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General Relativity and Quantum Cosmology

Comparison of various methods to extract ringdown frequency from gravitational wave data

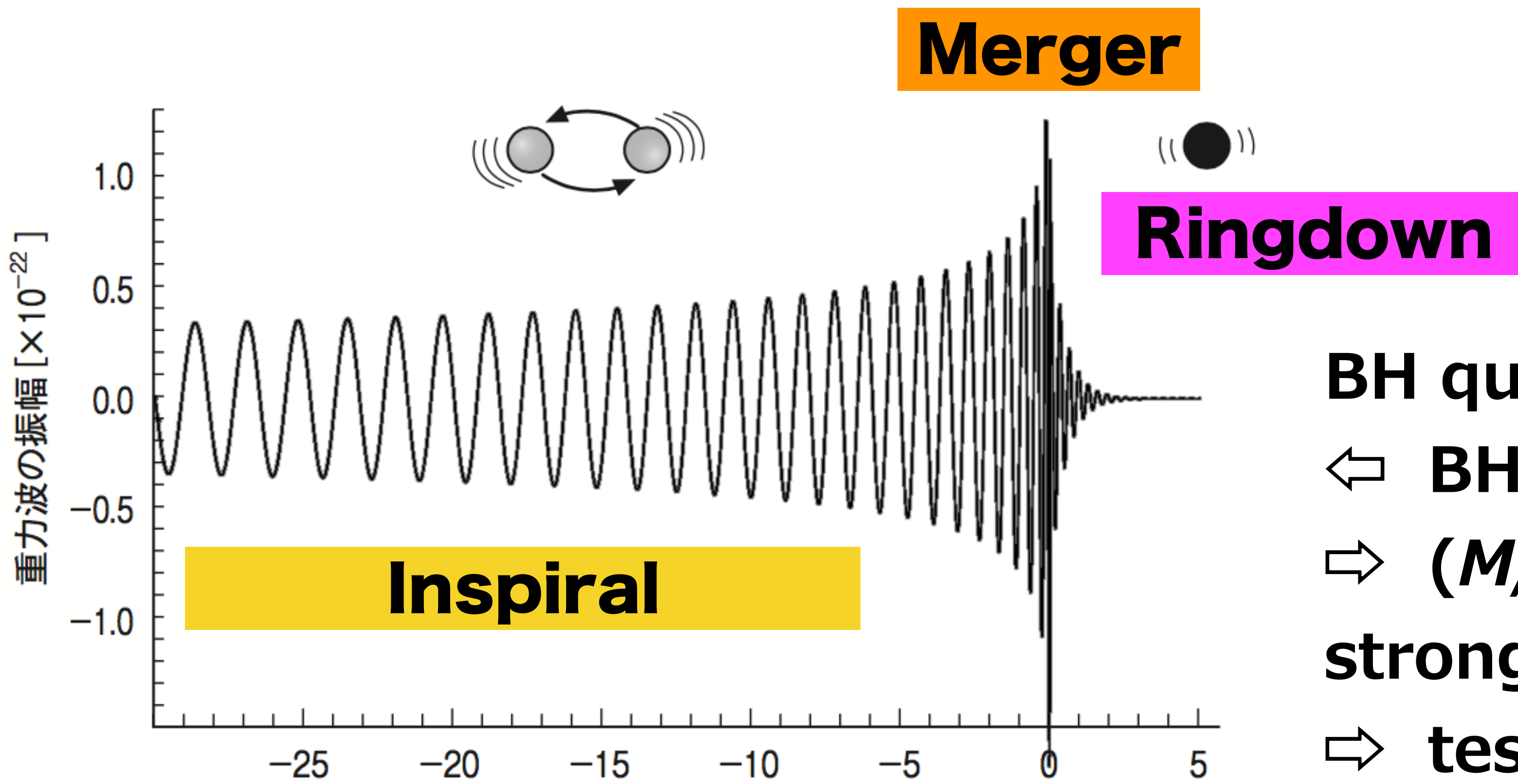
Hiroyuki Nakano, Tatsuya Narikawa, Ken-ichi Oohara, Kazuki Sakai, Hisa-aki Shinkai, Hirotaka Takahashi, Takahiro Tanaka, Nami Uchikata, Shun Yamamoto, Takahiro S. Yamamoto

(Submitted on 15 Nov 2018)

1. Matched-filtering method (ringdown)
2. Matched-filtering method (merger+ringdown)
3. Hilbert-Huang transformation method
4. Auto-Regressive method
5. Neural network method

https://gw-genesis.scphys.kyoto-u.ac.jp/ilias/goto_root_fold_669.html
http://www.oit.ac.jp/is/~shinkai/mockdatachallenge/

Towards testing gravity theories ⇒ Ringdown-part extraction is a key



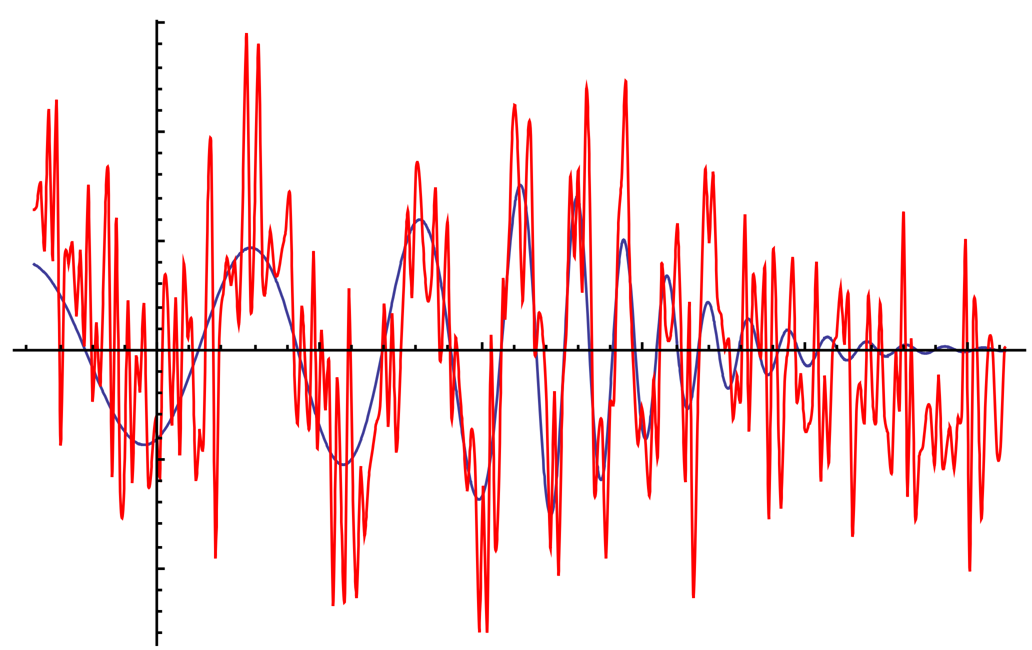
BH quasi-normal modes

⇦ BH perturbation theory

⇨ (*M*, *a*)

