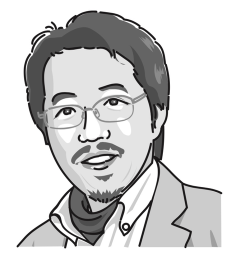
Autoregressive Approach to Extract Ring-down Gravitational Wave of Black-hole Merger

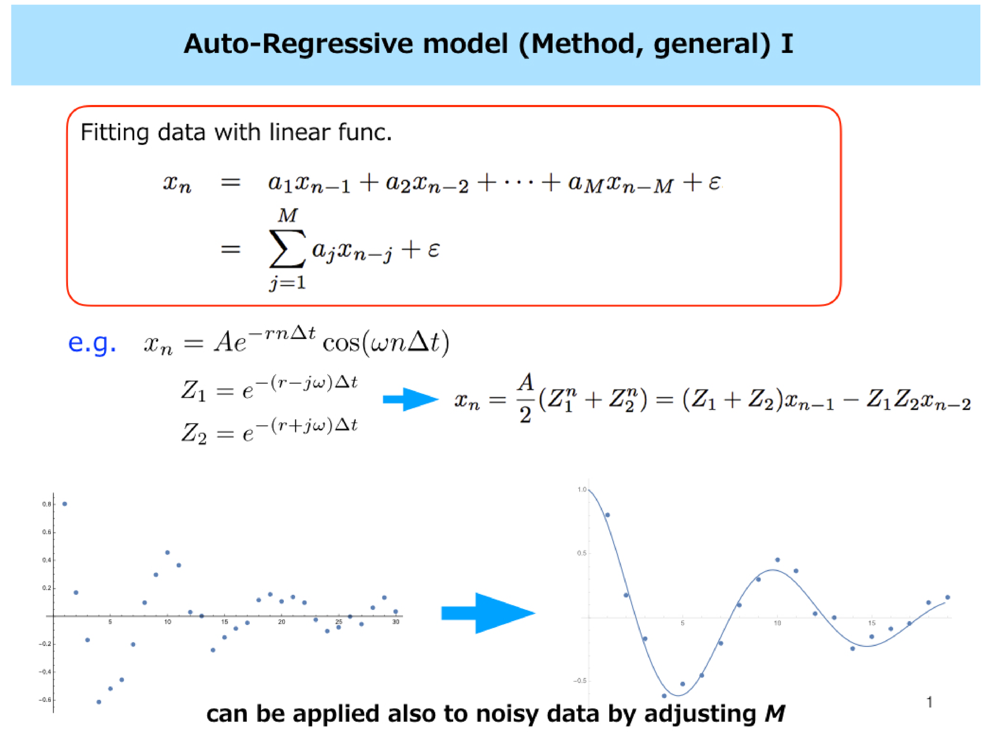


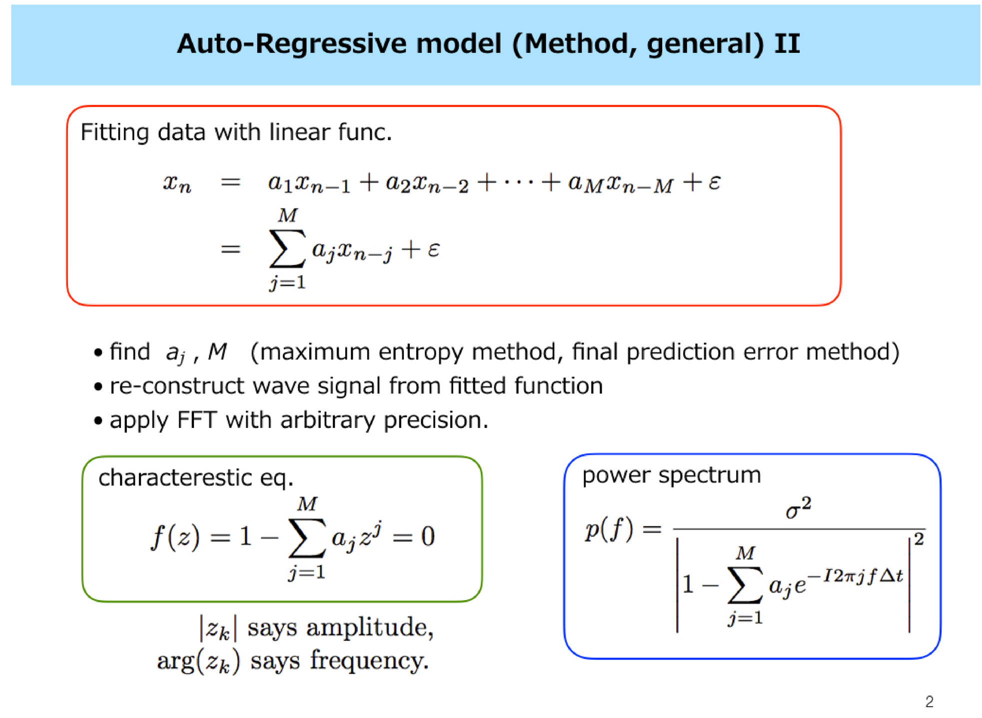
Hisa-aki Shinkai (Osaka Inst. Technology)
真貝寿明（大阪工業大学）

