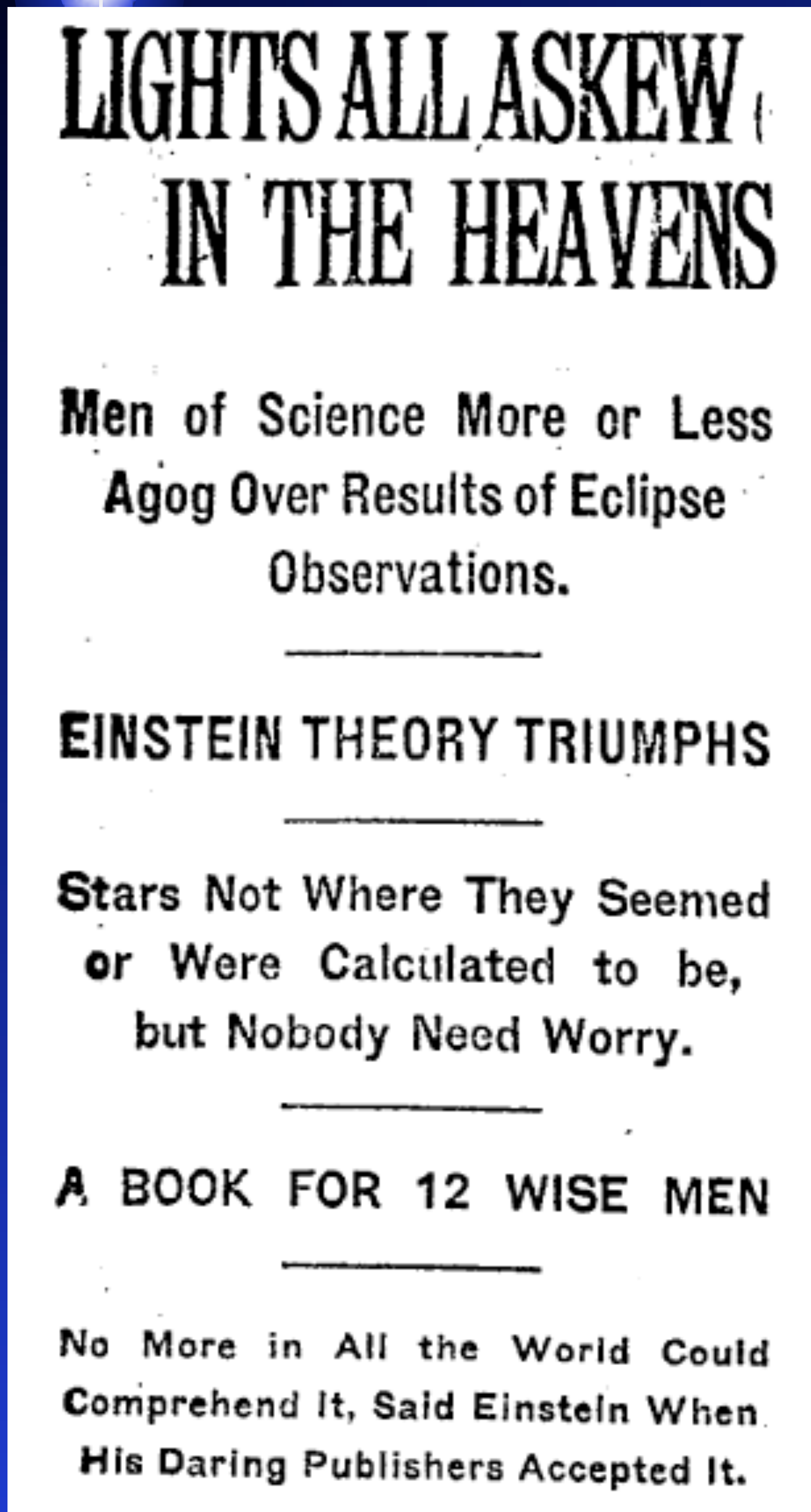
Prediction of General Relativity
—Bending of Light: gravitational lens—





Taken from the 22 November 1919 edition of the Illustrated London News.

Coverage in the (more excitable) New York Times.



LIGHTS ALL ASKEW IN THE HEAVENS

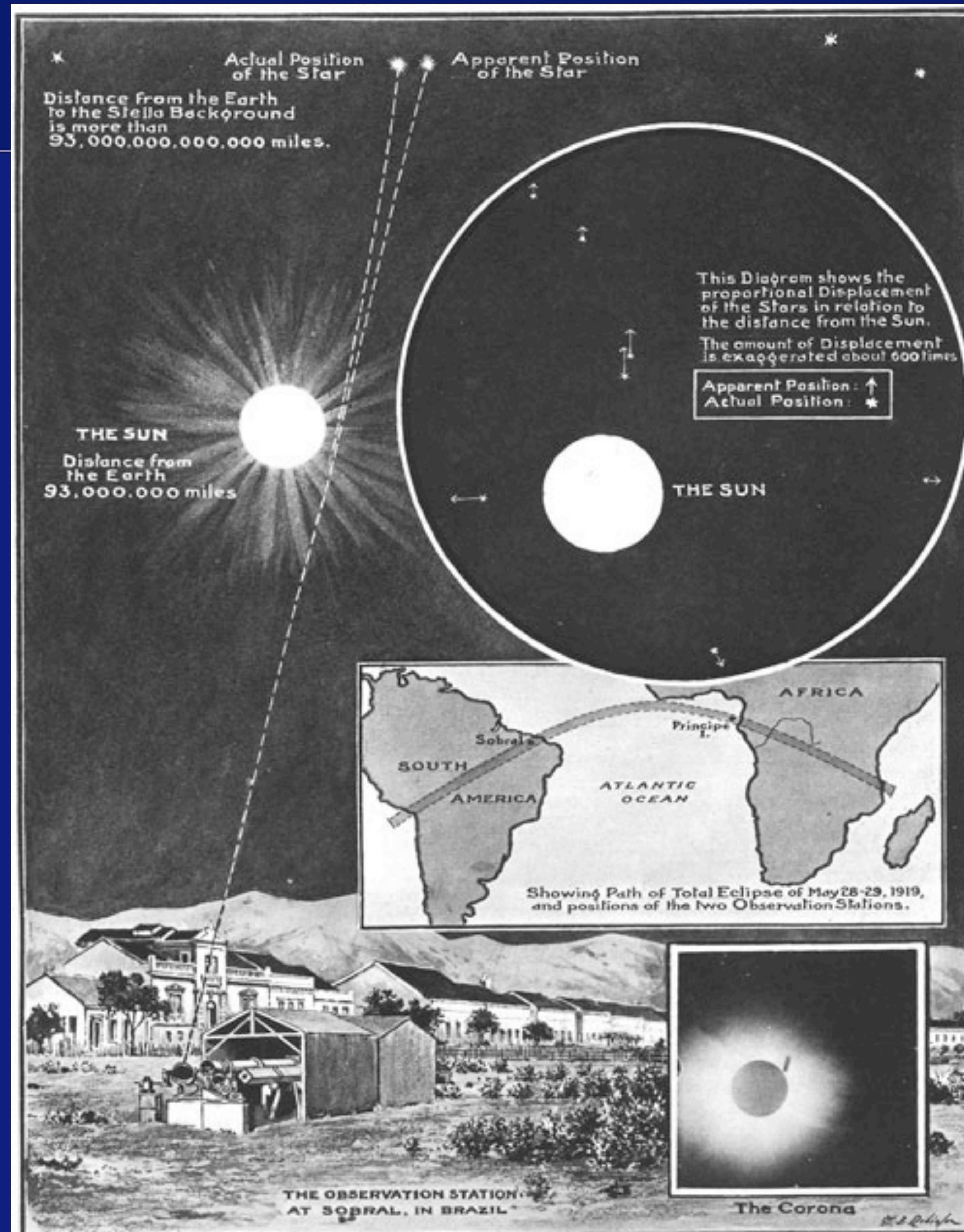
Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

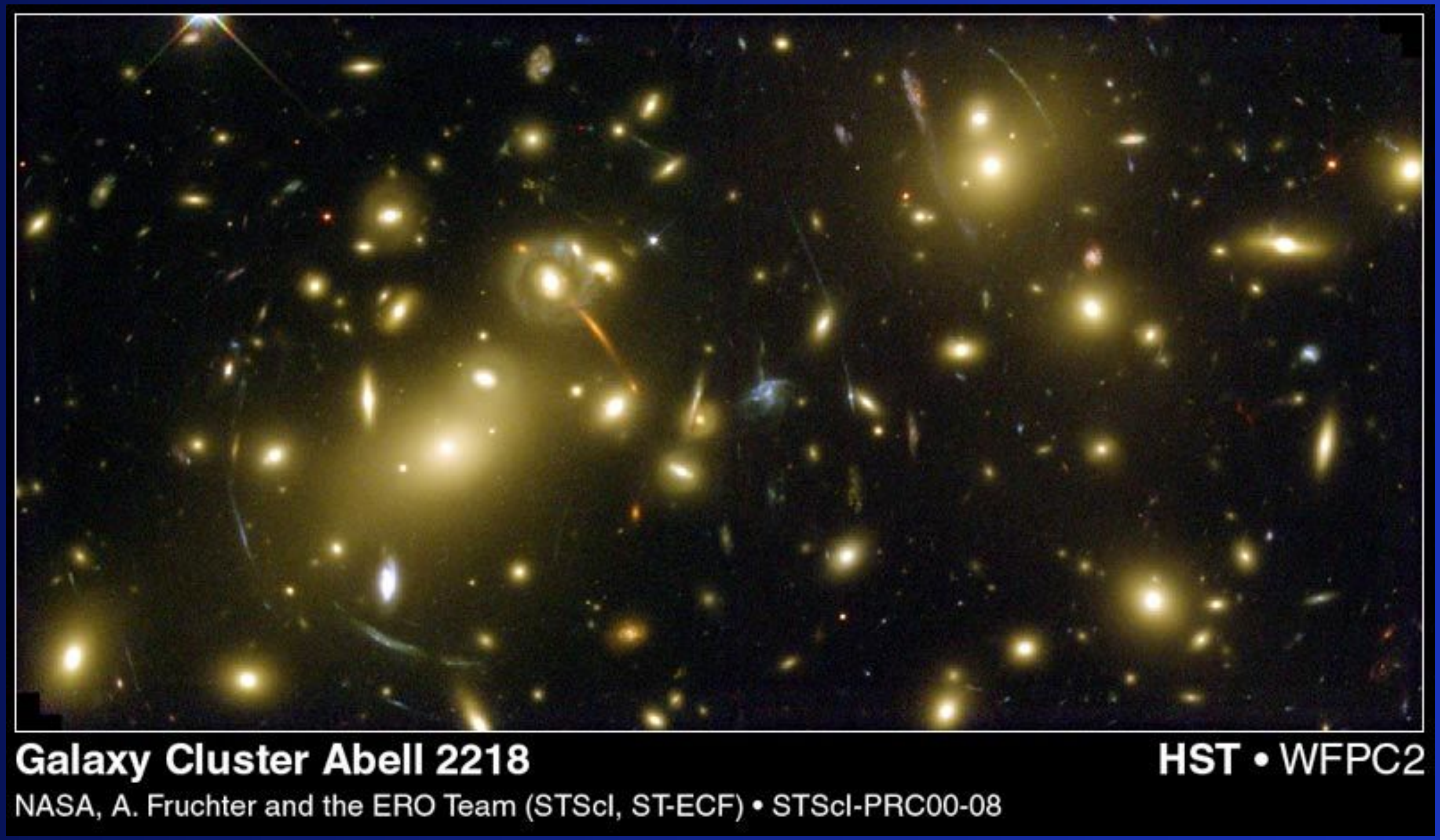


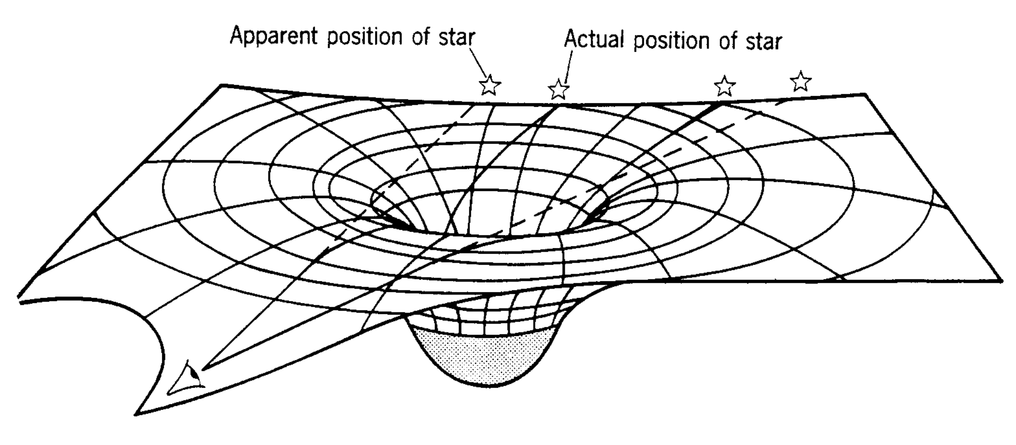
Arthur Stanley Eddington
(1882–1944)

