Apparent Horizon Formation in Five-dimensional Spacetime

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Abstract

We numerically investigated the formation of an apparent horizon in five-dimensional spacetime in the context of the cosmic censorship hypothesis. We modeled the matter by distributing collisionless particles both in spheroidal and toroidal configurations under the momentarily static assumption, and obtained the sequence of initial data by solving the Hamiltonian constraint equation. We found both S^3 and $\mathrm{S}^2\times\mathrm{S}^1$ horizons, only when the matter configuration is not steep. By monitoring the location of the maximum Kretchmann invariant, we guess an appearance of 'naked singularity' or 'naked ring' under the special situations.

1 Introduction

In general relativity, there are two famous conjectures concerning the gravitational collapse. One is the cosmic censorship hypothesis which states collapse driven singularities will always be clothed by event horizon and hence can never be visible from the outside. The other is the hoop conjecture [1] which states that black holes will form when and only when a mass M gets compacted into a region whose circumference in every direction is $C \leq 4\pi M$. These two conjectures have been extensively searched in various methods, among them we believe the numerical works by Shapiro and Teukolsky [2] showed the most exciting results; (a tendency of) the appearance of a naked singularity. This was reported from the fully relativistic time evolution of collisionless particles in a highly prolate initial shape; and the results of time evolutions are agree with those of the sequence of their initial data [3].

In recent years, on the other hand, gravitation in higher-dimensional spacetime is much getting a lot of attention. This is from an attempt to unify fundamental forces including gravity at TeV scale, and if so, it is suggested that small black-holes might be produced at the CERN Large Hadron Collider (LHC). The four-dimensional black-holes are known to be S^1 from the topological theorem, while in higher-dimensional spacetime quite rich structures are available including a torus black-hole ("black ring").

We, therefore, plan to reproduce the earlier numerical works in higher-dimensional spacetime, and this report shows our first trials to obtain the sequence of the initial data. As for the hoop conjecture, the modified version called *hyper-hoop* was proposed by Ida and Nakao [4] for the higher-dimensional spacetime, which was consistent with semi-analytic works [5]. We used the results of [5] as our code checks, and developed the code as we can investigate in more general situations.

2 Basic Equations

2.1 The Hamiltonian constraint equation

We consider the initial data sequence on a four-dimensional space-like hypersurface. A solution of the Einstein equations is obtained by solving the Hamiltonian constraint equation if we assume the moment of time symmetry. Applying a conformal transformation,

$$\gamma_{ij} = \psi^2 \hat{\gamma}_{ij}, \tag{1}$$

from conformally-flat base metric $\hat{\gamma}_{ij}$, the Hamiltonian constraint equation becomes

$$\hat{\Delta}\psi = -4\pi^2 G_5 \rho, \tag{2}$$

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where ρ is the mass density, G_5 is the gravitational constant in five dimensional theory of gravity. We solve (2) using the Cartesian coordinates, $ds^2 = \hat{\gamma}_{ij}dx^i dx^j = dx^2 + dy^2 + dz^2 + dw^2$, with various matter configurations (spheroidal and toroidal) by distributing 10^6 collisionless particles. We numerically solve (2) in the upper-half region ($x \geq 0,\ y \geq 0,\ z \geq 0,\ w \geq 0$) with 50^4 grids by setting the boundary conditions as

$$\nabla\psi = 0\,(\text{at } r = 0), \quad \psi = 1 + \frac{M}{2r^2}\,(\text{at } r \to \infty), \quad \text{where } r = \sqrt{x^2 + y^2 + z^2 + w^2}, \tag{3}$$

where M can be interpreted the total mass of the matter.

2.2 Apparent Horizons

For finding an apparent horizon, we follow [4] and [5]. Since we assume the matter is axially symmetrical distribution, the horizon will also be axially symmetric. Using the coordinate ξ and θ, where

$$\xi = \arctan\frac{\sqrt{x^2 + y^2 + w^2}}{z}, \quad \theta = \arctan\frac{\sqrt{x^2 + y^2}}{w}, \tag{4}$$

the axisymmetric apparent horizon is identified by solving

$$\begin{aligned}\ddot{r}_m - \frac{4\dot{r}_m^2}{r_m} - 3r_m + \frac{r_m^2 + \dot{r}_m^2}{r_m} \times \Bigg[\frac{2\dot{r}_m}{r_m}\cot\xi - \frac{3}{\psi}(\dot{r}_m \sin\xi + r_m \cos\xi)\frac{\partial\psi}{\partial z}\\ + \frac{3}{\psi}(\dot{r}_m \cos\xi - r_m \sin\xi)\left(\frac{\partial\psi}{\partial z}\sin\xi\cos\theta + \frac{\partial\psi}{\partial y}\sin\xi\sin\theta + \frac{\partial\psi}{\partial w}\cos\xi\right)\Bigg] = 0,\end{aligned} \tag{5}$$

with the boundary condition

$$\dot{r}_m = 0 \quad \text{at } \xi = \frac{\pi}{2}. \tag{6}$$

On the other hand, for the torus of the ring radius C, additional $S^2 \times S^1$ apparent horizon may exist. This marginal surface is obtained by solving the equation for $r(\xi)$, satisfying

$$\ddot{r} - \frac{3\dot{r}^2}{r} - 2r - \frac{r^2 + \dot{r}^2}{r} \times \left[\frac{\dot{r}\sin\xi + r\cos\xi}{r\cos\xi + C} - \frac{\dot{r}}{r}\cot\xi + \frac{3}{\psi}(\dot{r}\sin\xi + r\cos\xi)\frac{\partial\psi}{\partial x} - \frac{3}{\psi}(\dot{r}\cos\xi - r\sin\xi)\frac{\partial\psi}{\partial z}\right] = 0, \tag{7}$$