strongest gravity we can observe

⇨ test of gravity theories



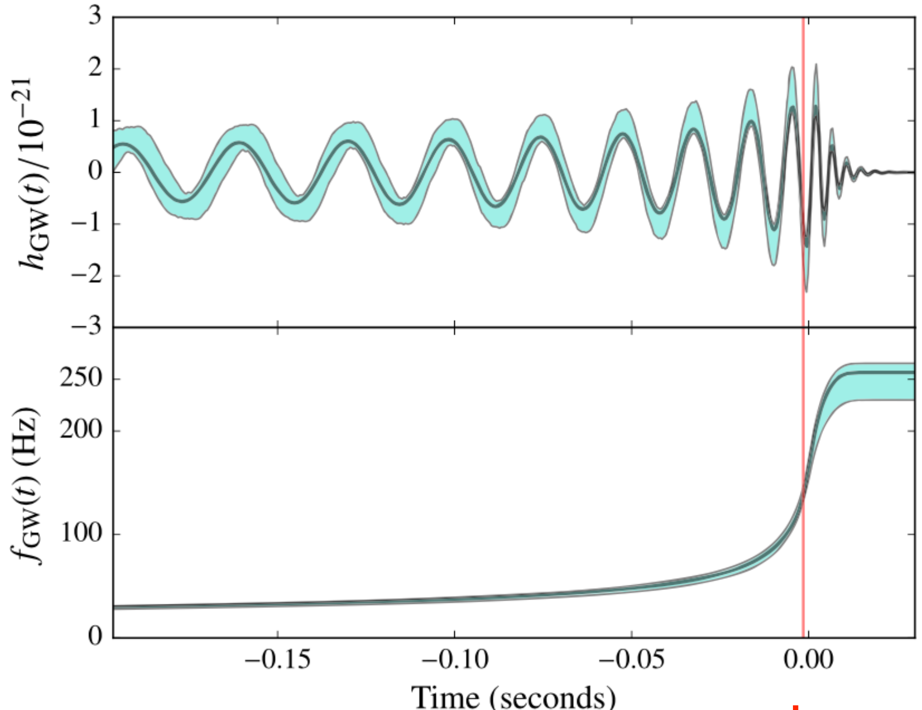
For *60M* BH of *a*=0.75,

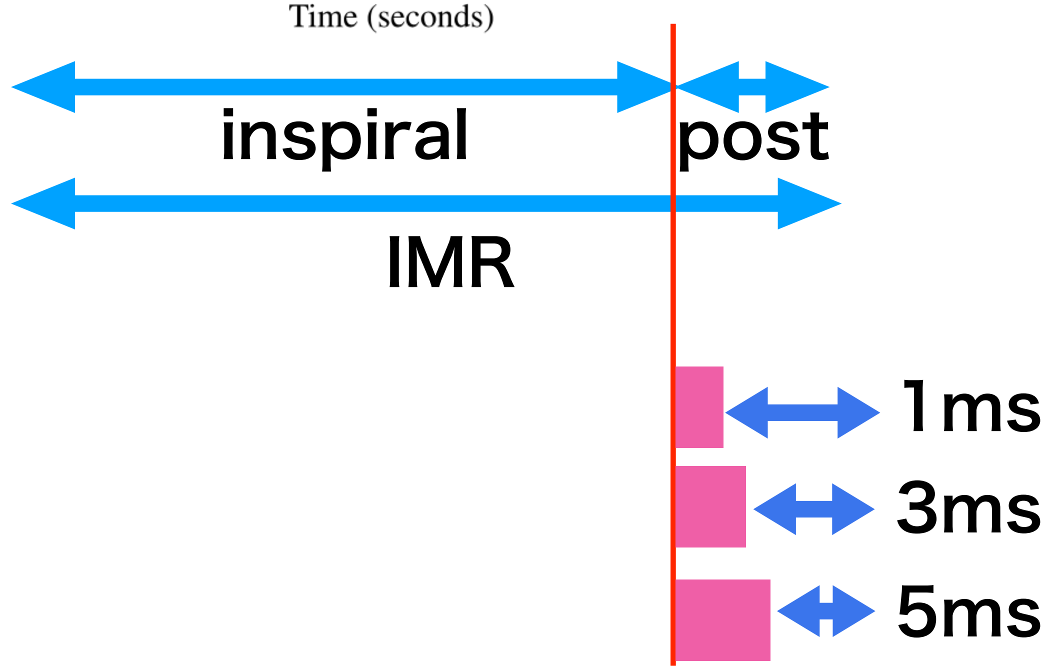
frequency = 300 Hz

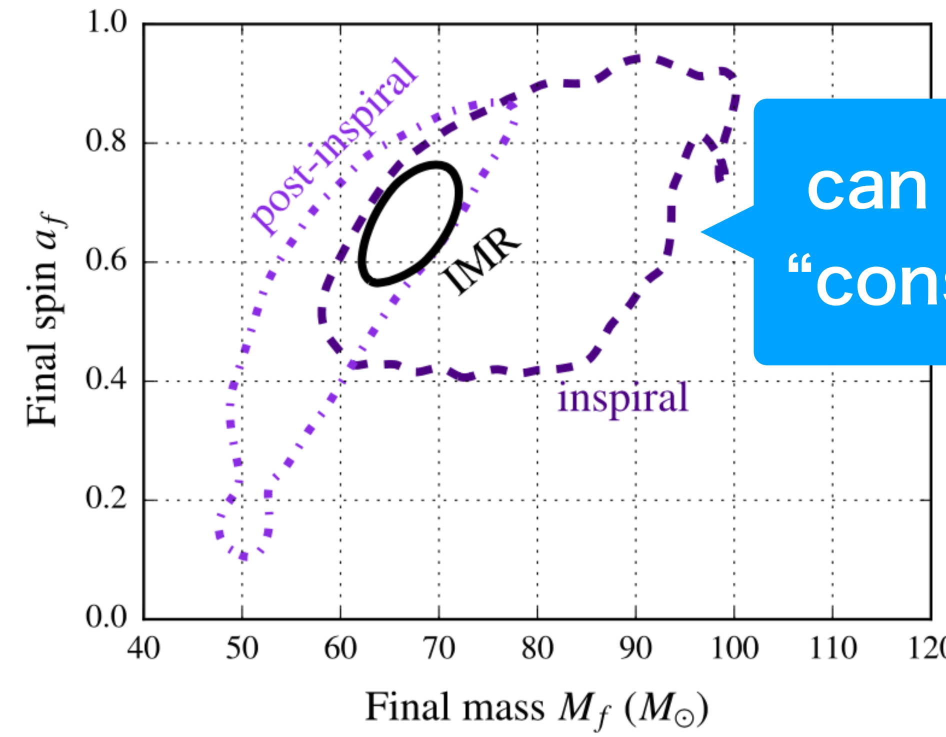
damping time scale = 3.7 ms

LIGO/Virgo, GW150914

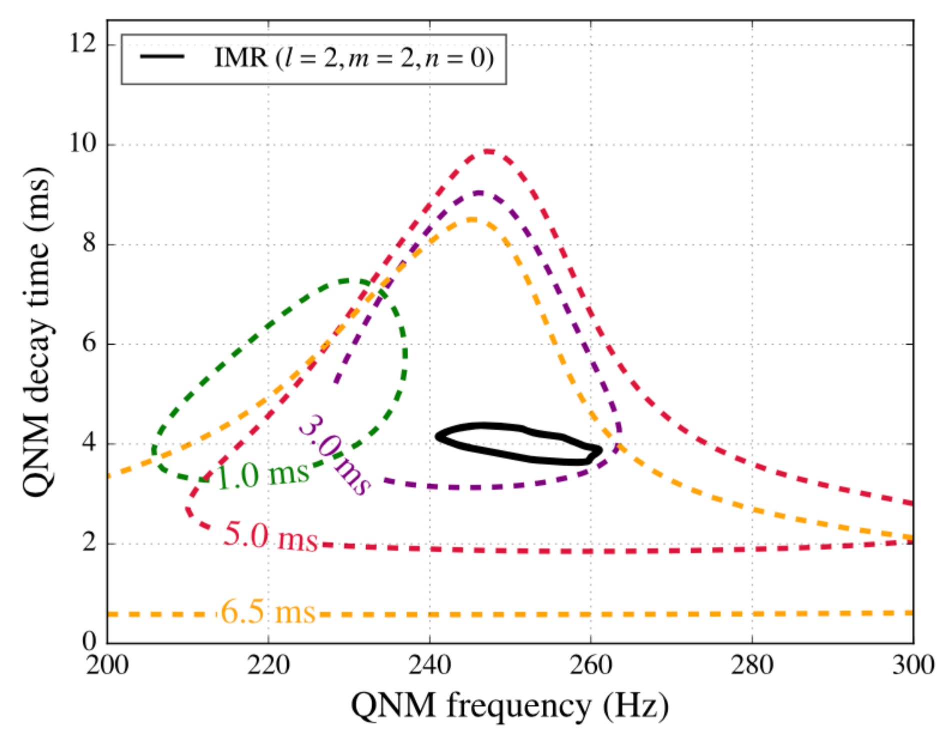
LIGO/Virgo, PRL 116 (2016) 221101; PRL 121 (2018) 129902E [1602.03841]







can say only “consistency”