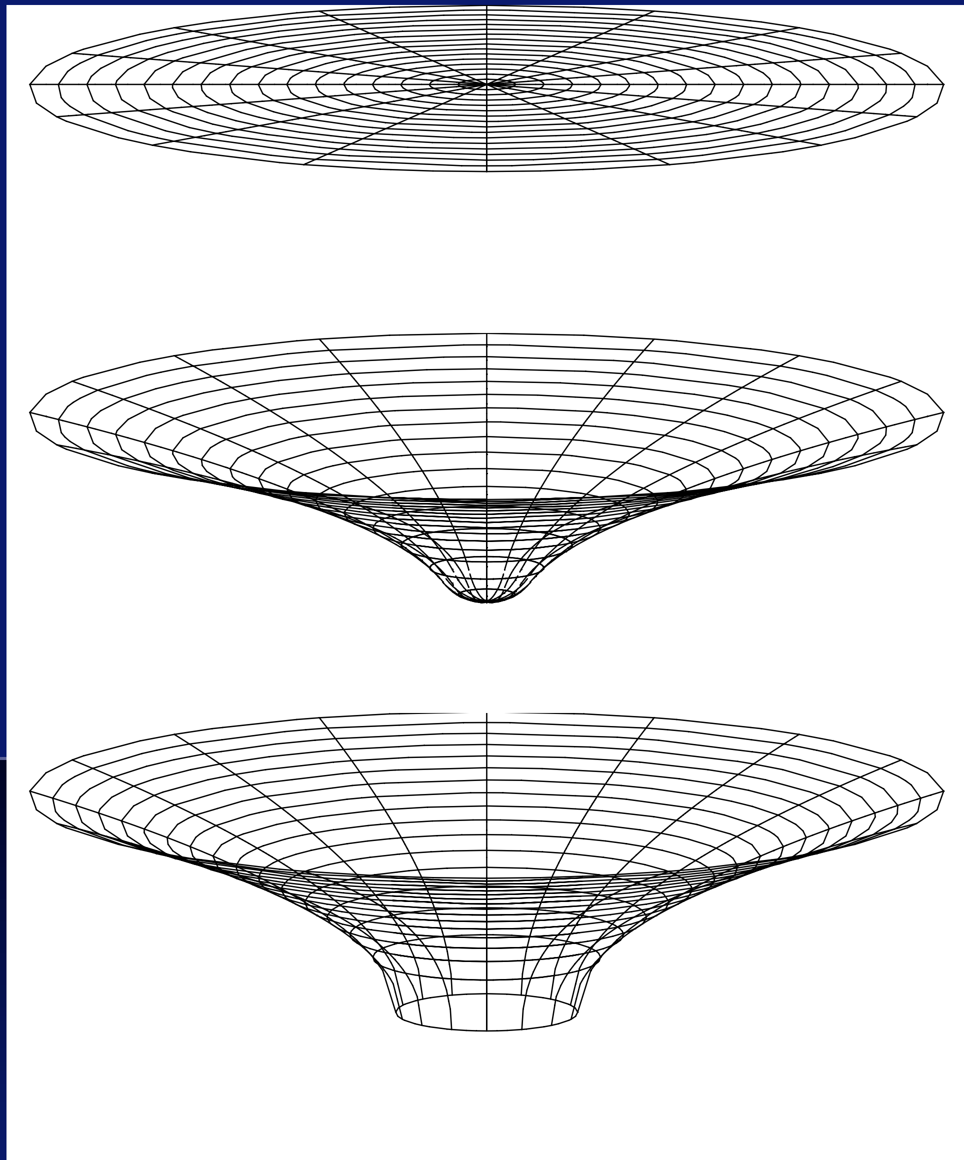
He was an early advocate of Einstein's General Relativity, and an interesting anecdote well illustrates his humour and personal intellectual investment: Ludwig Silberstein, a physicist who thought of himself as an expert on relativity, approached Eddington at the Royal Society's (6 November) 1919 meeting where he had defended Einstein's Relativity with his Brazil-Principe Solar Eclipse calculations with some degree of scepticism and ruefully charged Arthur as one who claimed to be one of three men who actually understood the theory (Silberstein, of course, was including himself and Einstein as the other two). When Eddington refrained from replying, he insisted Arthur not be "so shy", whereupon Eddington replied,

"Oh, no! I was wondering who the third one might be!"

Prediction of General Relativity
—Bending of Light: gravitational lens—





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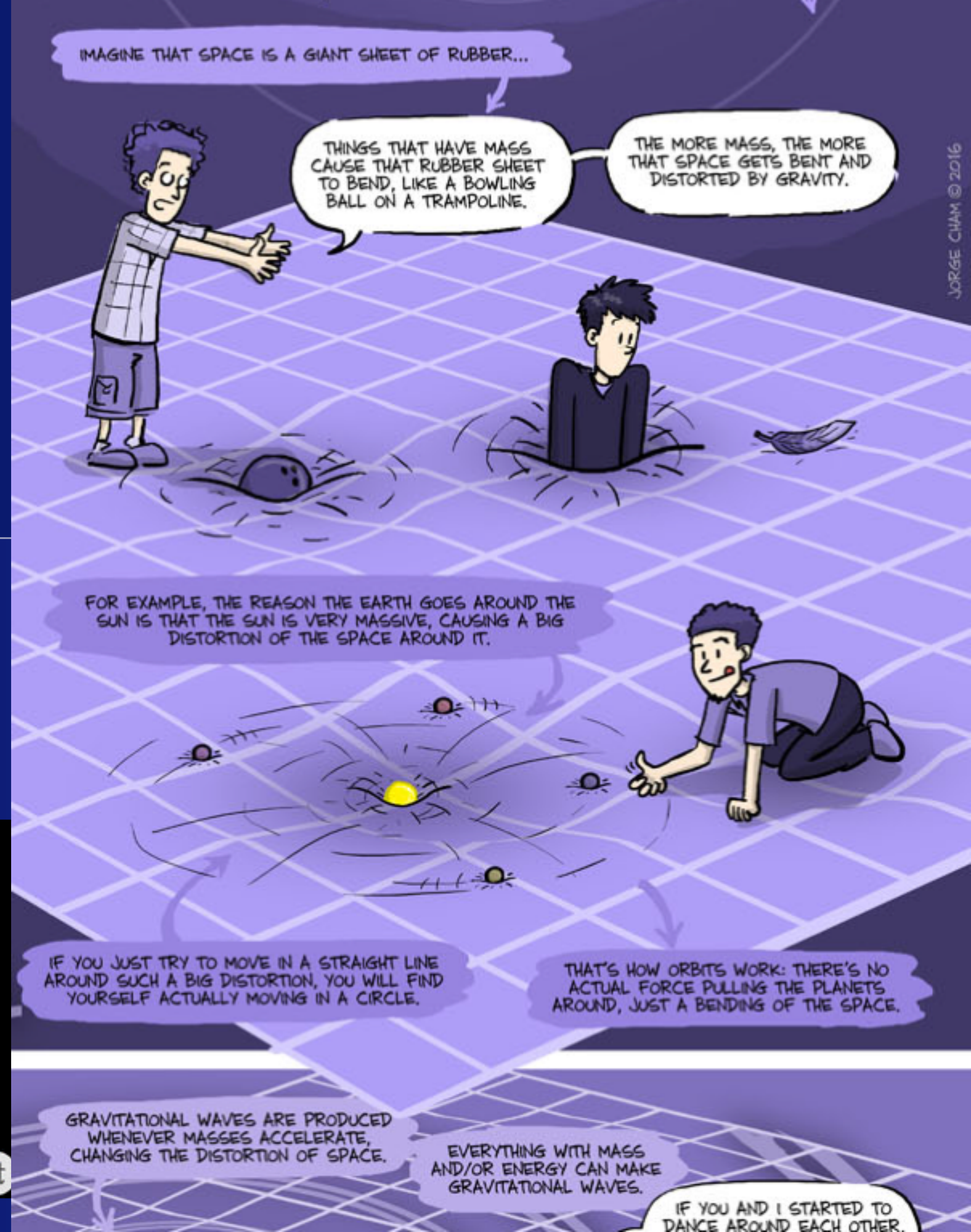
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JORGE CHAM © 2016
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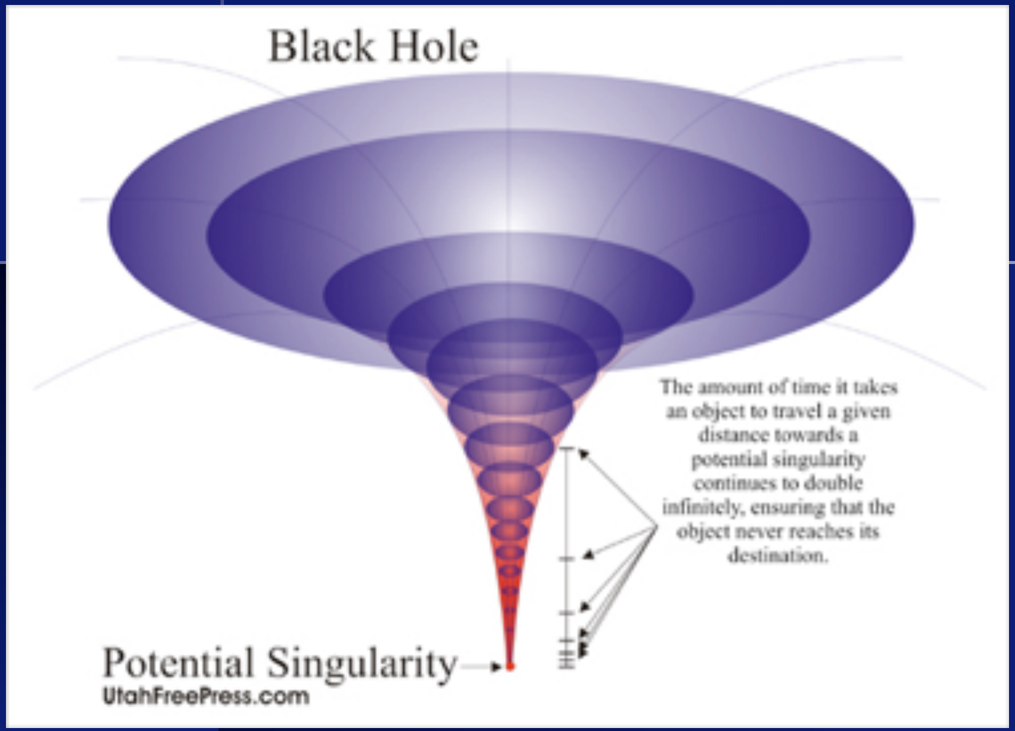
Black-hole


Black Hole
The amount of time it takes an object to travel a given distance towards a potential singularity continues to double infinitely, ensuring that the object never reaches its destination.
Potential Singularity
UtahFreePress.com


FailsShot


Subscribe

2.5 How and Why we know there is a black hole

Schwarzschild solution (black-hole solution)

The solution of the Einstein equation (2.18) of spherically symmetric, static, and vacuum space-time.

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \quad (2.19)$$

where G and c are gravitational constant and speed of light, M is the mass at the center, r is radial coordinate.

Figure 26: Karl Schwarzschild (1873–1916)

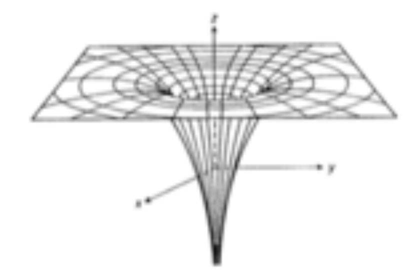


Figure 27: Black-hole is the bended trampoline to infinity.