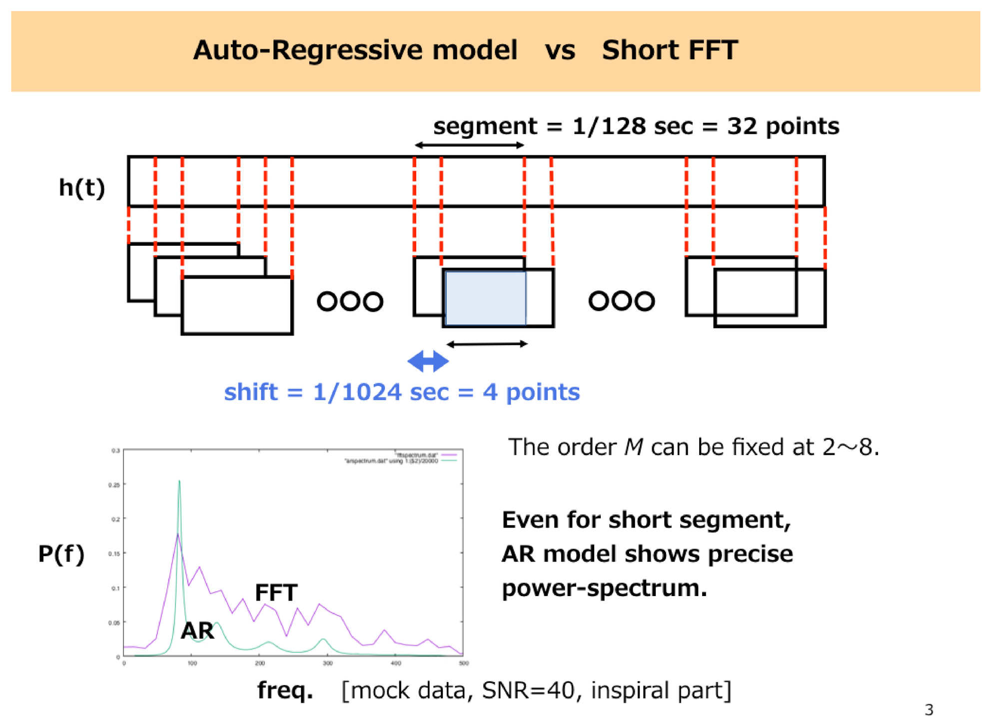
Outline & Summary

We apply an autoregressive (AR) model for identifying the ring-down part of gravitational wave of binary black-hole mergers.
This approach enables us to extract signals without templates, and is effective for short-period data.
After having experience of parameters using mock data, we apply to extract the ring-down frequency of the remnant black-hole of GW150914, GW170104, and GW170814, of which ring-down waves are expected to be included in LIGO data.
We find that AR analysis extracts ringdown part for GW150914 and GW170814 with consistent mass and spin of the remnant BH which were reported by LIGO/Virgo group. However, we failed for GW170104, which might be due to the small S/N (=13) compared to the others (S/N=23.7 & 18).