Mockdata preparation

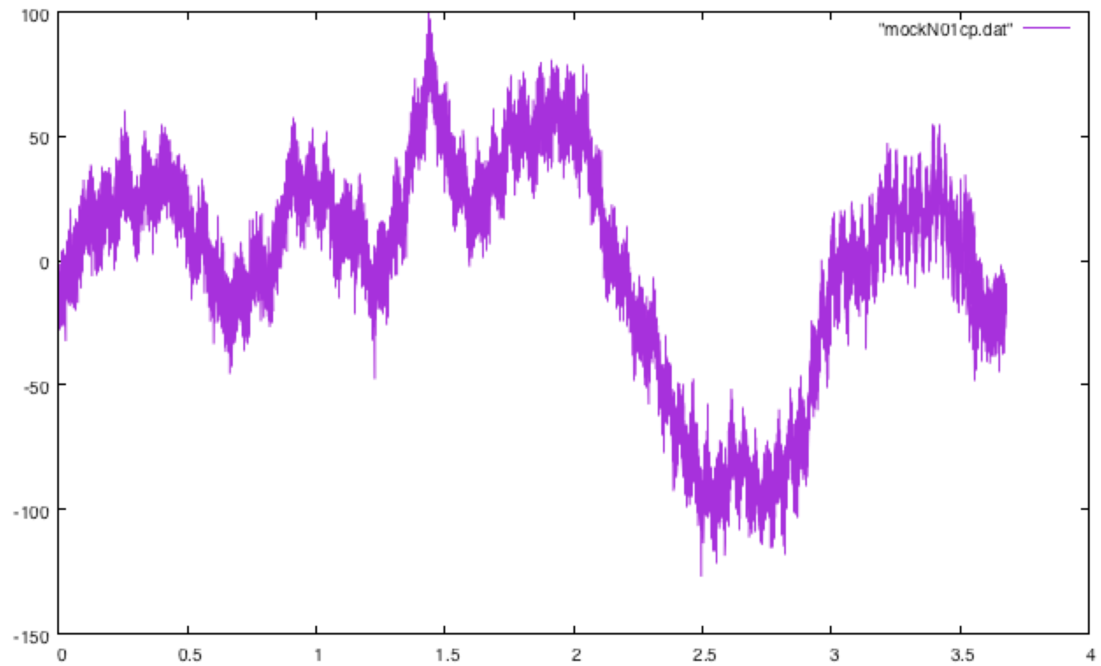
SXS data + shifted ringdown injection + aLIGO noise

modified after t_merger (set A)

modified before/after t_merger (set B)

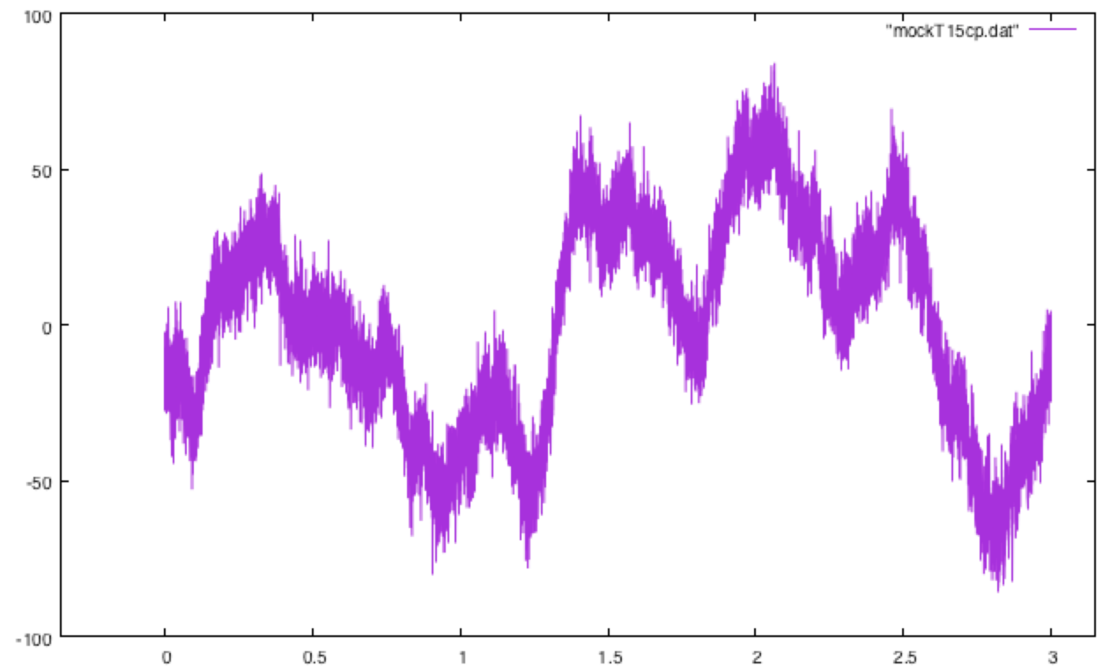
SNR=60 (total)

SNRrd = 13.81



SNR=20 (total)

SNRrd = 5.85



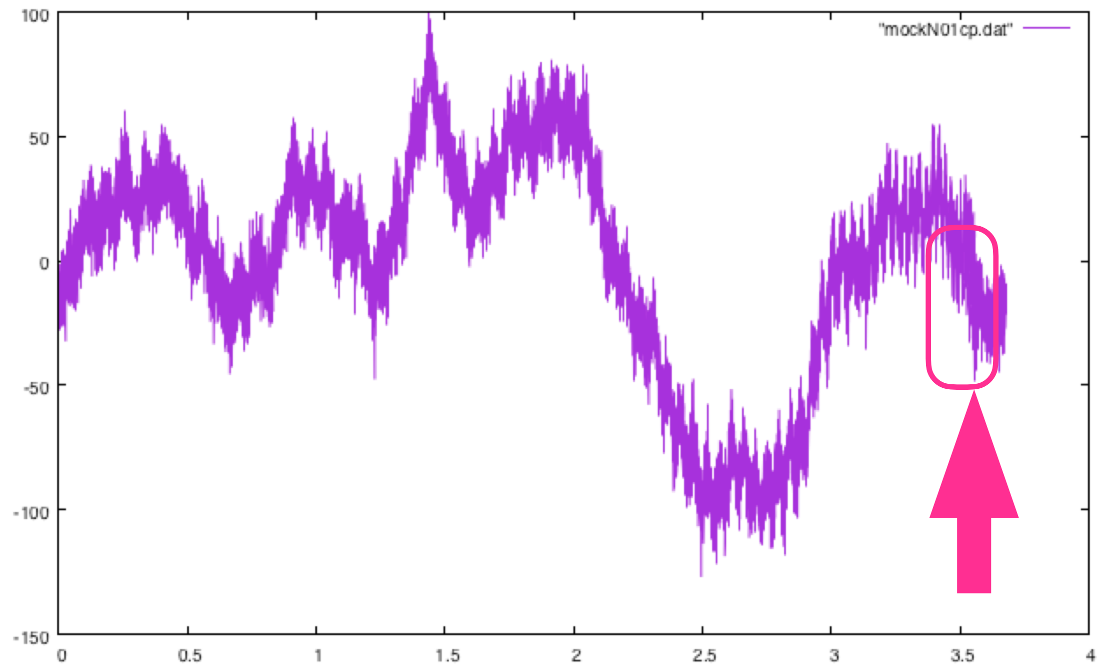
data	SNR		injected	
	ρ_{all}	ρ_{rd}	$f_R^{(inj)}$	$f_I^{(inj)}$
A-01	60.0	13.81	260.68	44.58
A-02	60.0	12.73	345.16	50.49
A-03	60.0	13.79	382.53	32.58
A-04	60.0	11.84	284.18	44.73
A-05	60.0	16.78	346.20	23.07
A-06	30.0	5.57	272.85	33.40
A-07	30.0	6.56	272.85	44.54
A-08	30.0	7.27	301.89	42.24
A-09	30.0	6.93	324.60	27.25
A-10	30.0	7.88	282.55	37.45
A-11	20.0	6.36	314.24	30.58
A-12	20.0	3.45	382.10	48.60
A-13	20.0	4.68	249.36	47.97
A-14	20.0	4.13	299.32	41.88
A-15	20.0	4.54	319.42	31.55
B-01	60.0	17.58	352.56	36.20
B-02	60.0	14.27	210.78	42.77
B-03	60.0	13.67	258.83	48.42
B-04	60.0	20.09	271.13	25.40
B-05	60.0	17.07	291.99	34.20
B-06	30.0	9.53	411.57	29.48
B-07	30.0	6.29	295.78	59.38
B-08	30.0	6.03	312.39	59.24
B-09	30.0	6.01	198.34	57.91
B-10	30.0	8.31	323.32	37.86
B-11	20.0	5.20	208.80	39.75
B-12	20.0	6.60	246.66	27.85
B-13	20.0	4.46	323.71	62.51
B-14	20.0	6.20	215.15	33.15
B-15	20.0	5.85	335.20	25.11