Exercise 3 The radius of black-hole, R_{BH} can be given by

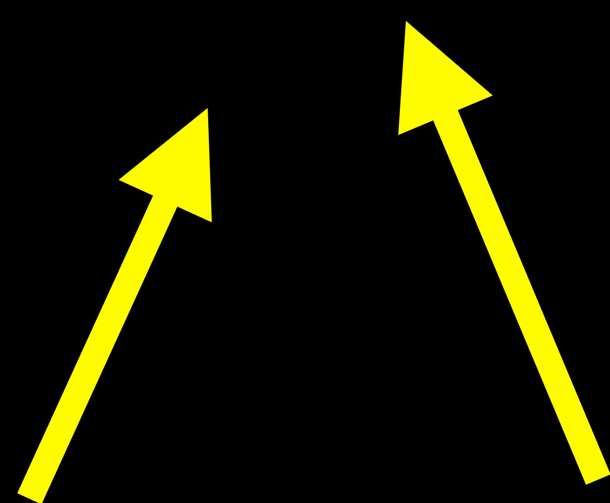
$$R_{BH} = \frac{2GM}{c^2}, \quad (2.20)$$

where M is the mass of black-hole, $G = 6.67 \times 10^{-11}$[m^3/kg/s^2] is gravitational constant, and $c = 3.0 \times 10^8$[m/s] is the speed of light. Calculate the size of

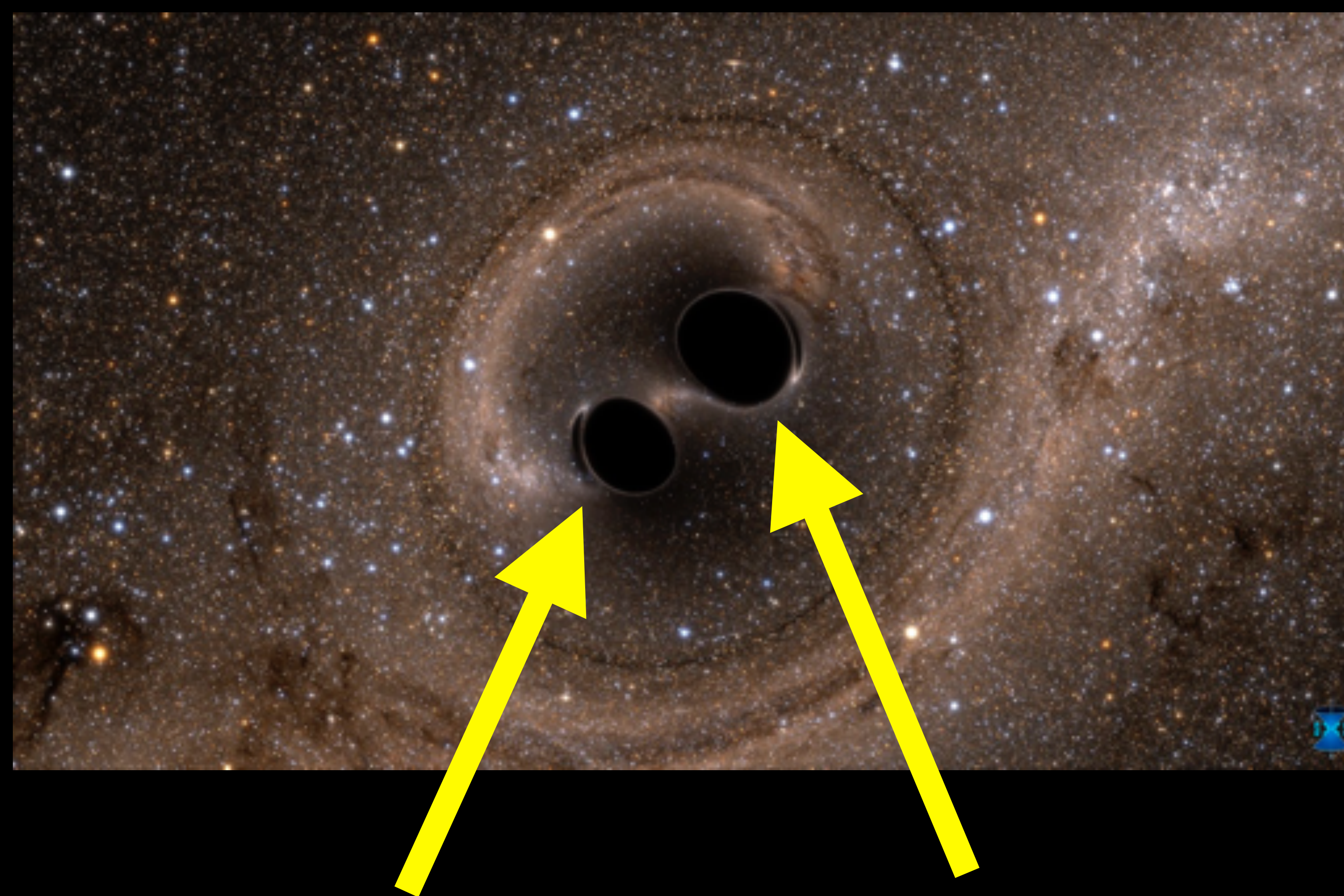
- an Earth-mass black hole ($M = 6.0 \times 10^{24}$[kg])
- a Sun-mass black hole ($M_\odot = 2.0 \times 10^{30}$[kg])
- a black-hole of the center of our galaxy ($M = 4.2 \times 10^6 M_\odot$)

Black hole ＝Too heavy object

Even the Light cannot escape

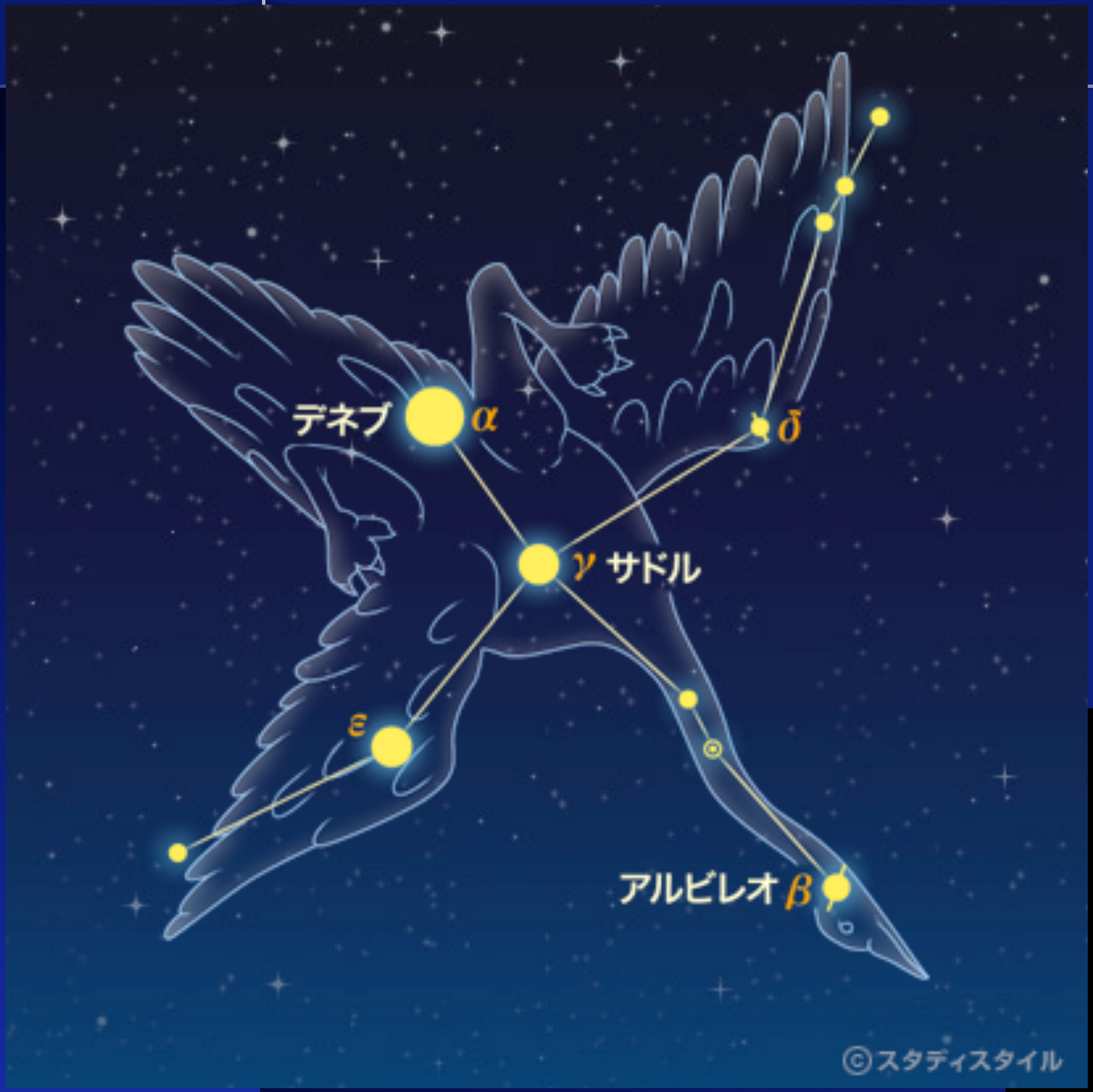


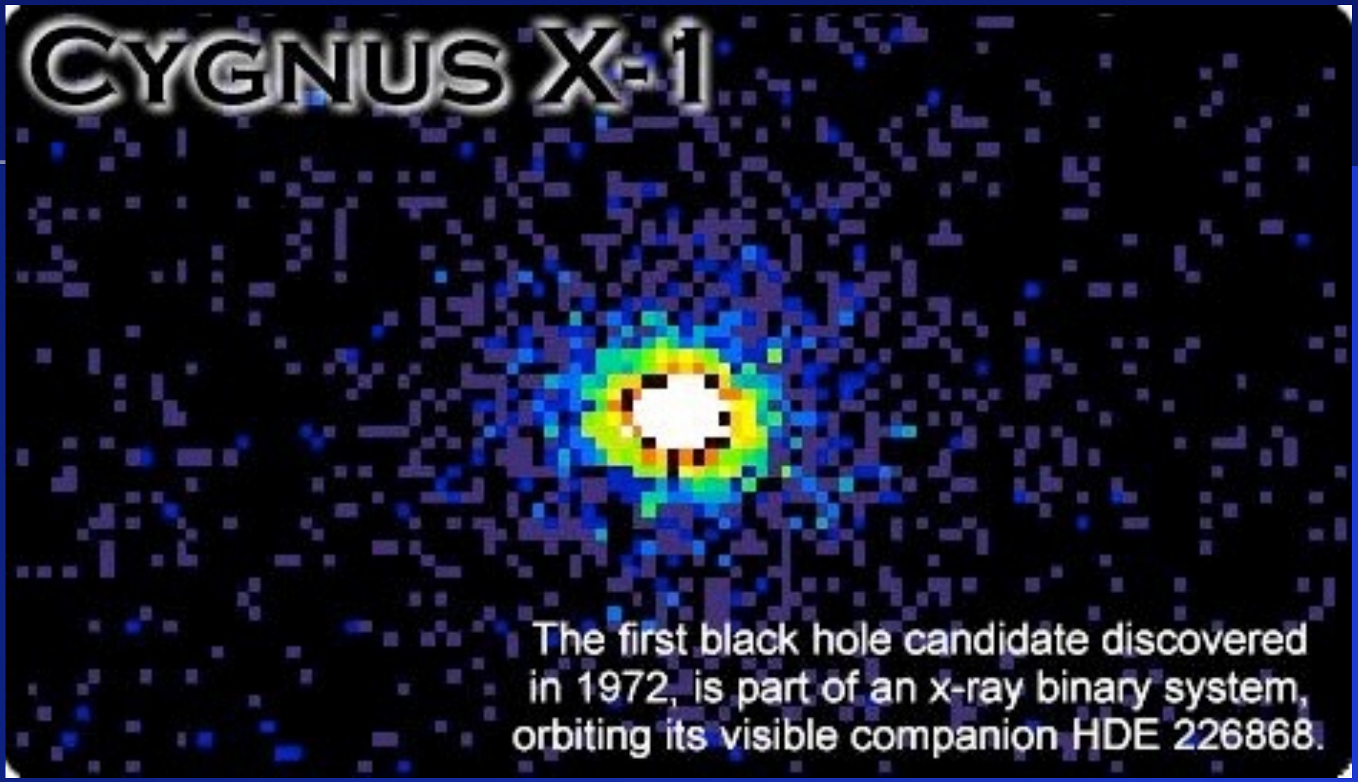
Black hole ＝Too heavy body
Even the Light cannot escape

