Differential Eqns. (Jan, 2010)

【Important】 Ask your lecturer whether you need to solve 選択問題 (option).

問題 **1** Make a differential equation by defining appropriate variables.

(1) Differential equation for curves which tangent are $\sin x$ at any point on xy plane.

(2) Differential equation for the number of bacteria which increases constantly with time.

(3) Differential equation for the number of atom which decays with time propotional to their number.

(4) Differential equation for the number of flu-infected patients which increases with time propotional to their squared number.

問題 **2** Show that $y(t) = e^{-\alpha t}$ satisfies

$$\frac{dy}{dt} = -\alpha y, \quad y(0) = 1,$$

where α is constant.

問題 **3** Solve

$$\frac{dy}{dt} + \alpha y = \beta, \quad y(0) = 0,$$

where α, β are constants.

問題 **4** Suppose $y = e^{\lambda t}$ (λ: complex) satisfies a differential equation

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + by = 0 \quad (a, b : \text{const.})$$

(1) Show the equation for λ.

(2) Find the general solution of the above differential equation.

問題 **5** Consider the differential equation for $y(t)$;

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 10\sin t.$$

(1) Find a special solution.

(2) Find the solution which satisfies the initial condition $y(0) = 0$, $\dfrac{d\, y(0)}{dt} = 1$.

選択問題 Consider the population model with a function of time $y(t)$.

$$\frac{dy}{dt} = (1 - ay)y,$$

where a is a positive constant. This model denotes that population increases propotional to y, but that rate is reduced with y^2. Find the general solution and discuss the behavior using a graph. Let $y(0) = y_0$ (> 0).

Hint: Apply the "separation of variables" method, and then decompose into partial fractions.