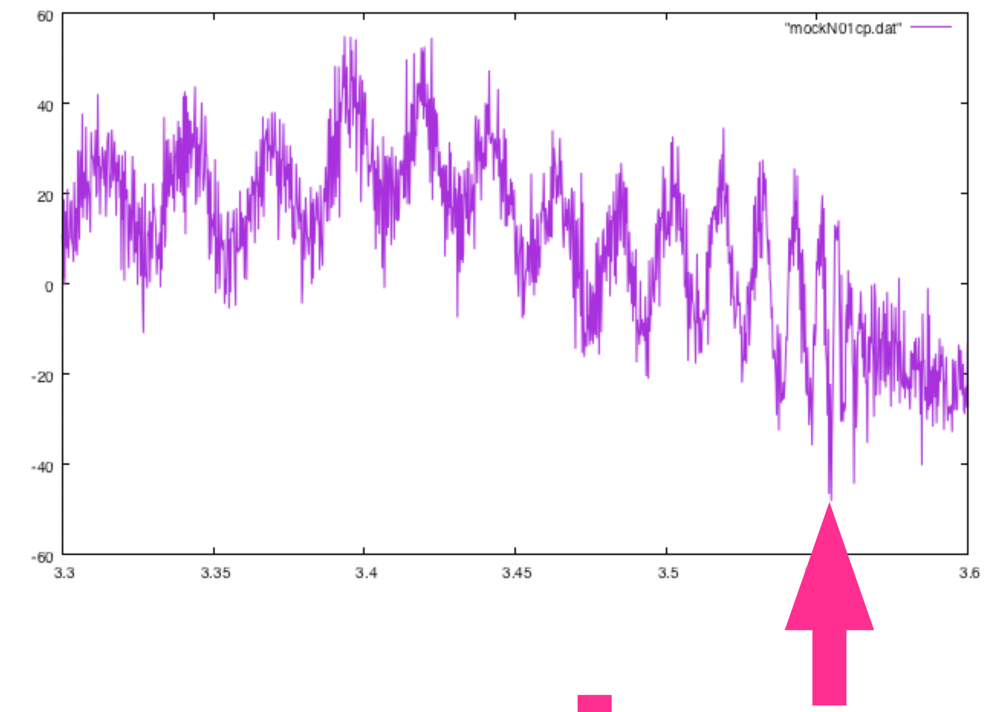
Mockdata preparation

SXS data + shifted ringdown injection + aLIGO noise

SNR=60 (total)

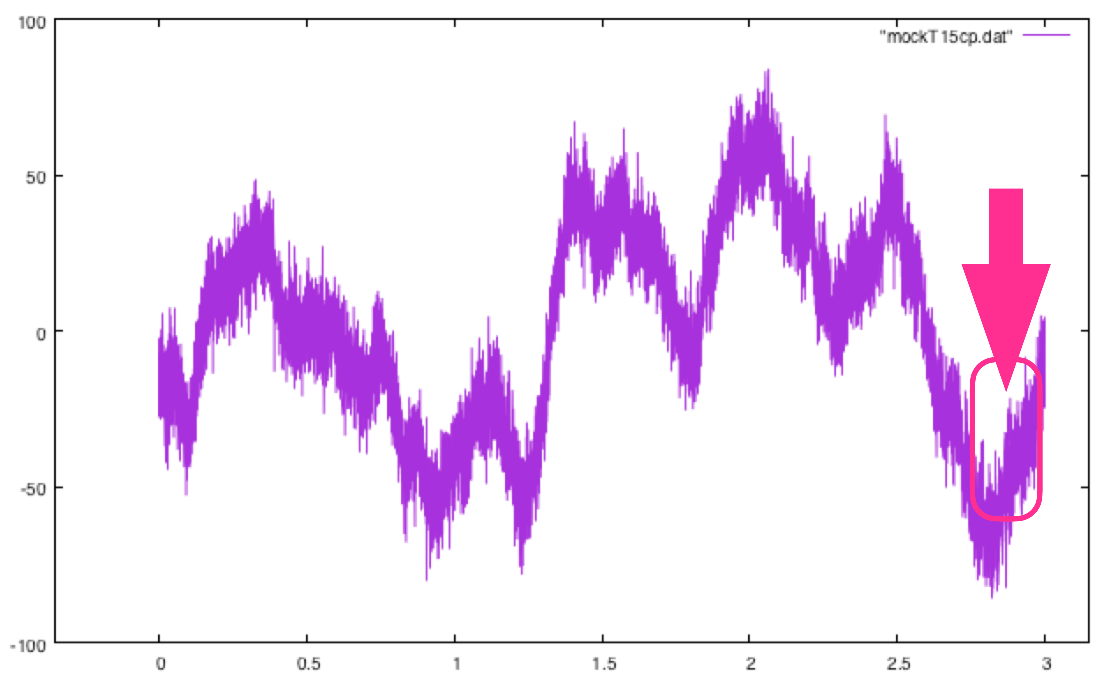
SNRrd = 13.81

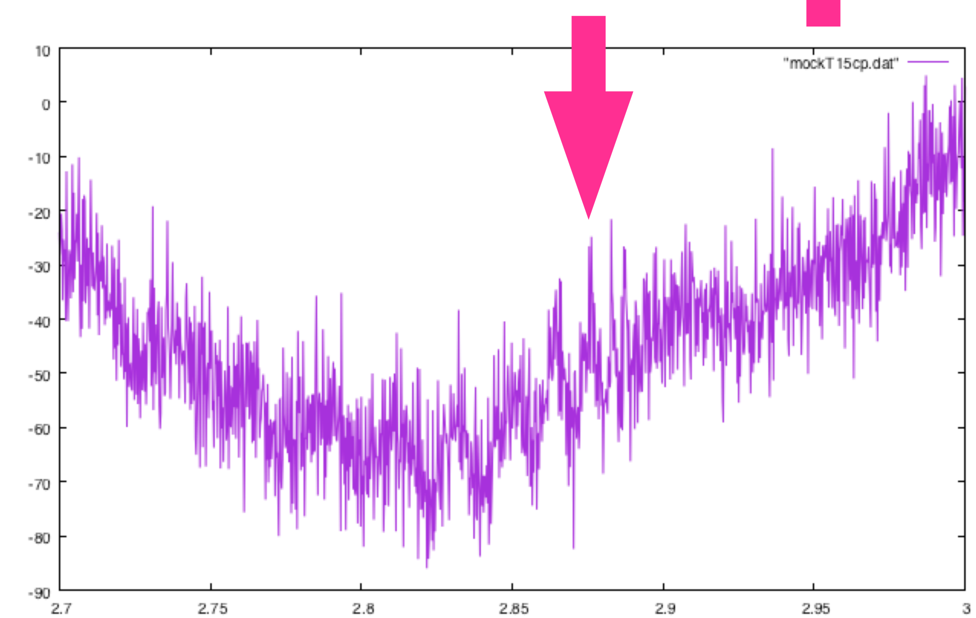




SNR=20 (total)