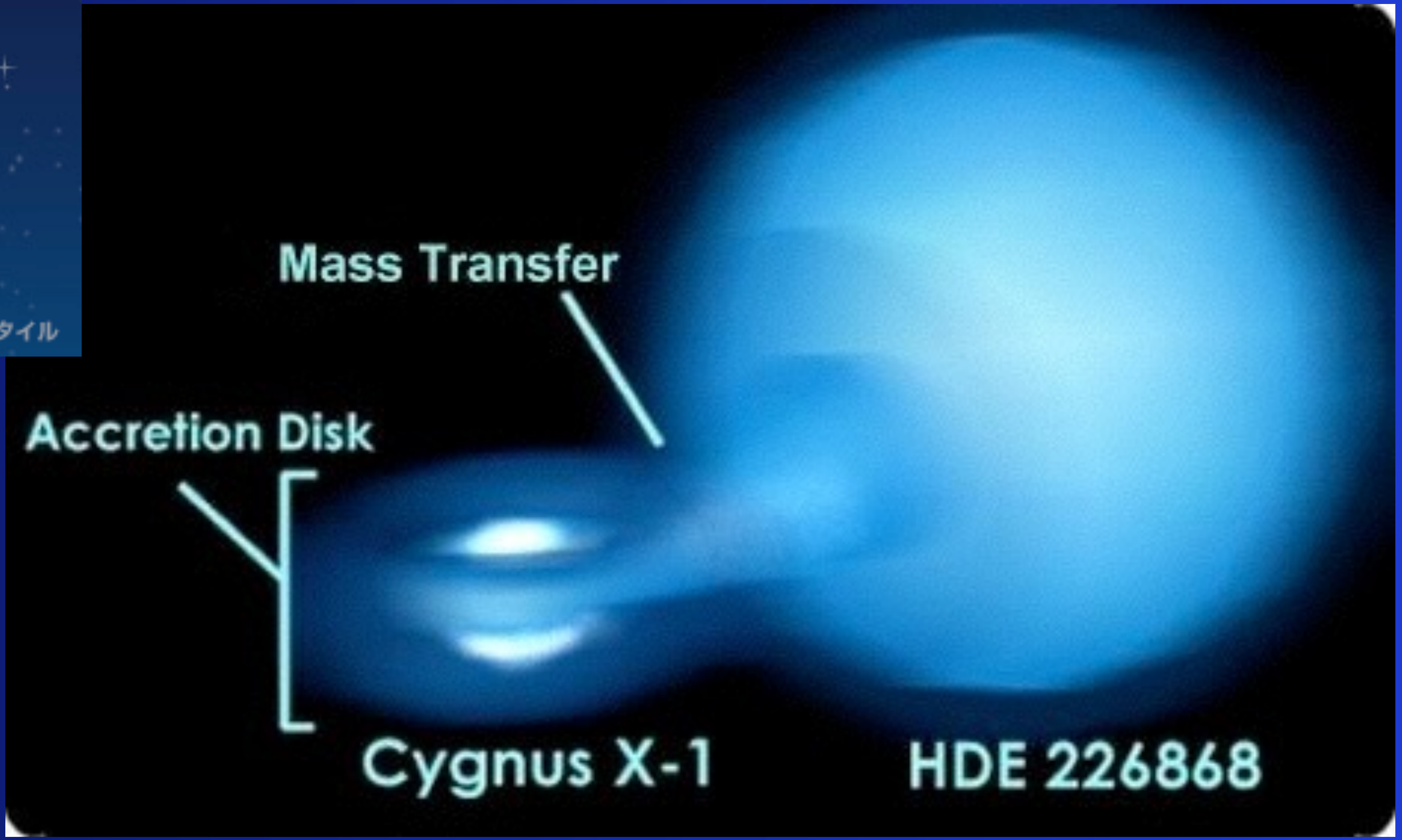


Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

BH candidate: Cygnus X-1







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Gravitational Lensing


MATTHEW McCONAUGHEY
ANNE HATHAWAY
JESSICA CHASTAIN
AND MICHAEL CAINE
GO FURTHER.
FROM THE DIRECTOR OF THE DARK KNIGHT TRILOGY AND INCEPTION
INTERSTELLAR
IN THEATRES AND IMAX
EVERYWHERE
NOVEMBER 7

Making of **Interstellar (2014)**

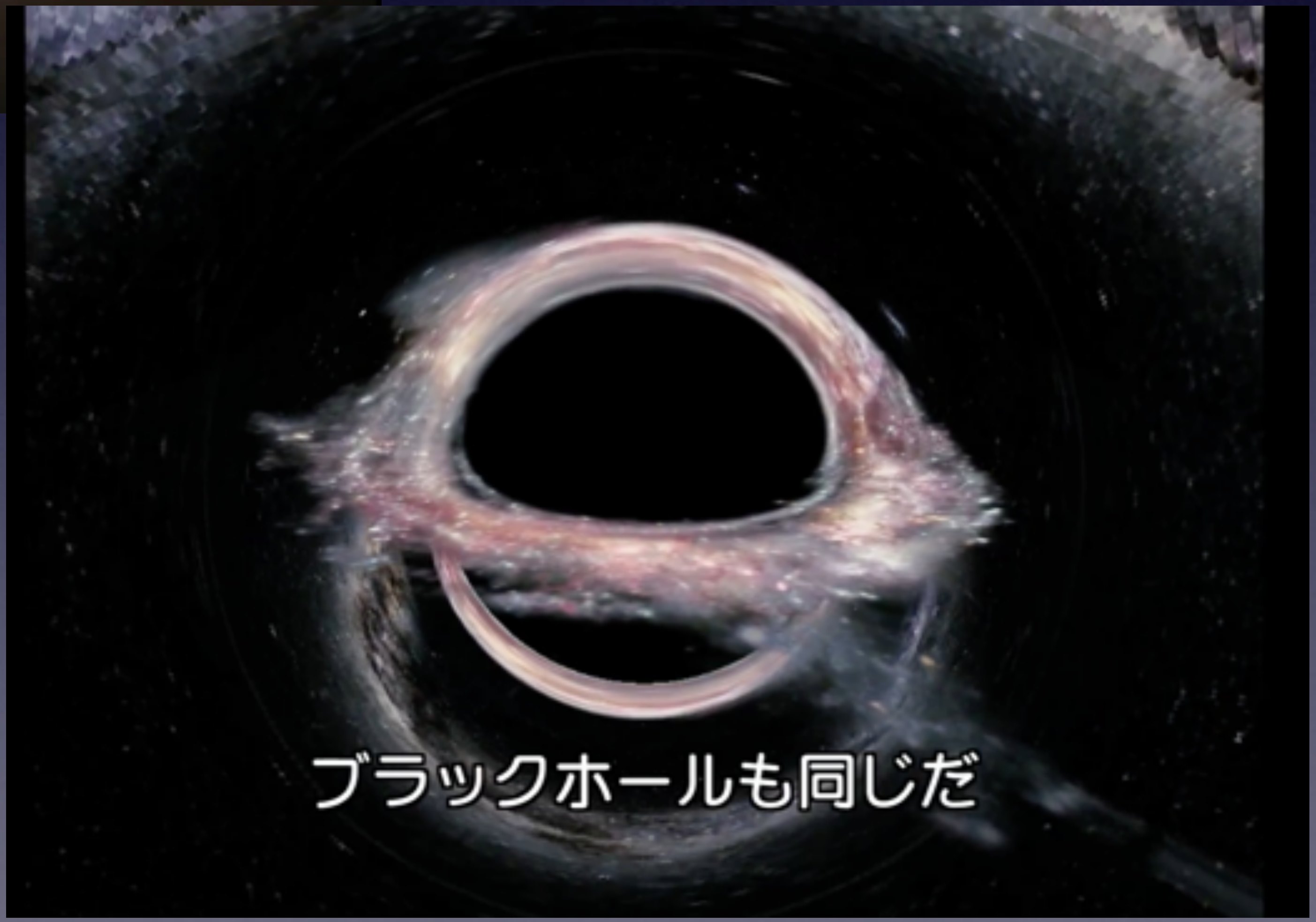
0’57”

キップ・ソーン
Kip Thorne

Interstellar (2014)

Executive Producer: Kip Thorne

https://www.youtube.com/watch?v=qZZ9jRan9eo



Interstellar (2014)

Executive Producer: Kip Thorne

https://www.youtube.com/watch?v=qZZ9jRan9eo

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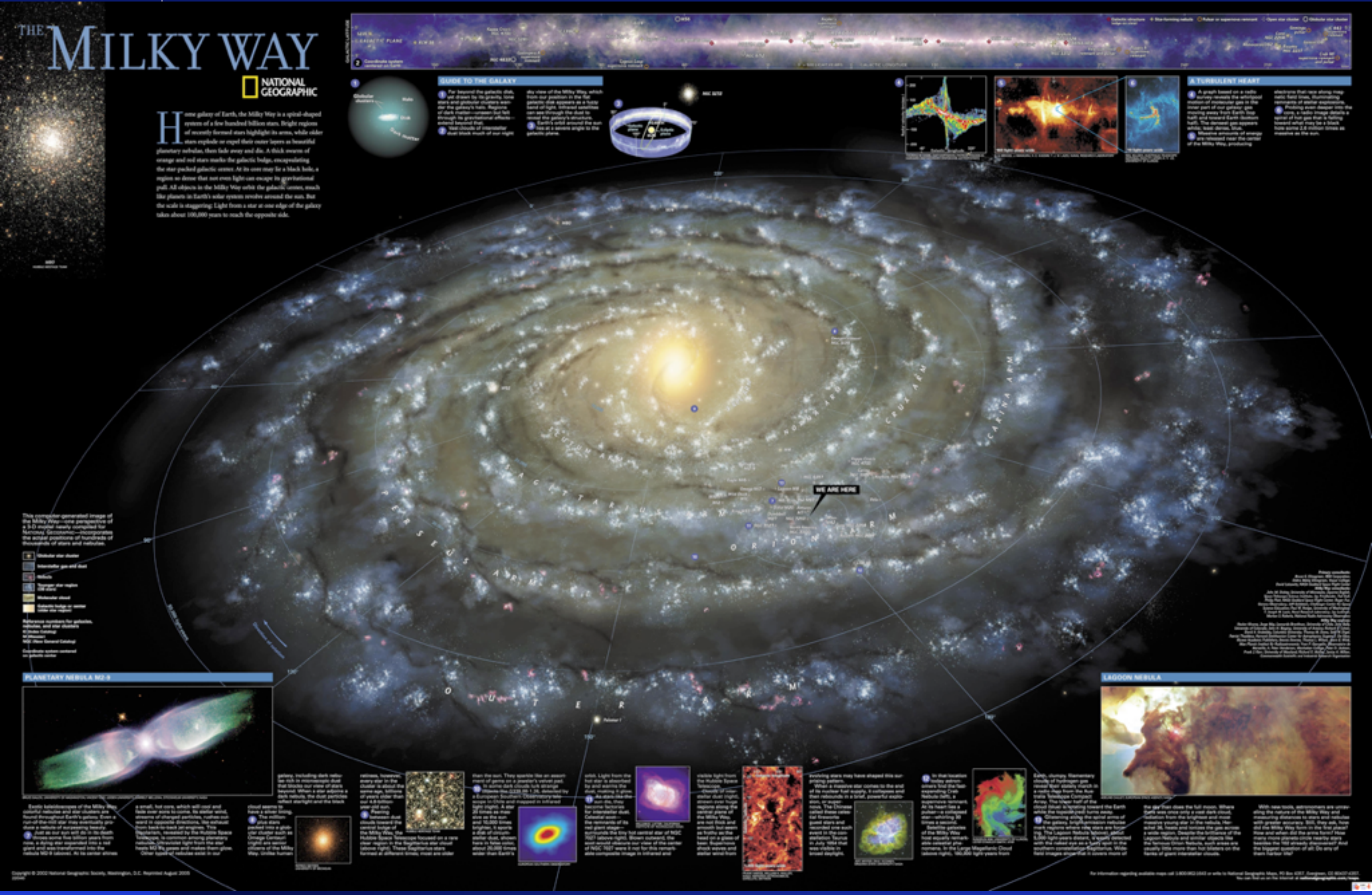
Kepler motion

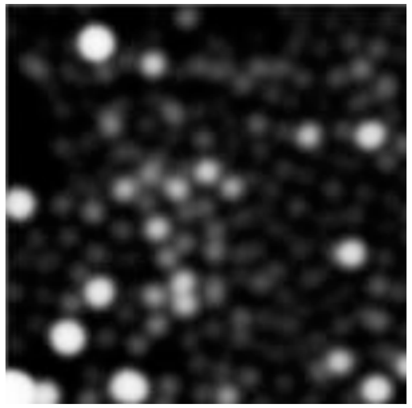
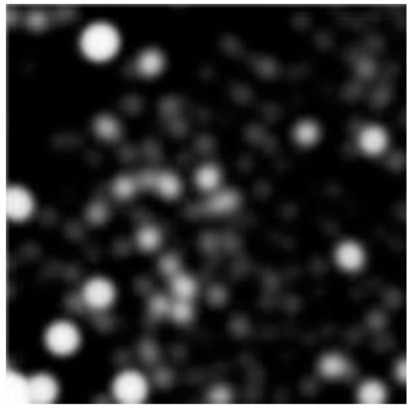
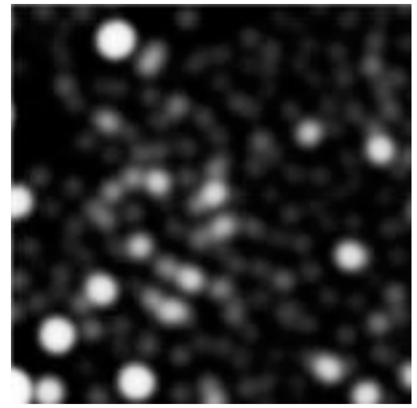
Supermassive BH in the center of Galaxy