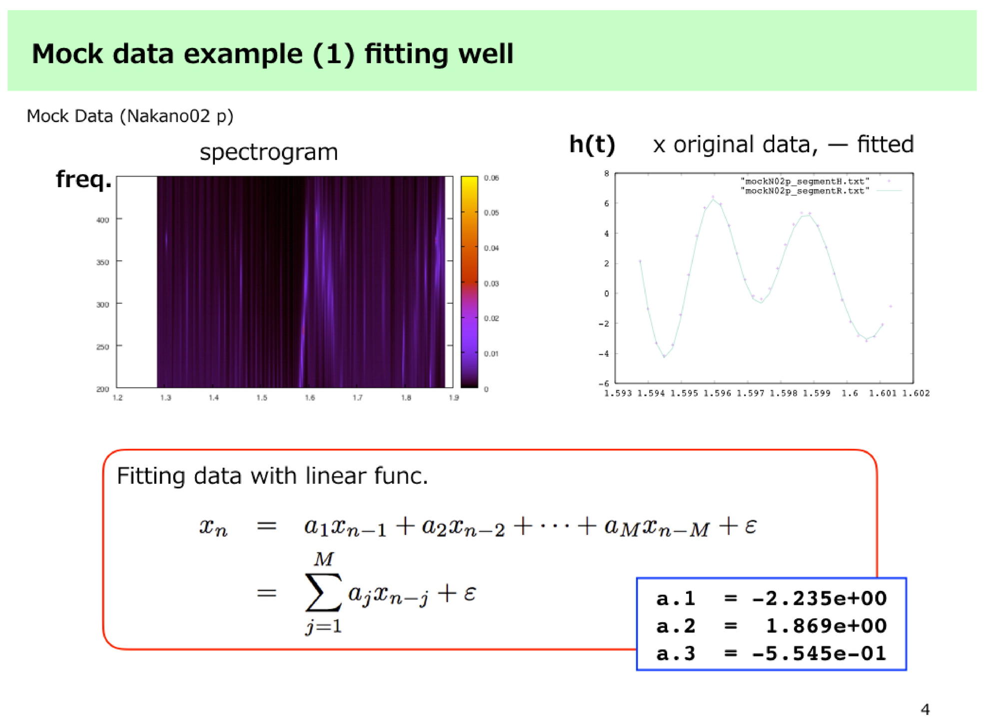
Method (general)