SNRrd = 5.85





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http://www.oit.ac.jp/is/~shinkai/mockdatachallenge/

method 1 :: Matched Filtering

template

$$(h_1|h_2) = 2\int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$$

data

noise spectral density

Matched Filtering (ringdown only) by Uchikata 内潟

template

$$\hat{h}(t) = \begin{cases} 0 & (t < t_0) \\ \frac{1}{N} e^{-\omega_{\rm I}(t-t_0)/Q} \cos[\omega_{\rm R}(t-t_0) - \phi_0] & (t \geq t_0) \end{cases}$$

Matched Filtering (merger + ringdown) by Tanaka 田中

smoothly connected templates

introducing two more parameters:

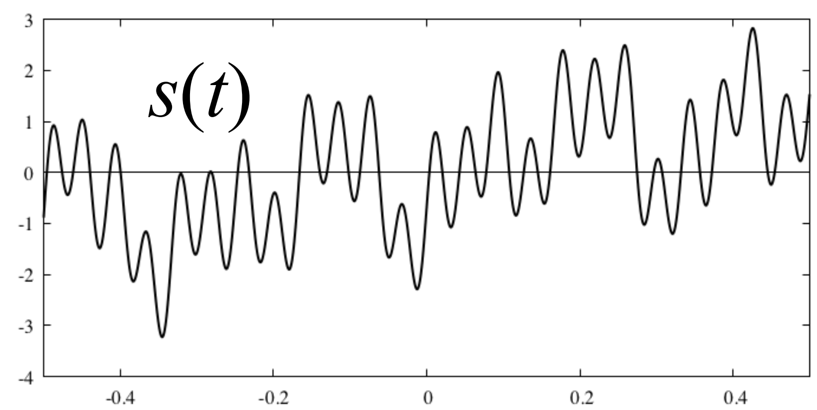
- relative amplitude of ringdown vs inspiral/merger
- transition rapidity of ringdown from merger

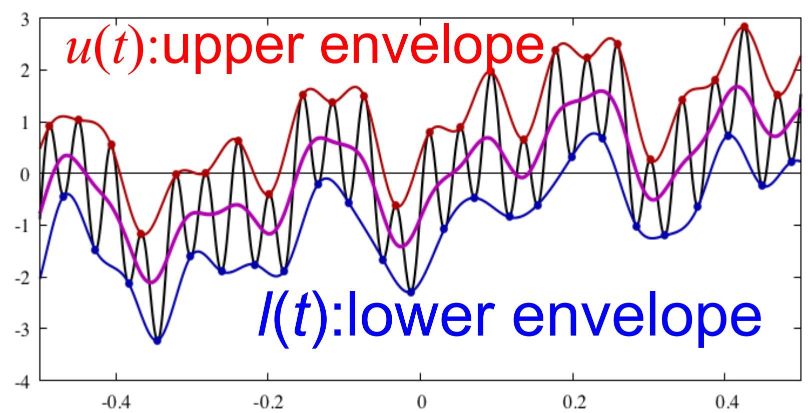
should have best performance for set B

method 2 :: Hilbert-Huang transformation (HHT)

by Oohara-Takahashi-Sakai 大原・高橋・酒井

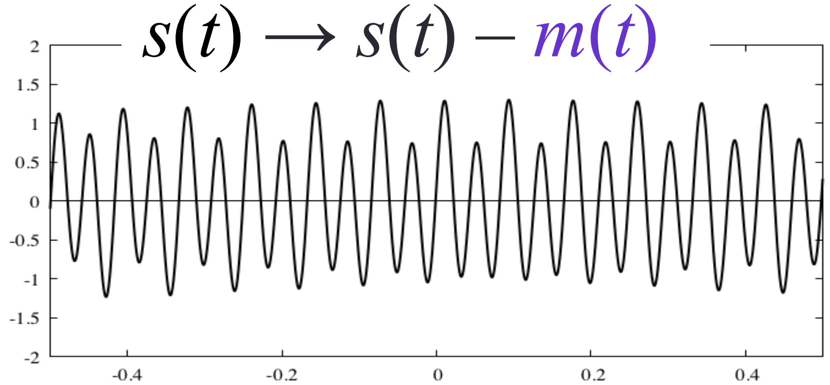
Empirical Mode decomposition





$m(t) := (u(t) + l(t))/2$

$s(t) \rightarrow s(t) - m(t)$



Hilbert-Spectral Analysis

$$v(t) = \frac{1}{\pi} P \int_{\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau$$

$$a(t) e^{i\theta(t)} = s(t) + iv(t)$$

We extract f_R (from $\theta(t)$) and f_I (from $a(t)$).

The choice of initial filtering band [f_L, f_H] is a little ad hoc.

We drop high and low frequency modes by filtering the data [f_L, f_H]

method 3 :: Auto-Regressive method (AR)

by Shinkai-S.Yamamoto 真貝・山本

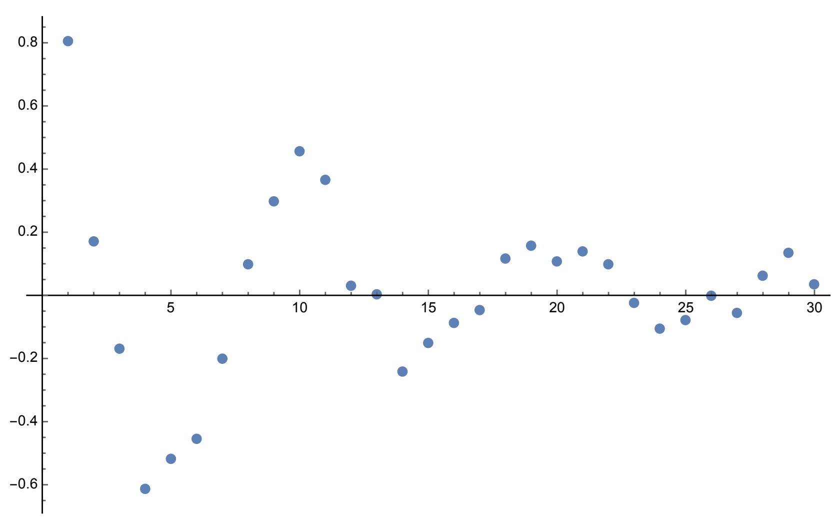
Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$
$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

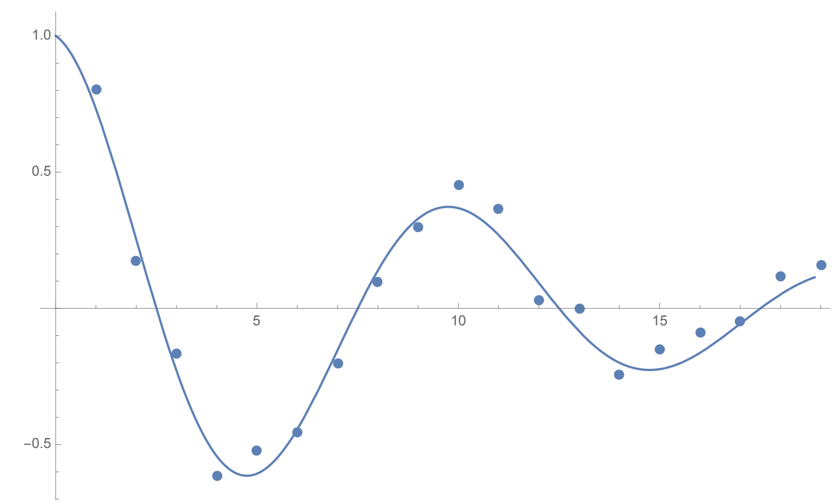
e.g. $x_n = Ae^{-rn\Delta t}\cos(\omega n \Delta t)$

$$Z_1 = e^{-(r-j\omega)\Delta t}$$
$$Z_2 = e^{-(r+j\omega)\Delta t}$$

→ $x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1 + Z_2)x_{n-1} - Z_1 Z_2 x_{n-2}$