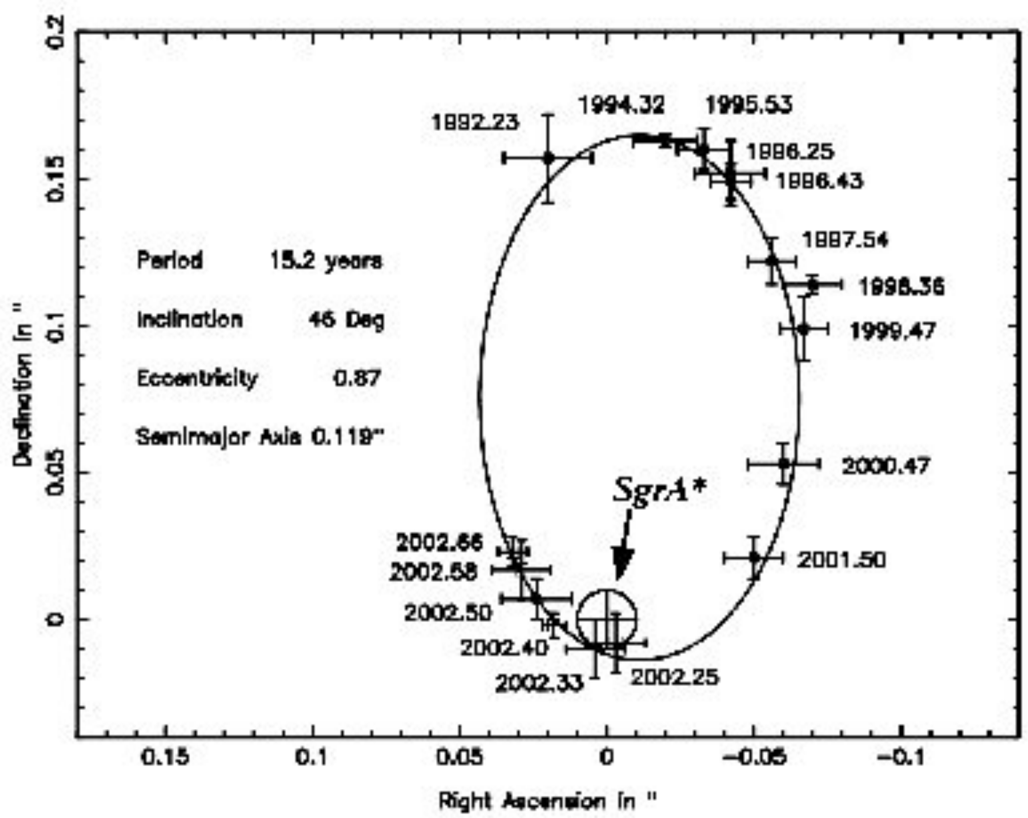


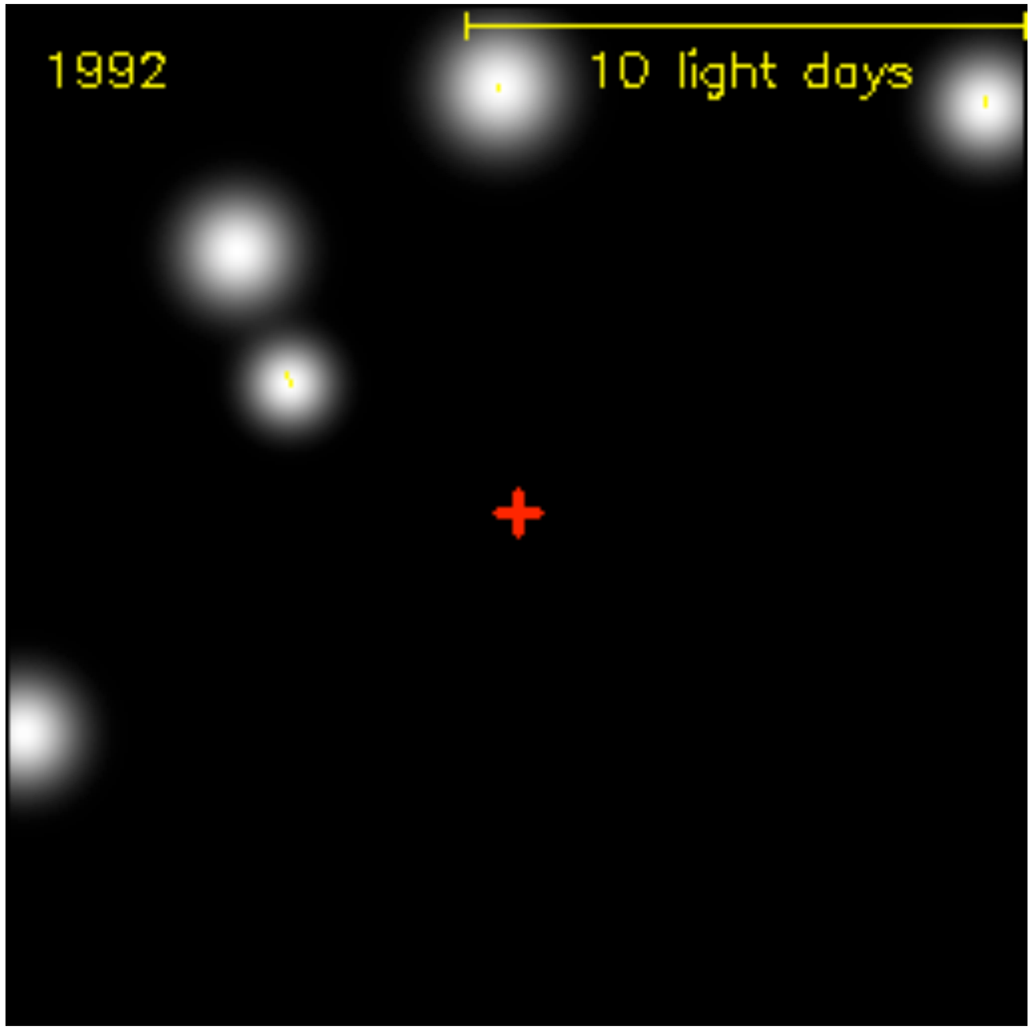
Zooming in the center of the Milky Way

http://www.youtube.com/watch?v=XhHUNvEKUY8 (1:15)