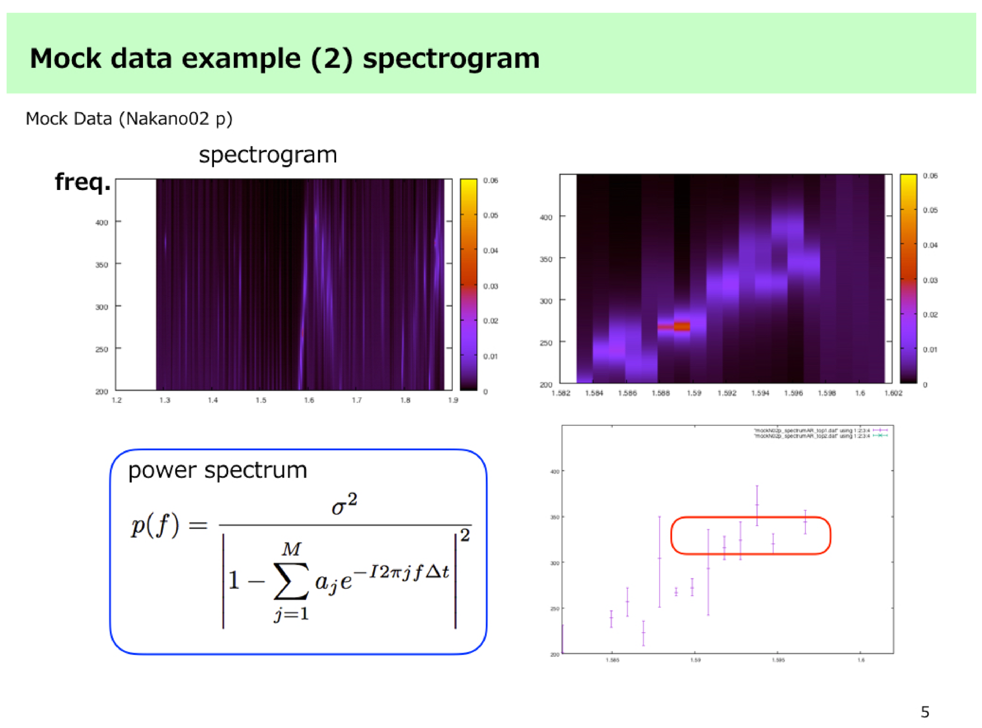


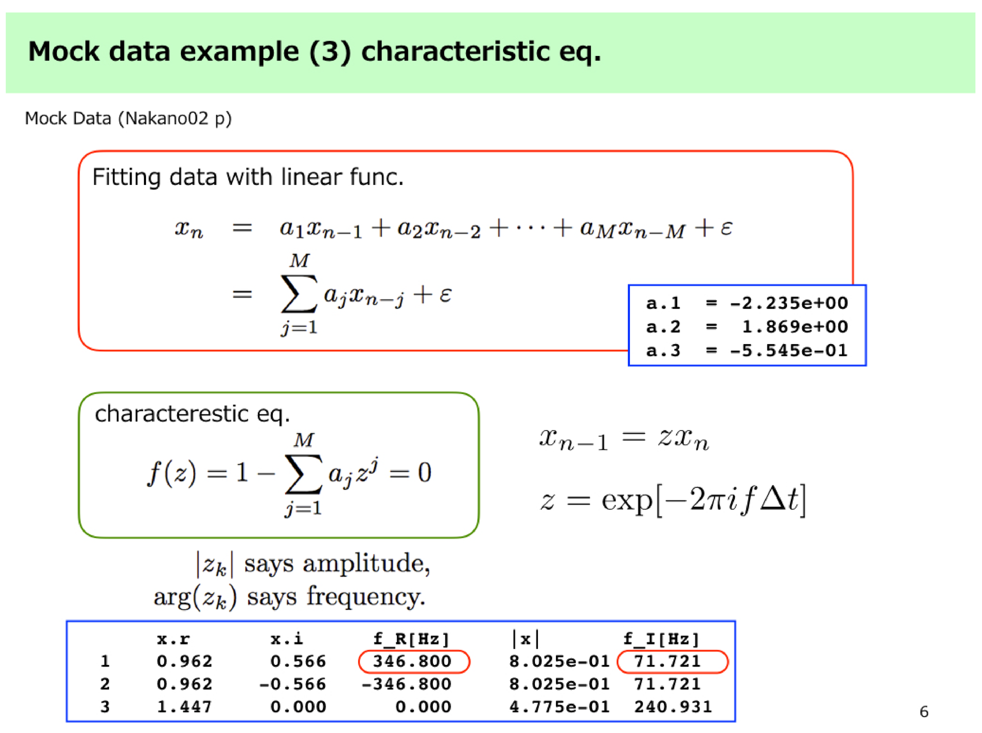


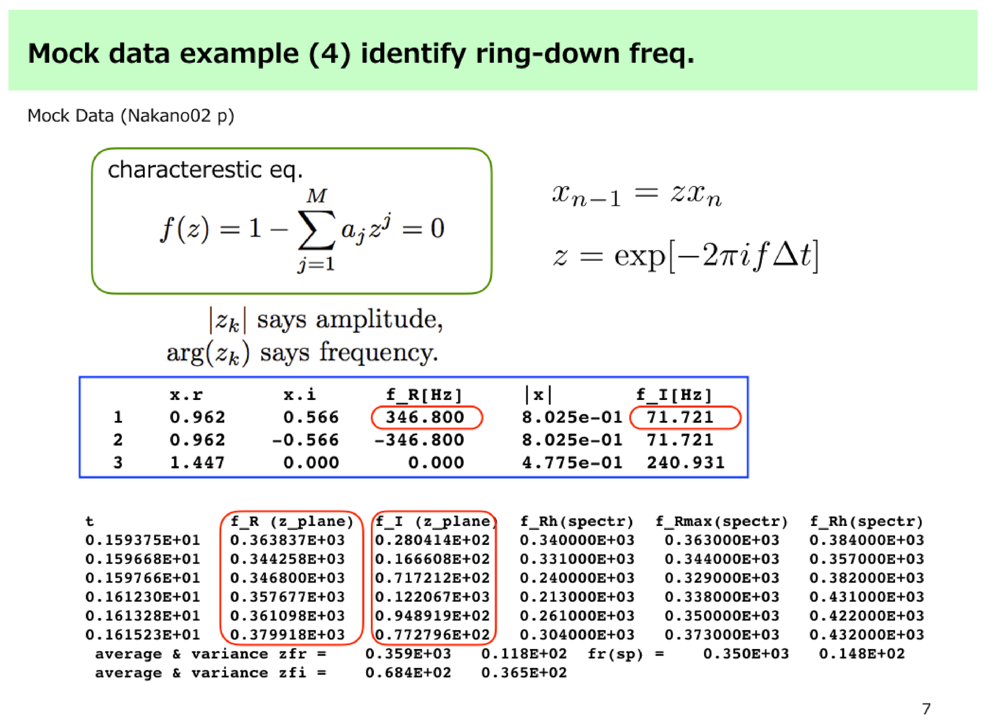


Example with mock data

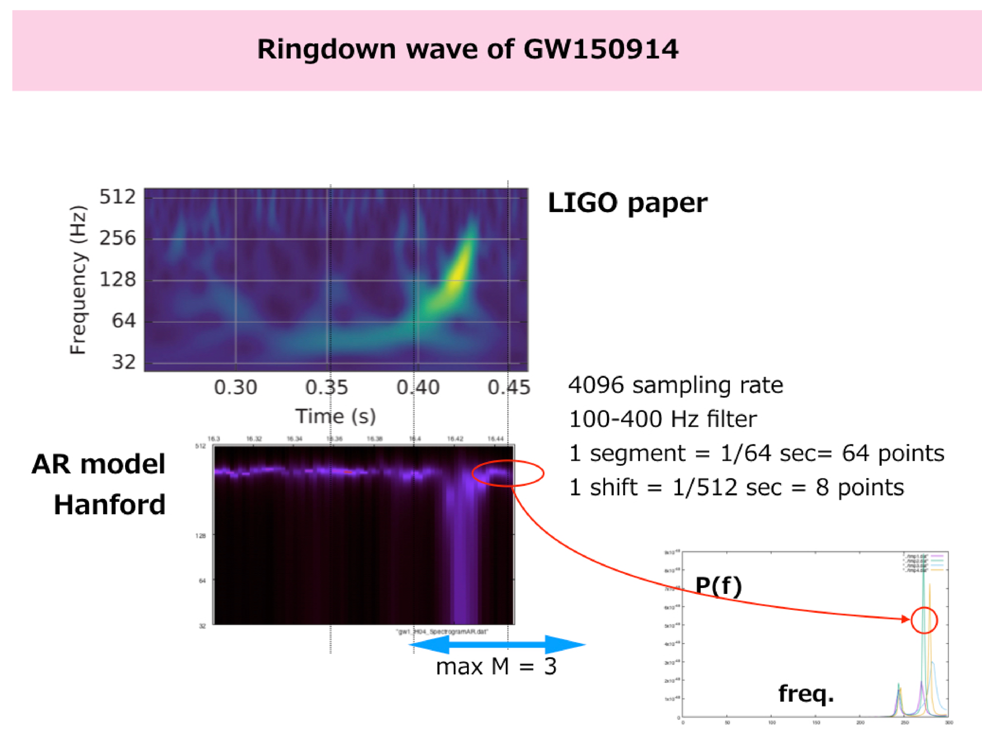


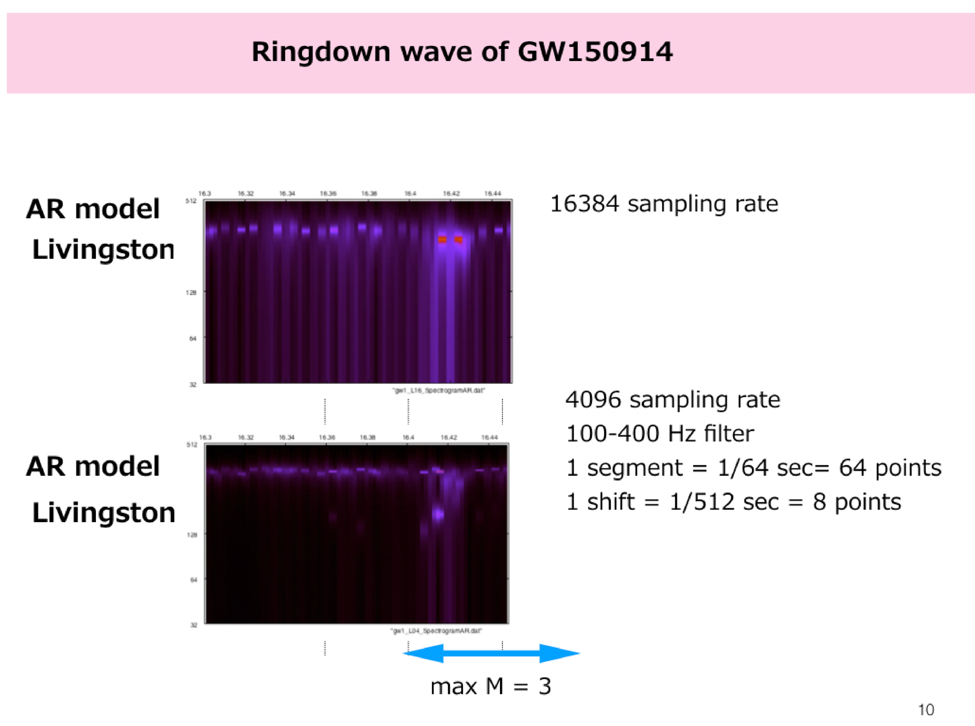






Application to GW150914, GW170104, GW170814





We apply AR method for GW150914. We downloaded the data from the Ligo site[2], of its 32-second data of 4096 sampling-rate taken at both Hanford and Livingston observatories. We used the data from $t = 8$ to $t = 24$. We applied a bandpass filter for 100 Hz $\sim$ 400 Hz, and made a segment of 1/64 second (64 data points) shifting them with 1/512 second (8 data points).

We then get the power-spectrum $p(f)$ from eq. (2) at each segment, and the number of its local maximum are less than 3 for the data $t = 16.4$ to $t = 17.4$. We name the frequencies $f_1, f_2, \cdots$ which give the local maximum of the spectrum. We also solve (4) at each segment (which is at most 10-th