→



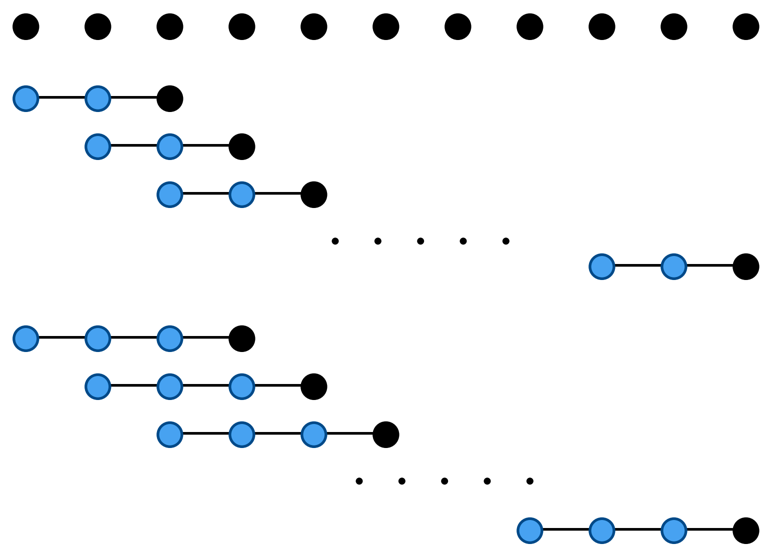
can be applied also to noisy data by adjusting *M*

method 3 :: Auto-Regressive method (AR)

Fitting data with linear func.

$$\begin{aligned} x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\ &= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \end{aligned}$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.



power spectrum

$$p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t} \right|^2}$$

method 3 :: Auto-Regressive method (AR)

Fitting data with linear func.

$$
\begin{aligned}
x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\
&= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon
\end{aligned}
$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

$$
p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t} \right|^2}
$$

characteristic eq.

$$
f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0
$$

$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

method 4 :: Neural Network method (NN)

by T.S.Yamamoto 山本

Convolutional layer

$$z'_{i,l'} = \sum_{l=1}^{L} \sum_{p=1}^{H} z_{i+p,l} h^l_{p,l'} + b_{i,l'}$$

Pooling layer

$$z'_i = \max_{k=1,\dots,p} z_{si+k}$$

Rectified linear unit (ReLU)

$$z' = h(z) = \max(z, 0)$$

Dense layer

$$z'_i = \sum_{j=1}^{N} w_{ij} z_j + b_i$$

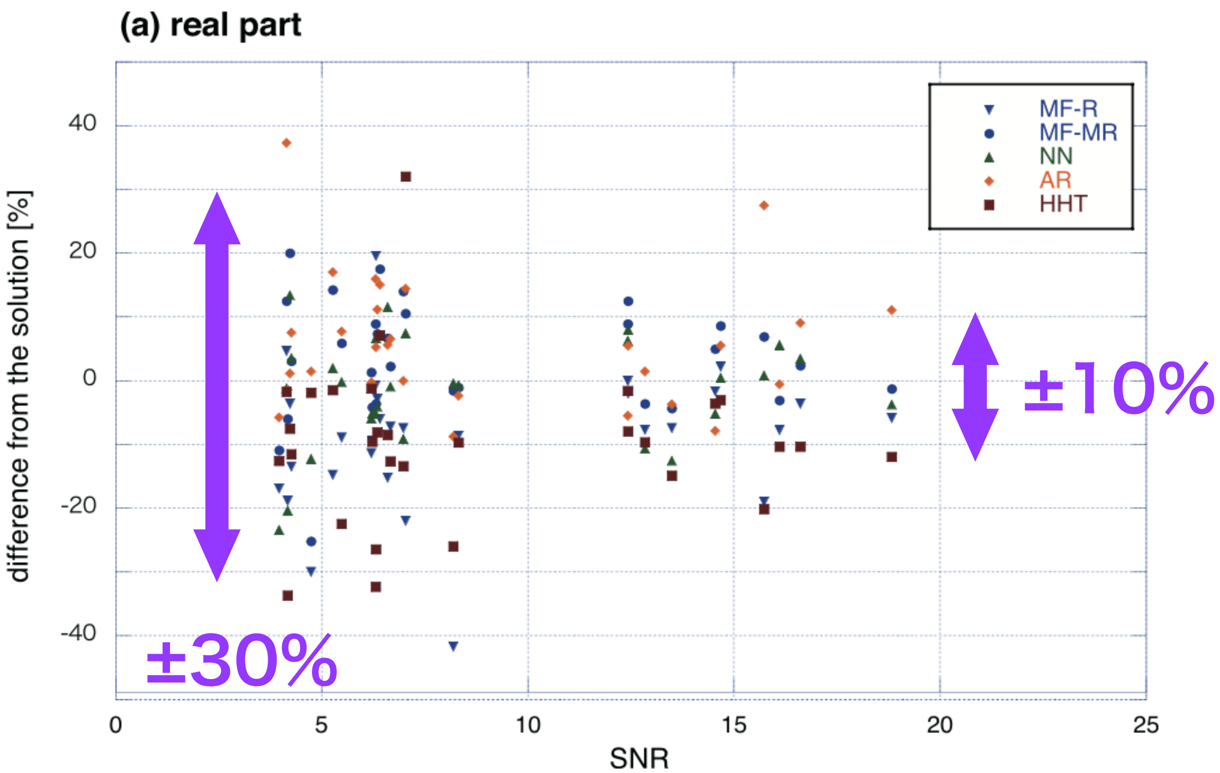
layer	dimension
Input	(256, 2)
Conv	(256, 64)
Pooling	(128, 64)
ReLU	(128, 64)
Conv	(128, 128)
Pooling	(64, 128)
ReLU	(64, 128)
Conv	(64, 256)
Pooling	(32, 256)
ReLU	(32, 256)
Conv	(32, 512)
ReLU	(32, 512)
Flatten	32×512
Dense	256
ReLU	256
Dense	2
Output	2