Supermassive BH in the center of Galaxy

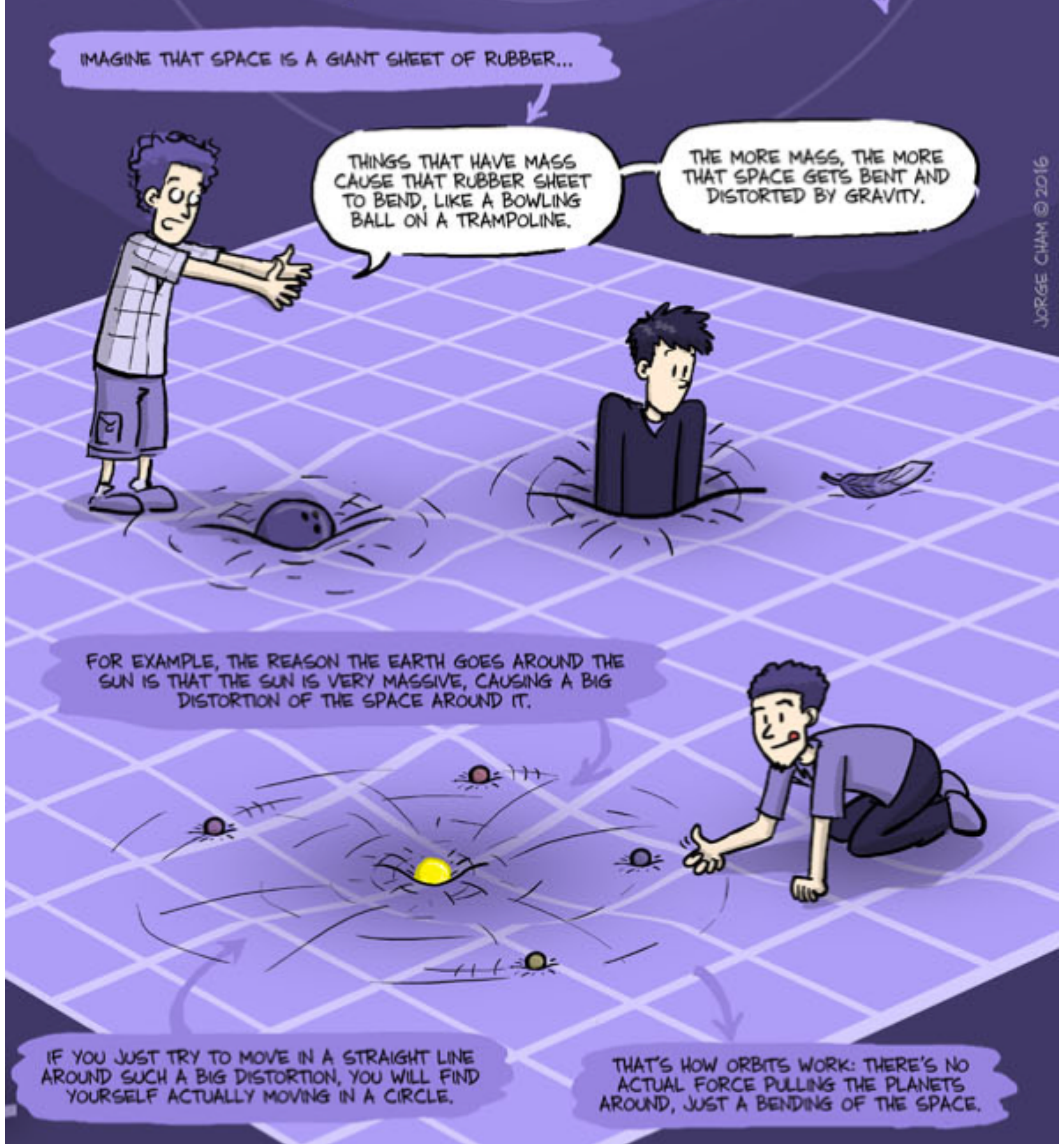



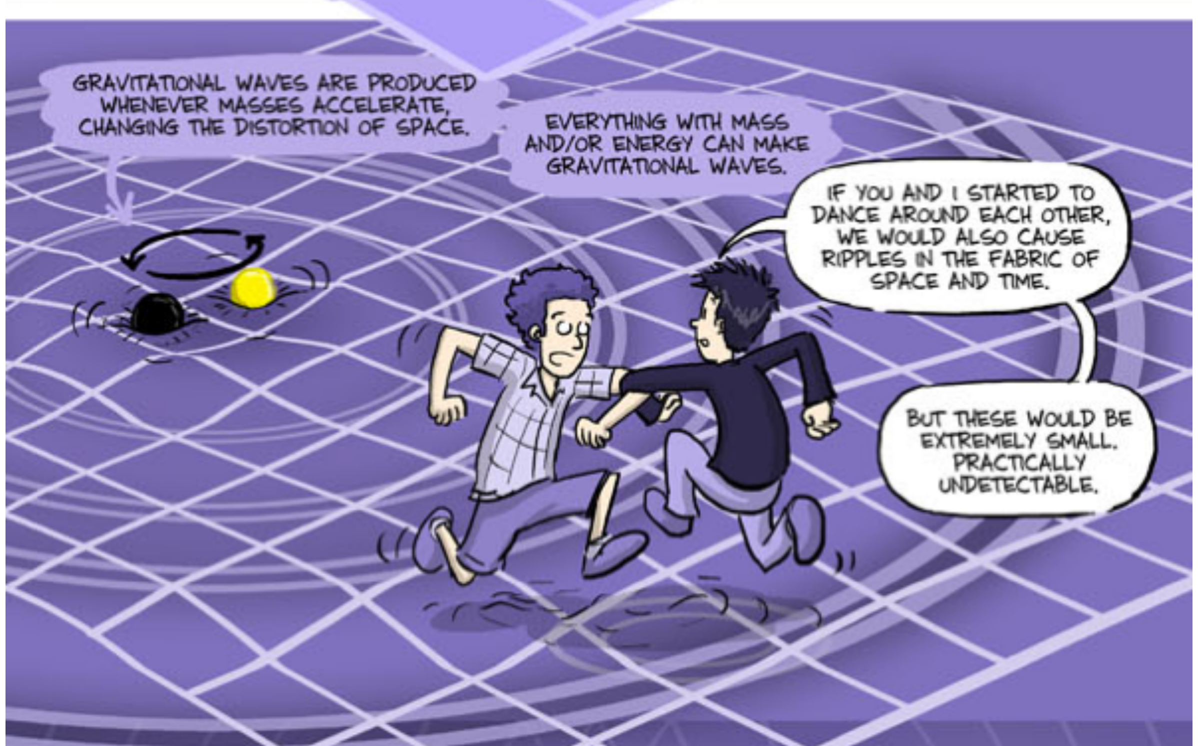


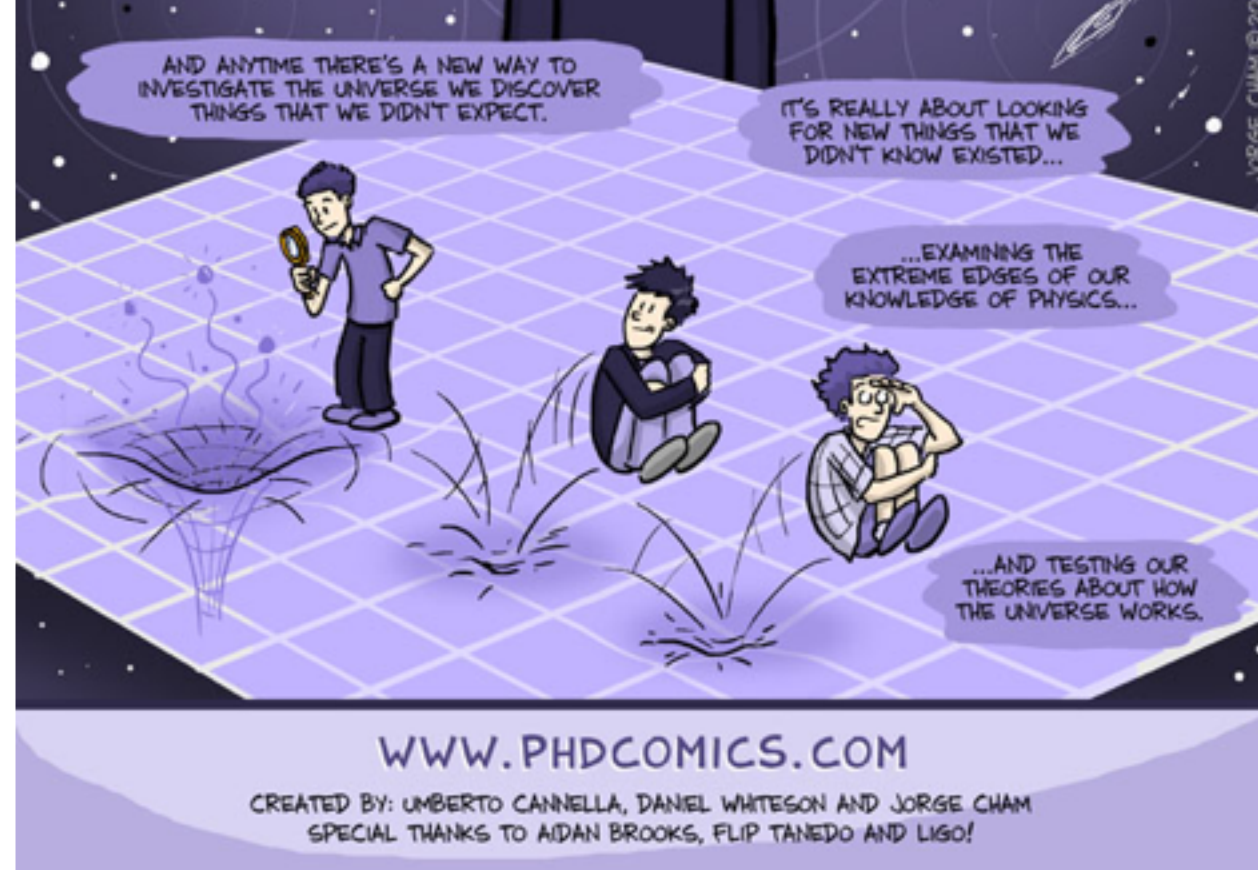
http://www.extinctionshift.com/SignificantFindings08.htm
http://www.brighthub.com/science/space/articles/13435.aspx#

Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Gravitational Wave

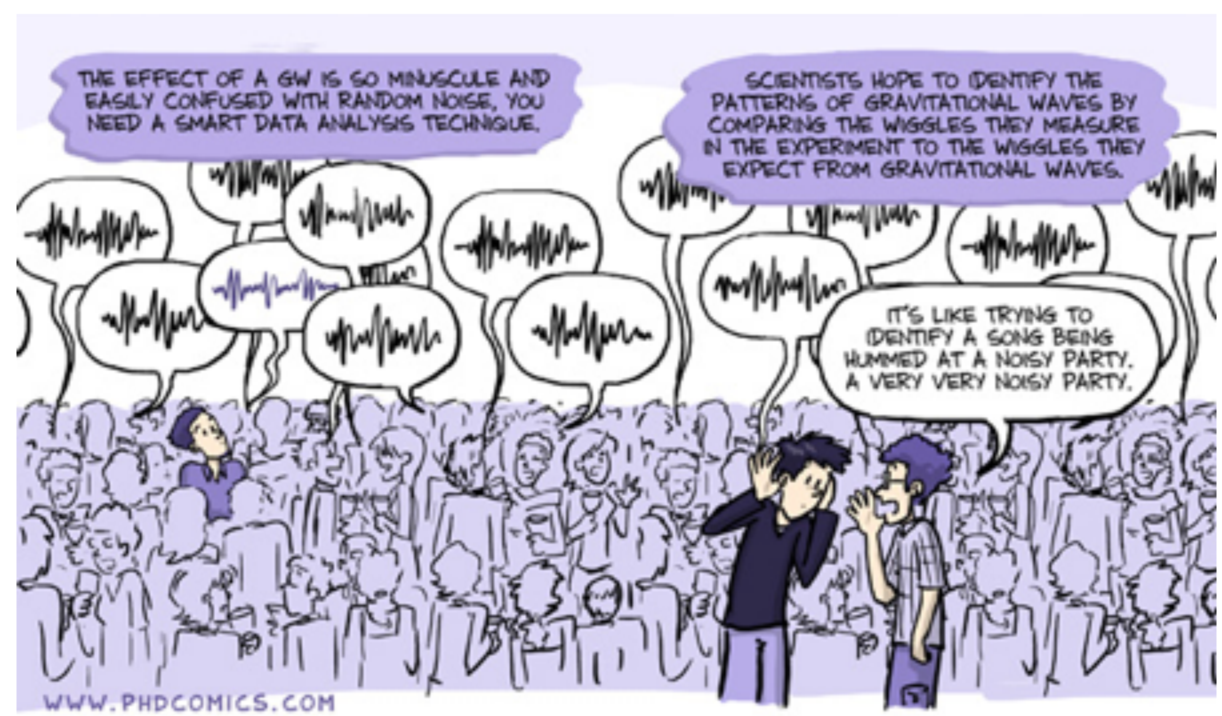




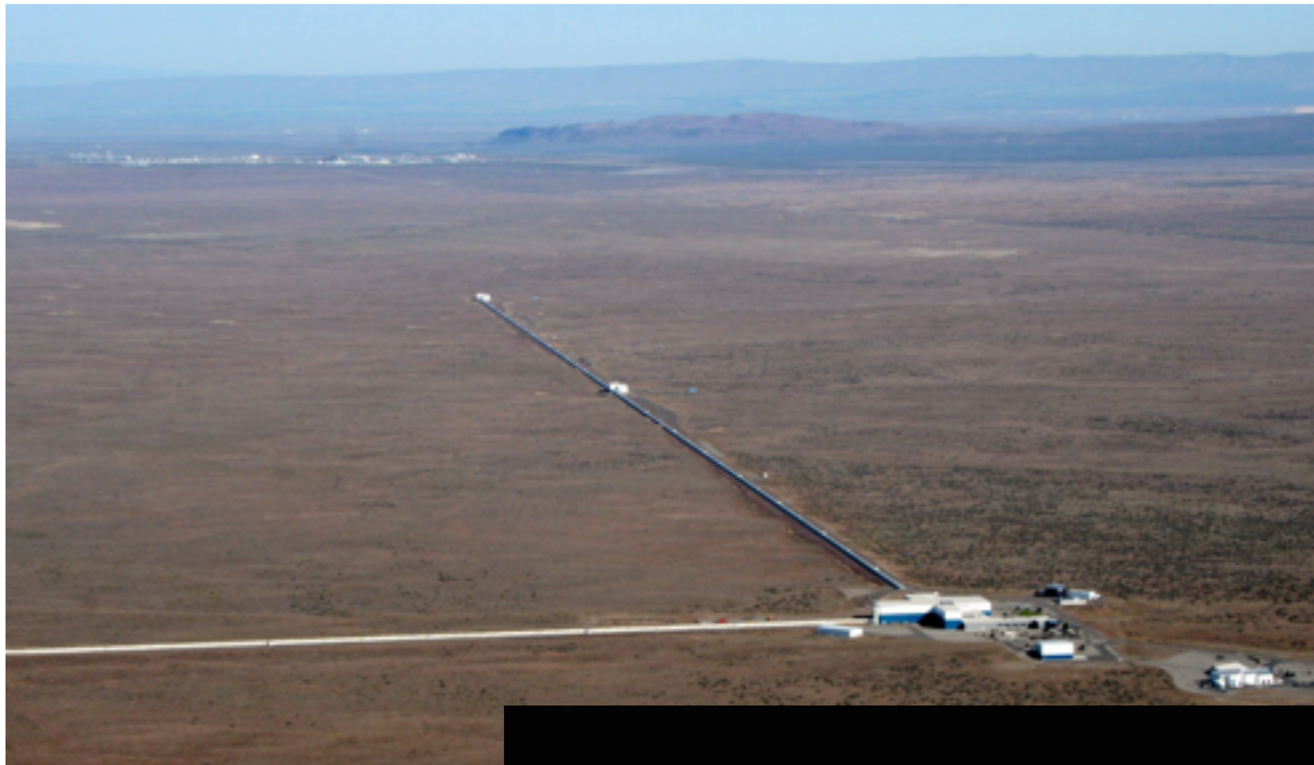


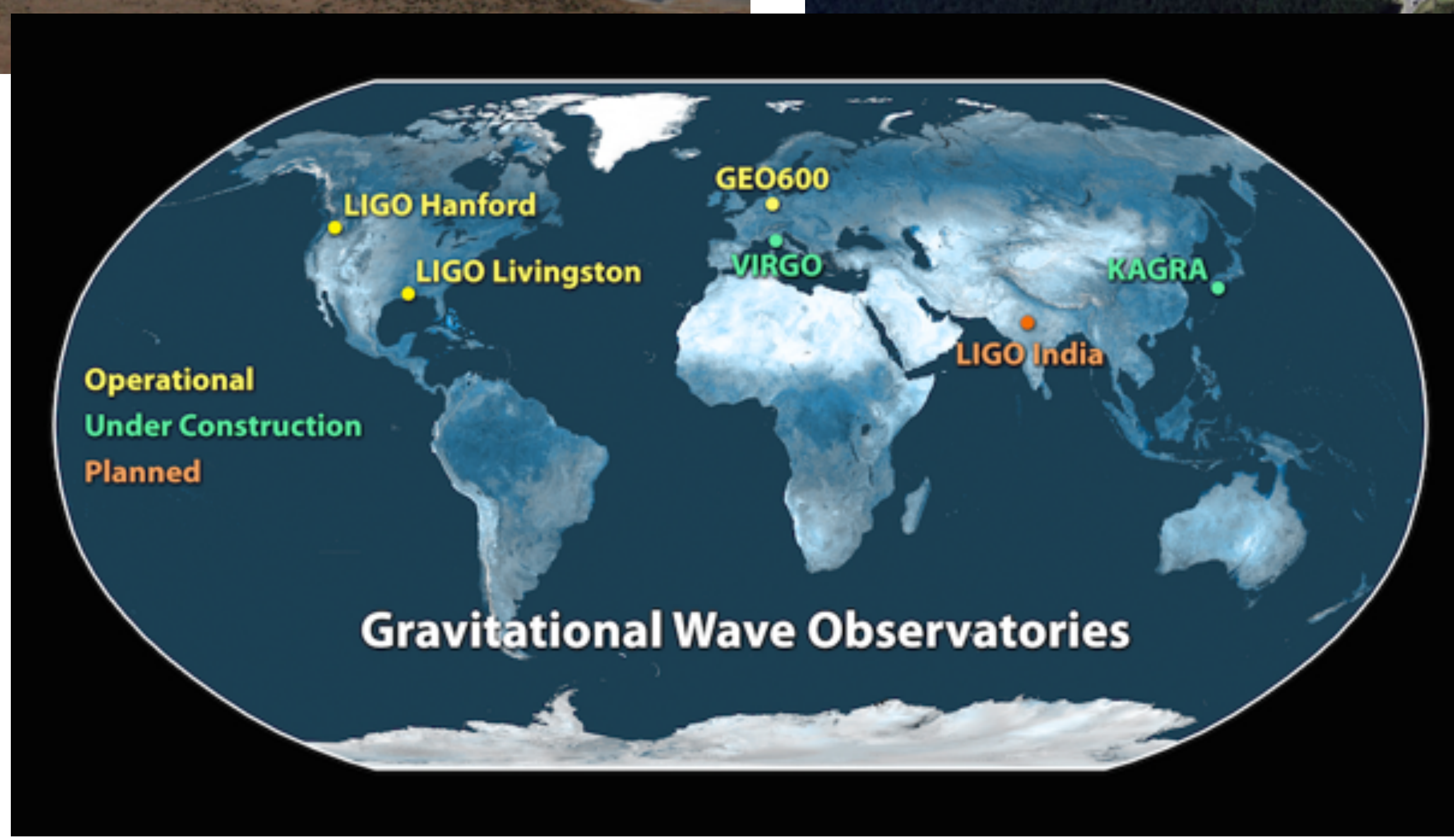
www.phdcomics.com

“gravitational waves explained”

