Wormhole Evolutions in n-dim Gauss-Bonnet gravity

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Previous Stories

(a) "Fate of Morris-Thorne (Ellis) wormhole" was numerically investigated in 2002. [HS & Hayward, PRD66, 044005].

　　The fate is either black-hole collapse or inflationary expansion, depending on the excessed energy.

(b) The n-dimensional GR Ellis wormhole solutions are obtained. Perturbation study suggests instability. [TT & HS, PRD88 (2013), 064023]. Numerical evolutions in 4-6 dim confirm its instability. [HS & TT, in preparation]

(c) The wormholes in anti-de Sitter spacetime are analyzed. Perturbation study suggests instability if throat is smaller than a half of AdS horizon. Numerical evolutions support this prediction. [TT & HS, in Poster B05.]

Outline & Summary

The dynamics of the simplest wormhole solutions in n-dimensional Gauss-Bonnet gravity are investigated numerically.
The solutions catch an unstable mode, and the throat begins inflate if GB coupling term is positive, while it turns into a black-hole if the coupling is negative. This horizon bifurcation can be seen easily in higher-dimensional spacetime. There exists the optimized positive coupling constant which maximizes the throat expansion.

Motivations

1. Motivation

Why Wormhole?

They make great science fiction -- short cuts between otherwise distant regions. Morris & Thorne 1988, Sagan "Contact" etc

1. Motivation

Dynamics in Gauss-Bonnet gravity?

- Action
$$S = \int_{\mathcal{M}} d^{N+1}x \sqrt{-g}\left[\frac{1}{2\kappa^2}(\mathcal{R} + \alpha_2 \mathcal{L}_{GB}) + \mathcal{L}_{matter}\right]$$
where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation
$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$
where $H_{\mu\nu} = 2\{\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}\} - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- has GR correction terms from String Theory
- has two solution branches (GR/non-GR).
- is expected to have singularity avoidance feature. (but has never been demonstrated.)

- new topic in numerical relativity.
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496

- much attentions in WH community
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

1. Motivation

Why Wormhole?

They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.

They are very similar to black holes --both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole = Hypersurface foliated by marginally trapped surfaces

BH & WH are interconvertible?

S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)

Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.

	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH → 1-way traversable	Temporal (timelike) outer TH → 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally. but constructible??

Field Eqs.

2. Field Equations

dual-null variables

n-dim., Spherical Symmetry, Dual-null coordinate
$$ds^2 = -2e^{-f(x^+,x^-)}dx^+ dx^- + r^2(x^+,x^-)d\Omega_3^{(n-2)}$$

Space-time Variables
$$\Omega = \frac{1}{r}$$
$$\vartheta_\pm \equiv (n-2)\partial_\pm r$$
$$\nu_\pm \equiv \partial_\pm f$$

We also define η as
$$Z \equiv k + \frac{2e^f}{(n-2)^2}\vartheta_+\vartheta_- \equiv k + W$$

matter variables

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$
$$T_{\mu\nu} = T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi)$$
$$= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right]$$

this derives Klein-Gordon equations
$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}.$$

Scalar field variables
$$\pi_\pm \equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi$$
$$p_\pm \equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi$$

Klein-Gordon eqs.
$$\Box\phi = -\frac{e^f}{r^2}\left(2r\phi_{uv} + (n-2)r_u\phi_v + (n-2)r_v\phi_u\right)$$
$$= -2e^f\phi_{uv} - e^f\Omega^2(\vartheta_-p_+ + \vartheta_+p_-)$$

Energy-momentum tensor
$$T_{++} = \Omega^2(\pi_+^2 - p_+^2)$$
$$T_{--} = \Omega^2(\pi_-^2 - p_-^2)$$
$$T_{+-} = -e^{-f}(V_1(\psi) + V_2(\phi))$$
$$T_{\pm\pm} = e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}(V_1(\psi) - V_2(\phi))$$

evolution equations (1)

x^+-direction

evolution equations (2)

x^--direction

This constitutes the first-order dual-null form, suitable for numerical coding.

initial data

- Static condition
$$(\partial_+ + \partial_-)\Omega = 0 \implies \vartheta_+ + \vartheta_- = 0$$
$$(\partial_+ + \partial_-)\psi = 0 \implies \pi_+ + \pi_- = 0$$
$$(\partial_+ + \partial_-)\phi = 0 \implies p_+ + p_- = 0$$
$$(\partial_+ + \partial_-)\vartheta_\pm = 0 \implies \vartheta_+\nu_\pm = \frac{1}{A}\Omega\kappa^2(\pi_+^2 - p_+^2) = \vartheta_-\nu_+ + \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2)$$

- Solve x^+ and x^- equations with the starting condition at the throat
$$\vartheta_+ = \vartheta_-(=0)$$
$$\nu_+ = \nu_-(=0)$$
$$-\kappa^2\Omega(\pi_+^2 - p_+^2)e^f = -\frac{1}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}k + \Lambda + \kappa^2(V_1 + V_2)\right] + \delta t\Omega^4\frac{(n-2)(n-5)}{2}k^2$$

If we assume only ghost field ϕ, then
$$p_+ = -p_- = \sqrt{\frac{1}{\kappa^2 e^f}\left[\alpha_1\frac{(n-2)(n-3)}{2}k - \frac{1}{\Omega^2}(\Lambda + \kappa^2 V_2) + \delta t\Omega^2\frac{(n-2)(n-5)}{2}k^2\right]}$$

- add perturbation
$$p_+(x^+ = x, x^- = 0) = p_+(\text{solution}) + a\exp[-100(x - 0.5)^2]$$

conformal factor　　scalar field　　expansion+

Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons

-- go to a Black Hole or Inflationary expansion

Figure 4: Partial Penrose diagram of the evolved space-time.
Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional positive energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Ghost pulse input -- Bifurcation of the horizons

Figure 3: Horizon locations, $\vartheta_\pm = 0$, for perturbed wormhole. Fig(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3,3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

ワームホールを通過できるか

図解雑学
タイムマシンと
時空の科学

Time Machine &
Science of Space-time
(HS, 2011)

WH evolution in 5/6/7-dim.

in GR

4d 5d 6d GR
ghost pulse (negative amp.) input

positive energy input --> BH formation

4d 5d 6d GR
ghost pulse (positive amp.) input

negative energy input --> throat inflates

in GB

5d GR vs Gauss-Bonnet　instability appears

$\alpha_{GB} < 0$　　$\alpha_{GB} > 0$

BH formation　　throat inflates

6d 7d Gauss-Bonnet　instability appears easily

$\alpha_{GB} > 0$　　$\alpha_{GB} > 0$

6 dim.　　7 dim.

$$S = \int_{\mathcal{M}} d^{N+1}x \sqrt{-g}\left[\frac{1}{2\kappa^2}(\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{GB}) + \mathcal{L}_{matter}\right]$$
where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$