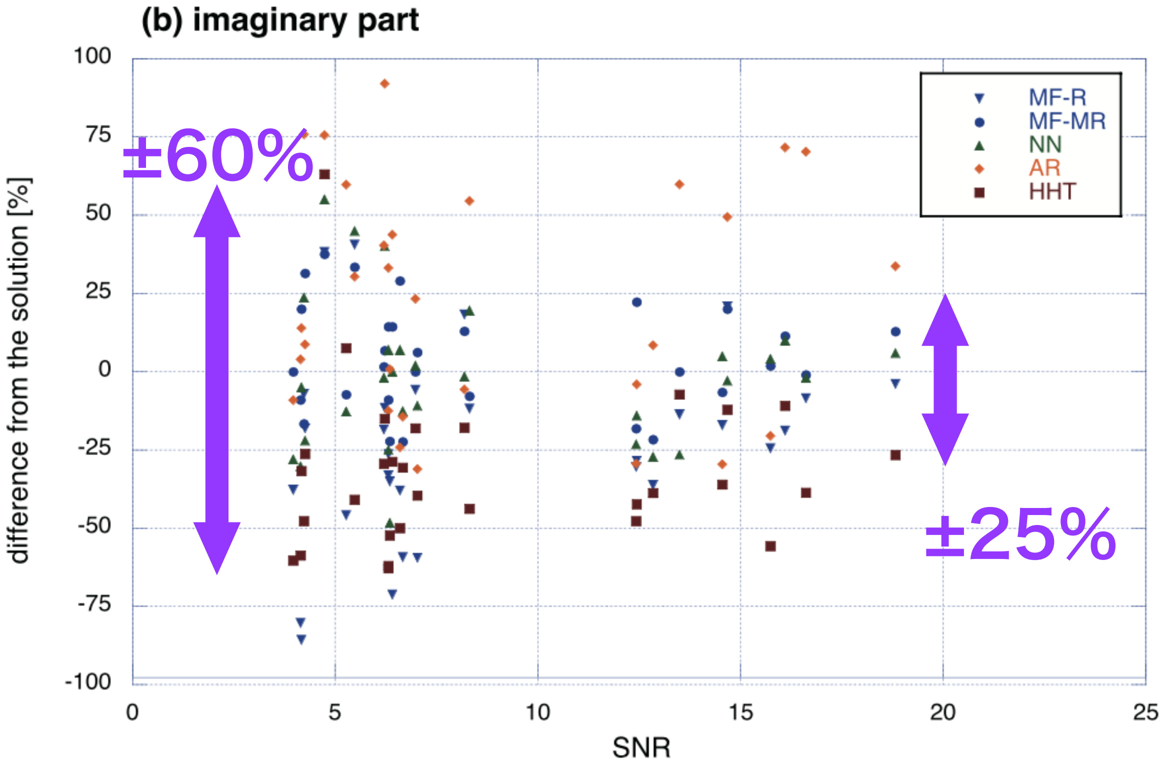
size x channels

Choose the coefficients
h, w and b
to minimize loss function

$$J(y, t) = \frac{1}{M} \sum_{m=1}^{M} (y_m - t_m)^2$$

Comparison





		$\overline{\delta \log f_R}(\%)$	$\sigma(f_R)(\%)$	$\overline{\delta \log f_I}(\%)$	$\sigma(f_I)(\%)$
MF-R	A	-5.96	13.27	-49.02	78.67
	B	-14.27	18.79	-26.63	42.05
MF-MR	A	2.99	12.57	0.55	17.49
	B	2.97	5.62	5.57	15.35
HHT	A	-12.73	18.58	-49.95	63.58
	B	-9.74	16.07	-38.58	44.57
AR	A	4.93	Excellent 10.66	12.85	27.44
	B	6.64	13.58	17.55	38.07
NN	A	-4.50	12.08	-13.48	29.34
	B	0.69	Excellent 4.75	3.36	16.67

- For matched filtering (MF-MR), Set-B template was used.
- Neural network (NN) was trained using set-B template.
- NN slightly exceeds the performance of MF-MR.
- For set A real part, AR showed the best performance.

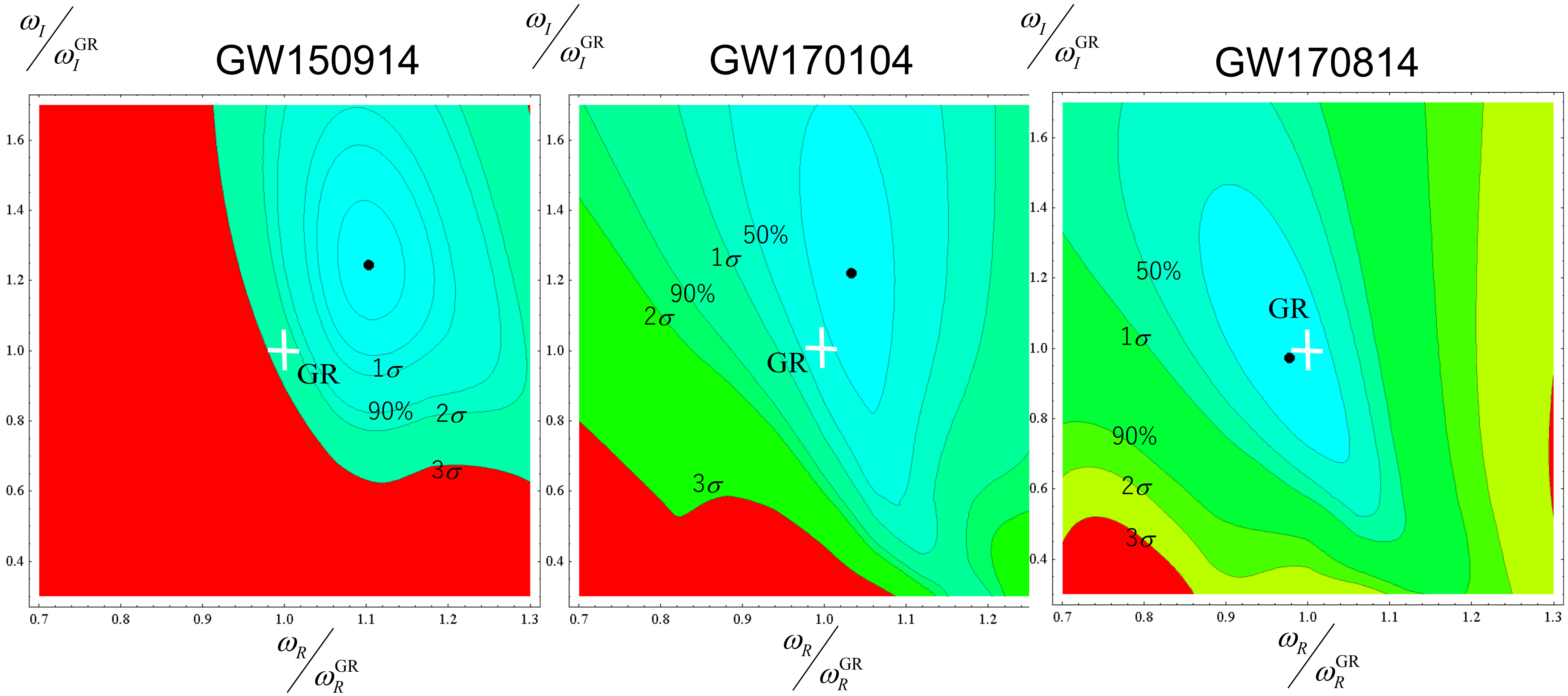
Application to the LIGO/Virgo data

List of Detected GW events

	ref.	M1+M2=Mf, Mdiff/Mtotal	spin a_final	Mpc z	SNR	deg^2
GW150914	PRL116, 061102 (2016/2/11)	36.2+29.1=62.3+3.0 **4.59%**	0.68	410Mpc 0.09	**23.7**	600
LVT151012	(2016/2/11)	23+13=35+1.5 **2.78%**	0.66	1000Mpc 0.20	9.7	
GW151226	PRL116, 241103 (2016/6/15)	14.2+7.5=20.8+0.9 **4.15%**	0.74	440Mpc 0.09	13.0	850
GW170104	PRL118, 221101 (2017/6/1)	31.2+19.4=48.7+1.9 **3.75%**	0.64	880Mpc 0.18	**13**	1300
GW170608	ApJ 851, L35 (2017/12/18)	12+7=18.0+1.0 **5.2%**	0.69	340Mpc 0.07	13	520
GW170814	PRL119,141101 (2017/10/6)	30.5+25.3=53.2+2.6 **4.66%**	0.70	540Mpc 0.11	**18**	60
GW170817	PRL119, 161101 (2017/10/16)	1.36〜1.60 + 1.17〜1.36 = 2.74 + ?	?	40Mpc	32.4	28