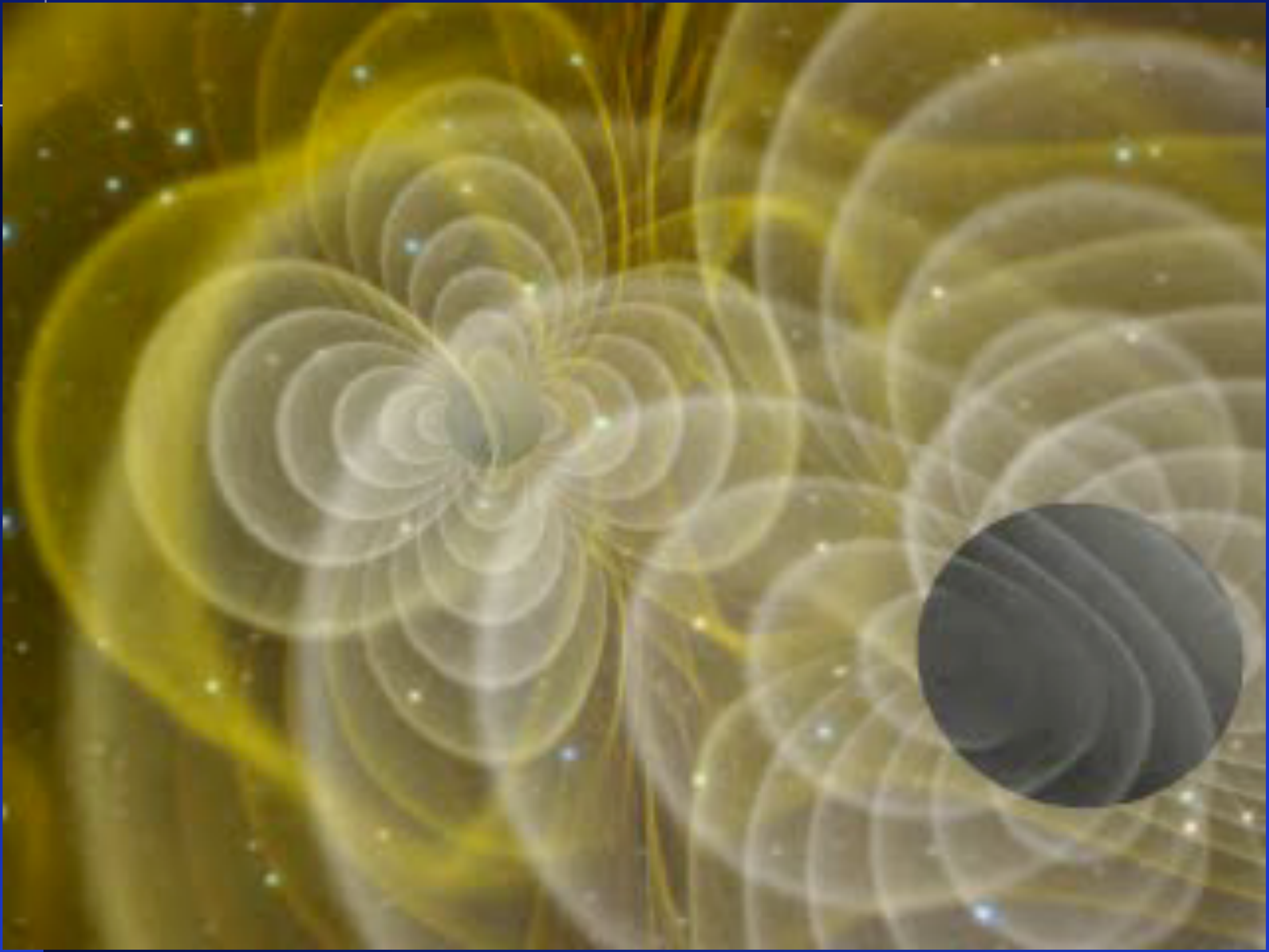


LIGO (Laser Interferemeter Gravitational-Wave Observatory)





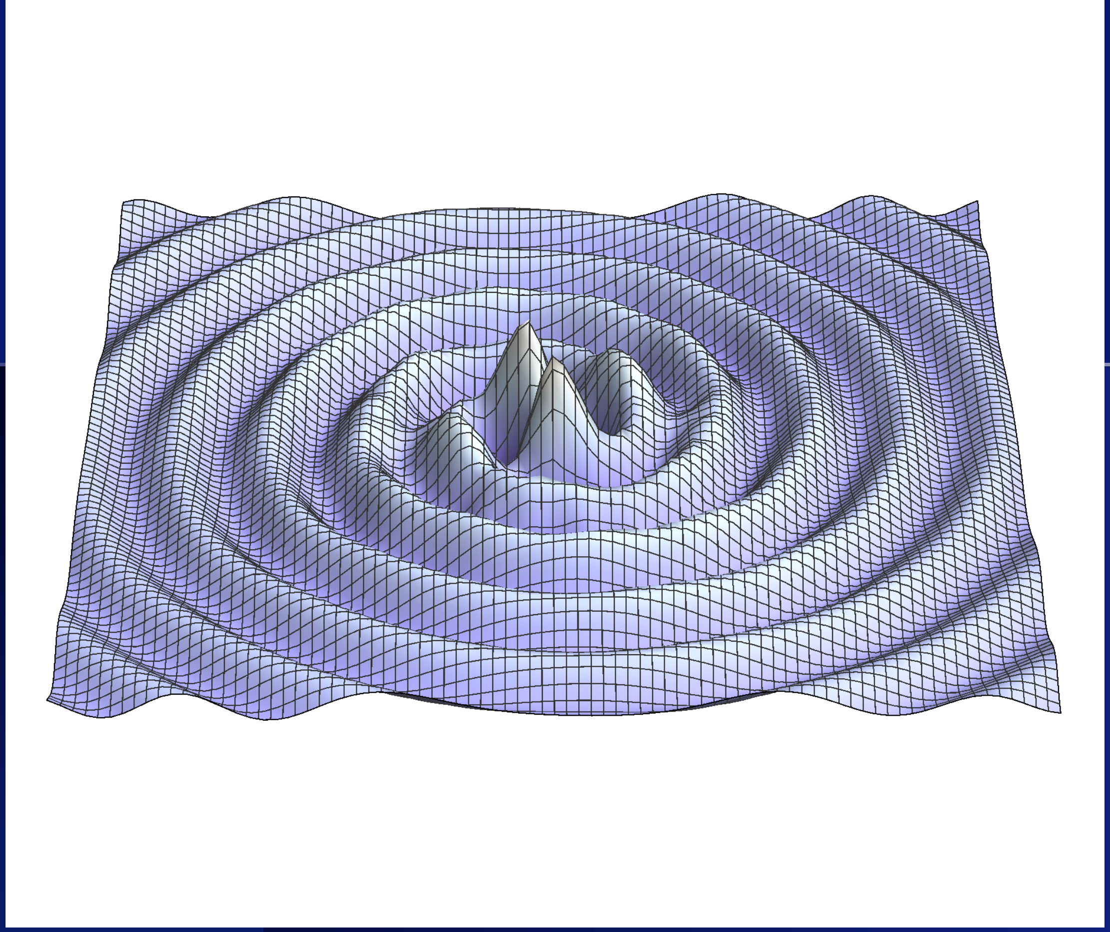
Collision of BH-BH binary



NCSA-AEI group (1998)

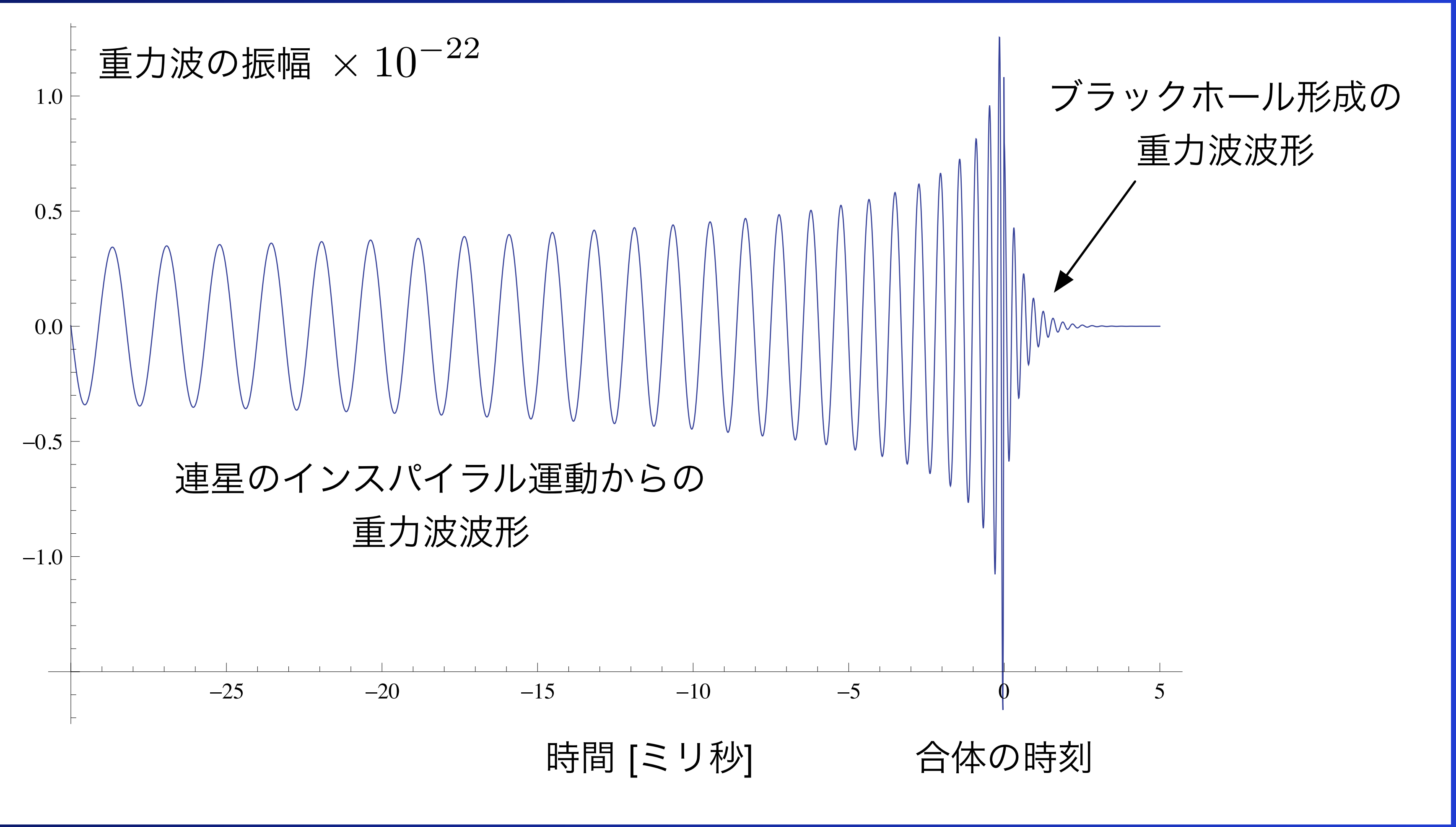
重力波の存在が間接的に確かめられた.