order polynomial equation), and identify the solution z_k of which real part of frequency is mostly close to the one obtained as $f_1, f_2, \cdots$. We list the solutions z_k of each segment which are candidates for ring-down modes, and check whether these candidates are within a close value over several segments. We found that sometimes a segment is full of noises and shows quite different numbers from continual segments. We excluded such data if it shows the peak frequency is one-sigma apart from the list.

In result, we conclude that 6 segments of Hanford data (from $t = 16.4258$ to 16.4312), and 8 segments of Livingston data (from $t = 16.4121$ to 16.4473) have a consistent frequency and damping rate. We obtain the ring-down frequency of the remnant BH of GW150914 as in Table 1. The results indicate the mass and spin of the BH, which are consistent with the values reported originally[1], as $62.2^{+3.7}_{-3.4}\ M_\odot$ and $a/M = 0.68^{+0.05}_{-0.06}$.

Ringdown wave of GW150914

t f real f imag

```
 4486     1   0.164258E+02   0.313508E+03   0.432791E+02   0.259000E+03   0.305000E+03   0.337000E+03
 4488xx   2   0.164297E+02   0.300353E+03   0.578188E+02   0.218000E+03   0.287000E+03   0.330000E+03
 4489     1   0.164316E+02   0.317507E+03   0.382182E+02   0.274000E+03   0.311000E+03   0.339000E+03
 4498     1   0.164492E+02   0.314336E+03   0.538556E+02   0.261000E+03   0.309000E+03   0.349000E+03
 4501     1   0.164551E+02   0.317640E+03   0.349751E+02   0.282000E+03   0.314000E+03   0.340000E+03
 4505     1   0.164629E+02   0.316355E+03   0.429281E+02   0.277000E+03   0.314000E+03   0.346000E+03
 4508     1   0.164688E+02   0.311752E+03   0.297619E+02   0.285000E+03   0.310000E+03   0.332000E+03
data points =     7
average & variance zfr =    0.313E+03   0.556E+01  fr(sp) =    0.307E+03   0.871E+01
average & variance zfi =    0.430E+02   0.926E+01
```

we see QNM at 300Hz, 0.003s after the merger.