Application to the LIGO/Virgo data : improved MF method

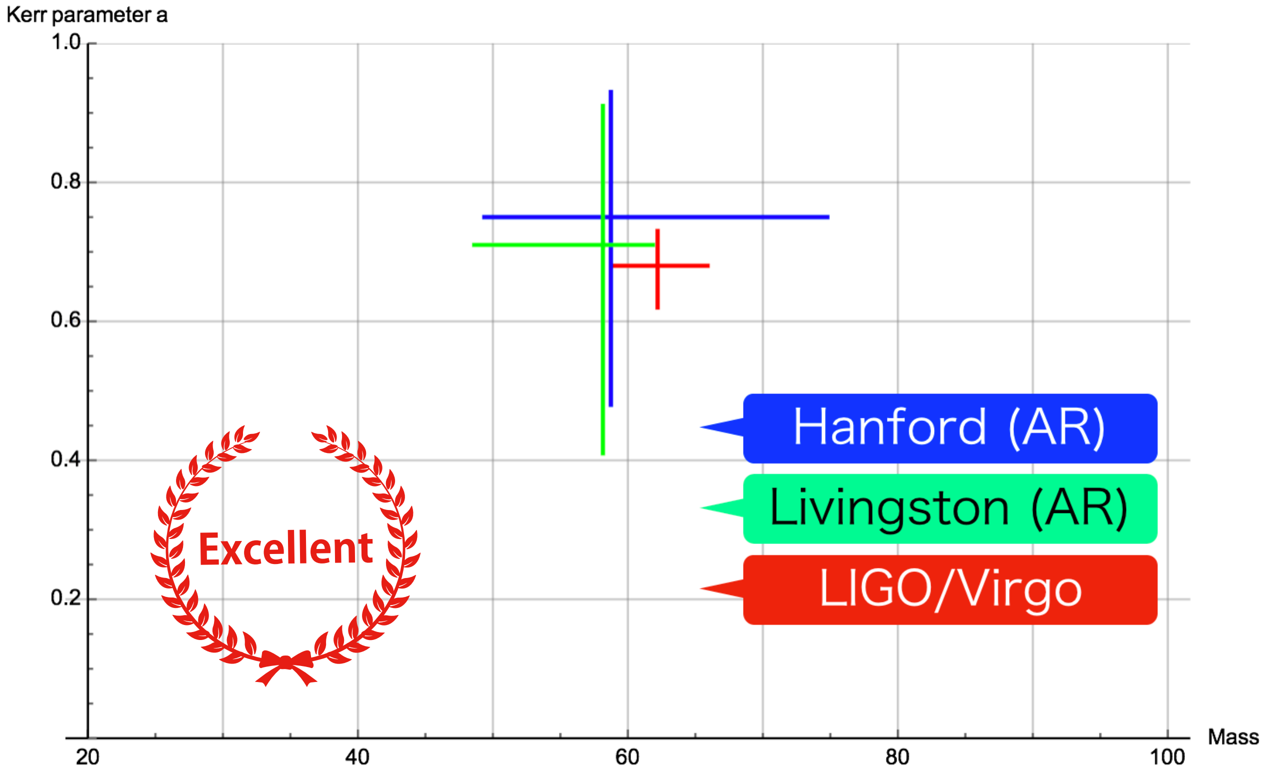
The analysis of real data using the improved matched filtering



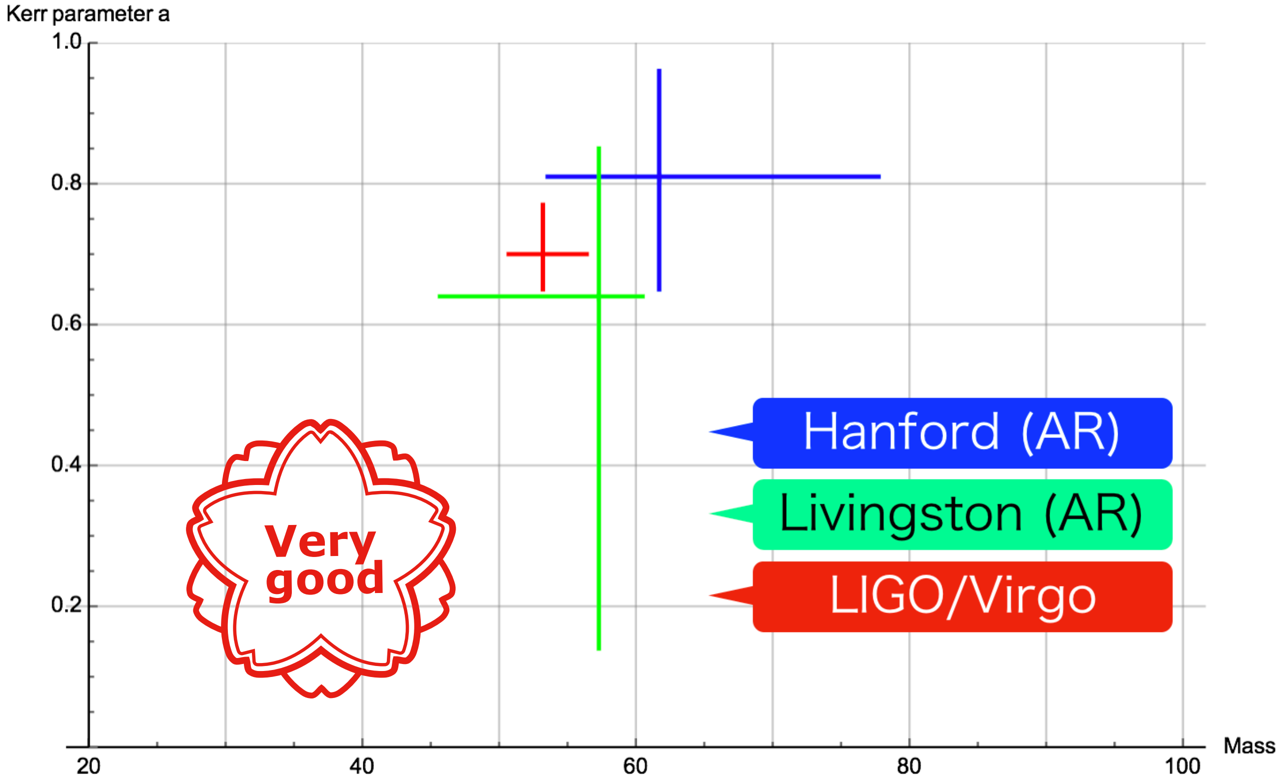
The estimate of the ringdown frequency is slightly inconsistent with the GR prediction obtained from the inspiral signal for GW150914.

Application to the LIGO/Virgo data : Auto-Regressive model

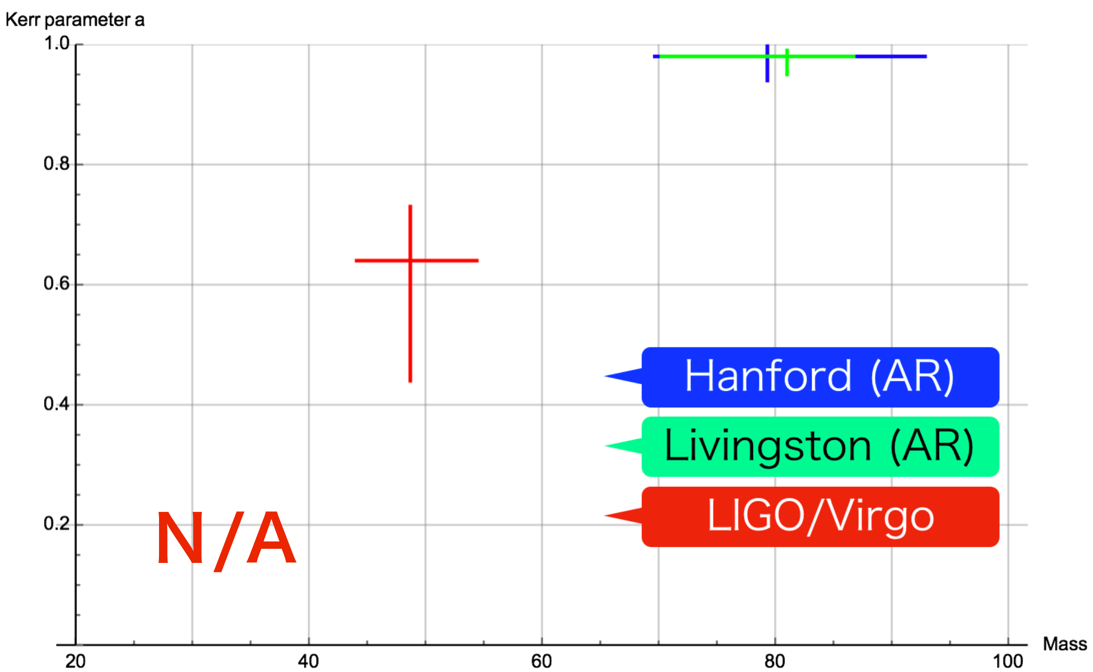
GW150914 (S/N = 23.7)



GW170814 (S/N = 18)



GW170104 (S/N = 13)



What's going now

Ringdown extraction methods are ready

If SNR>10, we can estimate f_real within 10% at our current methods.
But still hard for estimating f_imag.

Mockdata challenge shows several future improvements of each methods

AR: try with EMD, try with another approaches

HHT: improve mode-splitting problem in EMD

NN: estimate the errors

Application to the LIGO/Virgo data

is in progress

Preparation for O3 data (2019 Feb〜) analysis

is in progress