重力波の直接観測をしたい！



連星中性子星

連星ブラックホール



David Reitze, LIGO Director

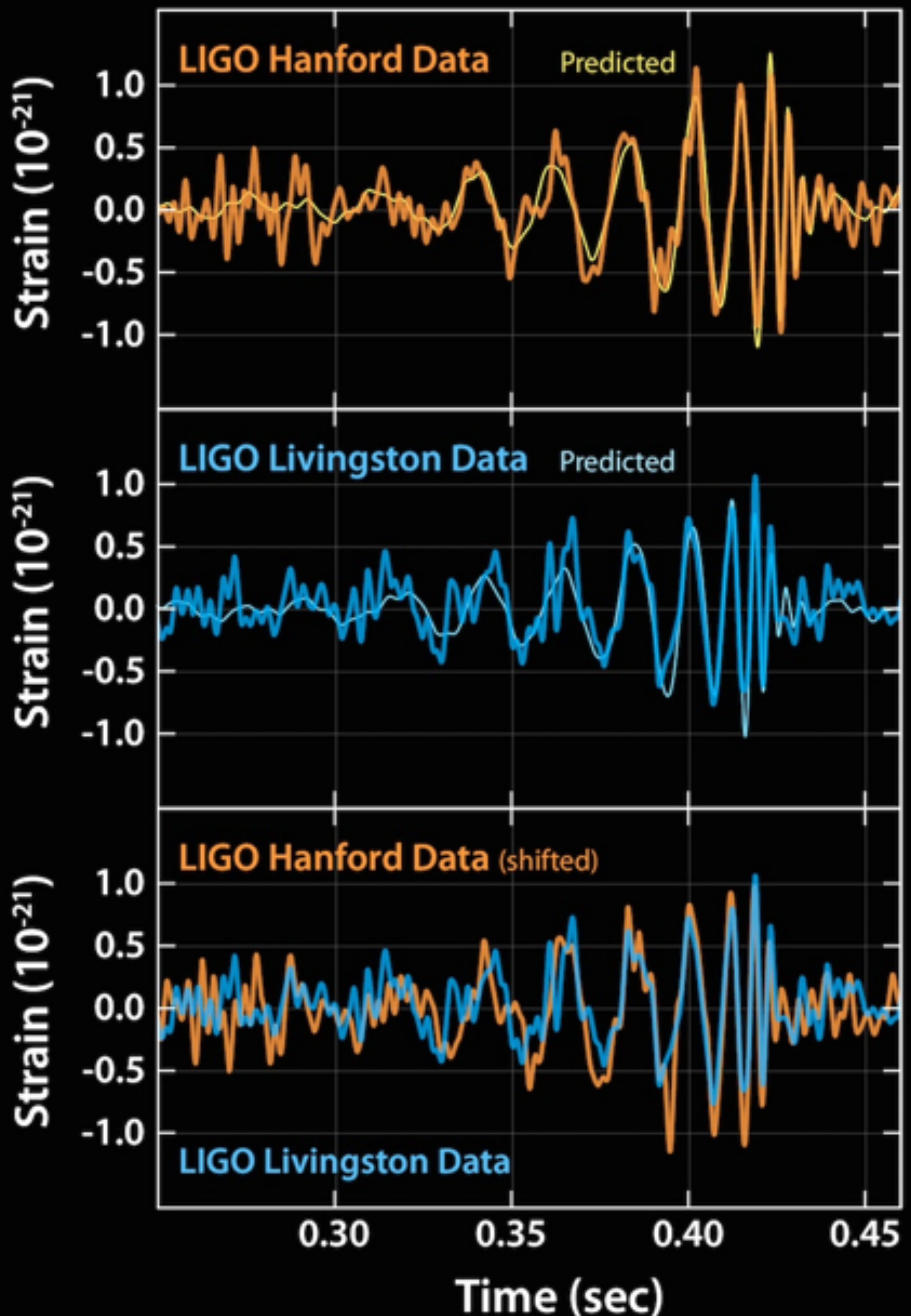
Feb 11, 2016

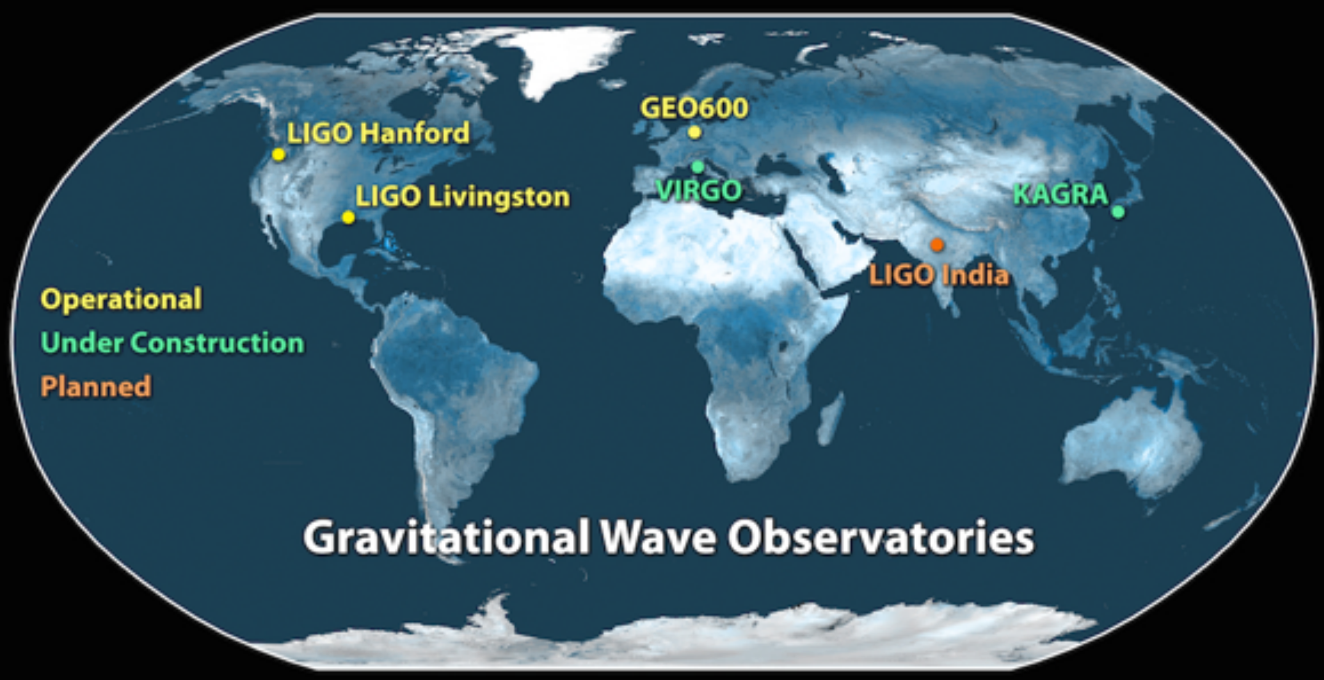
“We had detected gravitational waves.　We did it. ”

“我々は，重力波を検出した．　やり遂げたのだ. ”

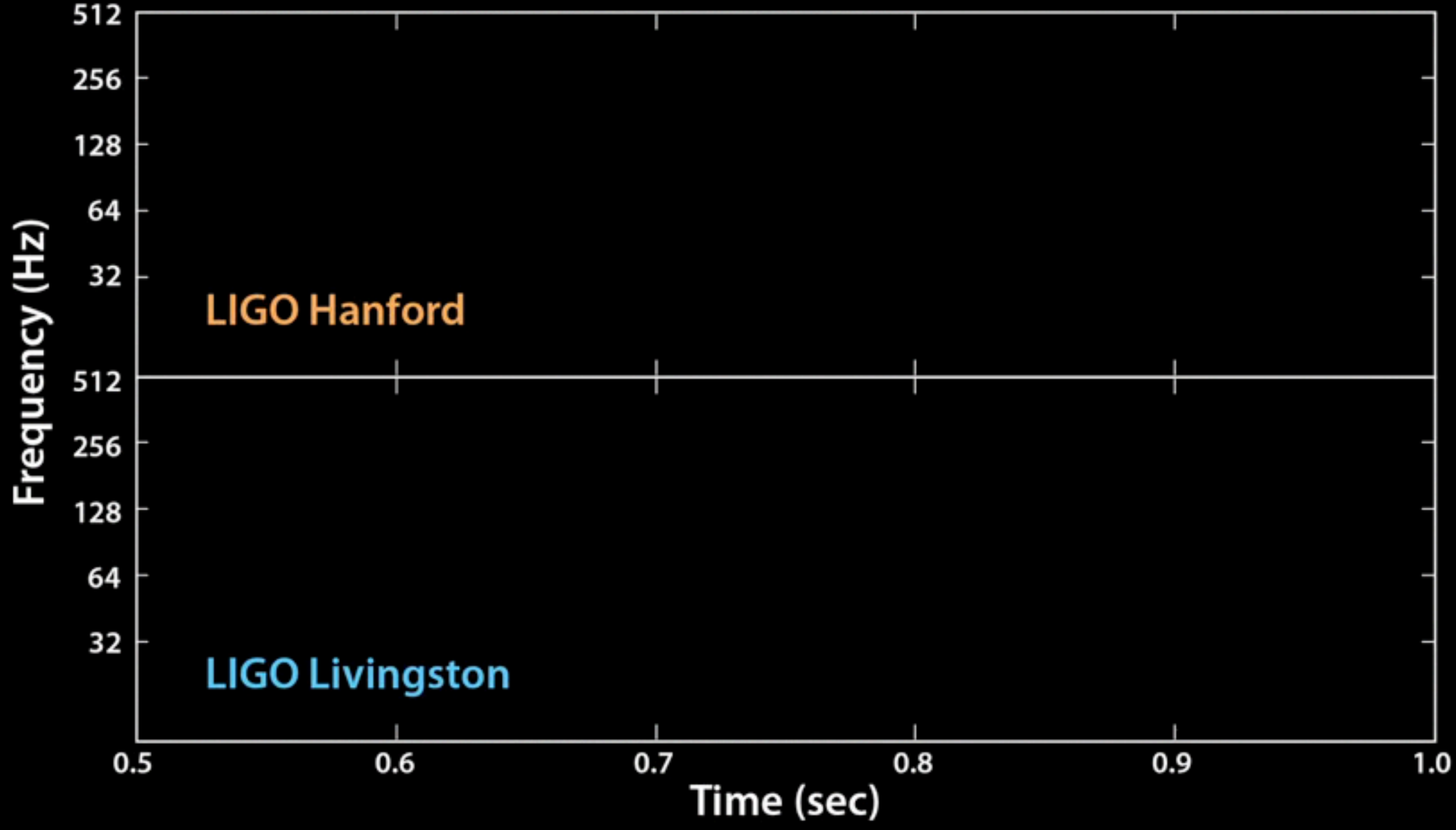
https://www.youtube.com/watch?v=aEPIwEJmZyE

Sep 14, 2015


LIGO Hanford Data
Predicted
LIGO Livingston Data
Predicted
LIGO Hanford Data (shifted)
LIGO Livingston Data
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)


LIGO Hanford
LIGO Livingston
GEO600
VIRGO
KAGRA
LIGO India
Operational
Under Construction
Planned
Gravitational Wave Observatories

Hear the sound of BH-BH collision, 12 billions yrs ago



始め2回は実周波数，後の2回は聞えやすいように+400Hz

https://mediaassets.caltech.edu/gwave

ブラックホール 解明に期待

る精度を目指す。実現すれば七億光年の範囲にある連星からの重力波を捉えられる。一年で十回ほどキャッチできる計算だという。

重力波の大きな特徴はブラックホールからも放出されること。連星が合体してブラックホールが生まれる瞬間を観測できると期待される。時間と空間をねじ曲げてすべてをのみ込むブラックホールは、光も電波も出さないため直接には観測できなかった。その謎に迫る強力な手段になる。

また、弱まらずに遠くまで届くので、遠い星の観測や誕生直後の宇宙の姿を知るのにも役立つと期待されている。

大阪工業大の真貝寿明教授（理論物理学）は「重力波を天文学に使うには、発生した場所を割り出す必要がある。それには望遠鏡が四カ所必要だ。KAGRAを含めた世界的なネットワークが重要」と話す。

Tokyo Shimbun 2016/2/12

重力波 初の直接観測

「研究者勇気づけた」

大阪市大院・神田教授 学生らに解説

米国を中心とした国際研究チームが「重力波」を初めて直接観測したとの発表を受け、重力波の研究が専門の神田展行・大阪市立大大学院教授(51)は12日、発表内容についての説明会を大阪市住吉区の同大杉本キャンパスで開いた。成果を詳しく理解してもらう狙い。学生ら約100人が参加し、真剣な表情で聴き入った。【畠山哲郎】

神田教授は、岐阜県飛騨市の大型低温重力波望遠鏡「KAGRA」のプロジェクトでもデータ管理グループのリーダーを務める。説明会では観測されたデータの見方などを解説し「我々にとっても勇気づけられるものだった」と語った。発表を受けて、研究室の学生が締め切り間際の論文を慌てて書き換えたエピソードを披露すると、会場は笑いに包まれた。

同大大学院理学研究科2年の和知慎吾さん(23)は「重力波だけでなく、ブラックホールも直接観測したことになると分かり、ためになった」と話していた。

重力波は、質量を持った物体が動いた時に周囲の時空にゆがみが生じ、そのゆがみが光速でさざ波のように宇宙空間に伝わる現象。物理学者のアインシュタインが「一般相対性理論」で存在を予言し、世界中の研究者が観測に挑戦していた。

重力波観測について解説する大阪市立大学大学院理学研究科の神田展行教授＝大阪市住吉区で、川平愛撮影

大阪工大 真貝教授「予想通りで驚いた」

大阪工業大情報科学部の真貝寿明教授（一般相対性理論）は「祝・重力波の直接検出」と題して、研究室のウェブページに一般向けの緊急解説記事を掲載した。昨年には一般向けの解説書「ブラックホール・膨張宇宙・重力波 一般相対性理論の100年と展開」を出版している。「こんなにも予想通りのものが見つかるのかと驚いた。素晴らしい発見だ」と感想を語った。

今後の研究については「日本でもKAGRAを使い、改めて重力波を確認したり、海外のチームと協力して重力波がどこから来たものなのかを調べたりしていくことが重要だ」と話した。【畠山哲郎】

Mainichi Shimbun 2016/2/13

GW exists !

GW was detected !

BHs exist !

BH binaries exist !

GR is right !

Discussion Black holes, therefore, do not show themselves. Then, how can we know there is a black hole? And why we believe there are black holes? Let's think on these issues at the session.

Gravitational Lensing

Kepler motion

Gravitational Wave

After 100 years, we are finally in the era of Astronomy with General Relativity.