Table 1. Results of frequency and damping rate of ring-down gravitational wave of GW150914.

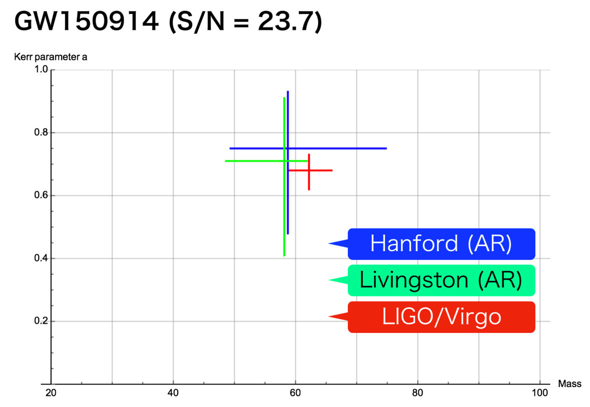
data	f_{real}[Hz]	f_{imag}[Hz]	mass ($M/M_\odot$)	Kerr parameter a/M
Hanford	$305.94^{+18.68}_{-27.82}$	$43.55^{+13.00}_{-17.99}$	$58.74^{+16.03}_{-9.37}$	$0.75^{+0.18}_{-0.27}$
Livingston	$300.02^{+17.49}_{-27.21}$	$44.94^{+12.88}_{-18.30}$	$58.15^{+16.49}_{-9.53}$	$0.71^{+0.20}_{-0.30}$

LIGO paper says $62.2^{+3.7}_{-3.4}\ M_\odot$ $a/M = 0.68^{+0.05}_{-0.06}$

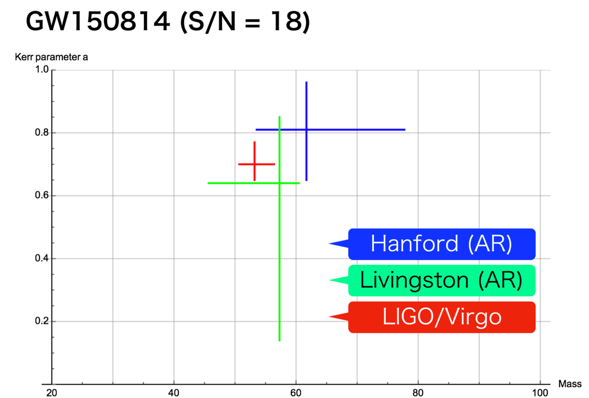
List of Detected GW events

	ref.	M1+M2=Mf, Mdiff/Mtotal	spin a_final	Mpc z	SNR	deg^2
GW150914	PRL116, 061102 (2016/2/11)	36.2+29.1=62.3+3.0 **4.59%**	0.68	410Mpc 0.09	**23.7**	600
LVT151012	(2016/2/11)	23+13=35+1.5 **2.78%**	0.66	1000Mpc 0.20	9.7	
GW151226	PRL116, 241103 (2016/6/15)	14.2+7.5=20.8+0.9 **4.15%**	0.74	440Mpc 0.09	13.0	850
GW170104	PRL118, 221101 (2017/6/1)	31.2+19.4=48.7+1.9 **3.75%**	0.64	880Mpc 0.18	**13**	1300
GW170608	ApJ 851, L35 (2017/12/18)	12+7=18.0+1.0 **5.2%**	0.69	340Mpc 0.07	13	520
GW170814	PRL119,141101 (2017/10/6)	30.5+25.3=53.2+2.6 **4.66%**	0.70	540Mpc 0.11	**18**	60
GW170817	PRL119, 161101 (2017/10/16)	1.36〜1.60 + 1.17〜1.36 = 2.74 + ?	?	40Mpc	32.4	28

GW150914 (S/N = 23.7)



GW150814 (S/N = 18)



GW170104 (S/N = 13)