with the boundary condition

$$\dot{r} = 0 \quad \text{at } \xi = 0, \pi \tag{8}$$

3 Results

3.1 Spheroidal configurations

First, we show the cases with homogeneous spheroidal matter configurations,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{w^2}{a^2} + \frac{z^2}{b^2} \leq 1. \tag{9}$$

where a and b are constants. In Figure 1, we show particle distributions and shape of the apparent horizon on our numerical grid. When $a = b$, the horizon is spherically symmetric and located at Schwarzschild radius. When $b = 3a$, the horizon becomes prolate. When $b = 5a$, on the other hand, we can not find an apparent horizon. The behavior is the same with [3] and [5]. The asterisk in Fig.1 is the location of the largest Kretchmann invariant, $I_{\max} = \max\{R_{abcd}R^{abcd}\}$, where R_{abcd} is the four-dimensional Riemann tensor. For all cases, we see the location of $I_{\max}$ is always outside the matter on the axis. We show the contours of $I = R_{abcd}R^{abcd}$ in Figure 2.

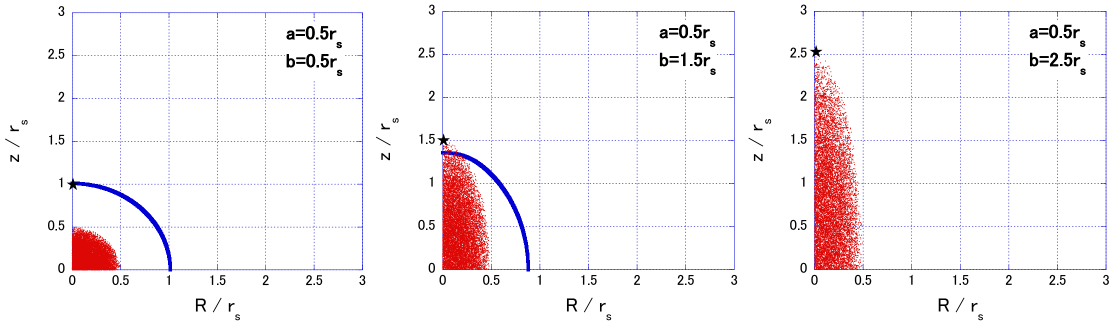

Figure 1: Matter distributions and apparent horizons for spheroidal matter distributions. We can not find an apparent horizon for highly spindle cases.

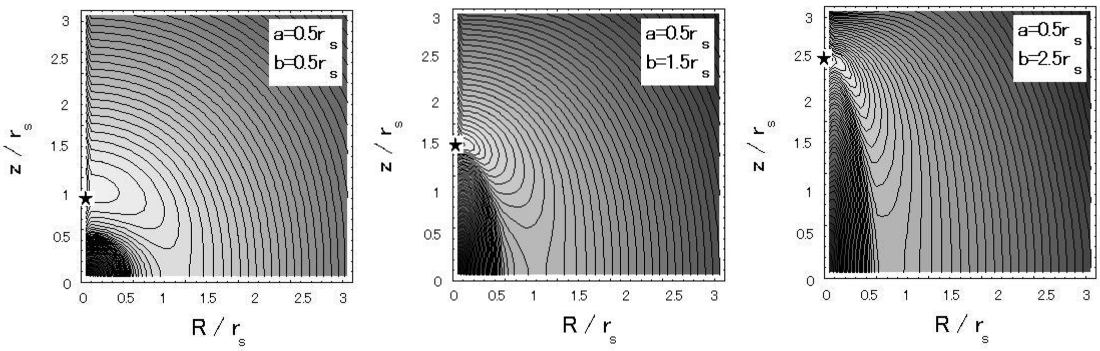

Figure 2: Contours of Kretchmann invariant, $I = R_{abcd}R^{abcd}$, corresponding to Fig.1.

In Figure 3, we plot $I_{\max}$ as a function of b/a. Fig.3 shows that the spindle cases have larger $I_{\max}$, that suggests the possibility of appearance of a naked singularity as in the four-dimensional case.

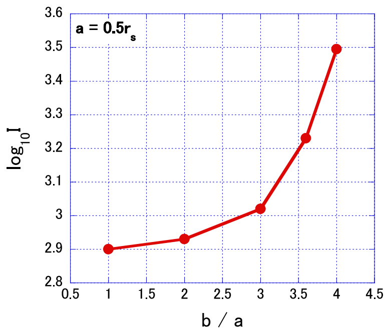

Figure 3: $I_{\max}$ as a function of b/a.

3.2 Toroidal configurations

Next, we consider homogeneous toroidal matter configurations as

$$\left(\sqrt{x^2+y^2}-C\right)^2+\left(\sqrt{w^2+z^2}\right)^2 \le r^2, \tag{10}$$

where C is the circle radius of torus, and r is ring radius of torus. Figure 4 shows the results of searches for apparent horizons. When C is $1.65r_s$, both S^3 and $S^2 \times S^1$ apparent horizons exist. On the other hand, when C is larger ($C = 1.78r_S$), only the $S^2 \times S^1$ ring horizon is observed. The value of $I_{\max}$ appears at the outside as well as spheroidal cases. Interestingly, $I_{\max}$ is not hidden inside the ring horizon when C is $2.55r_s$.

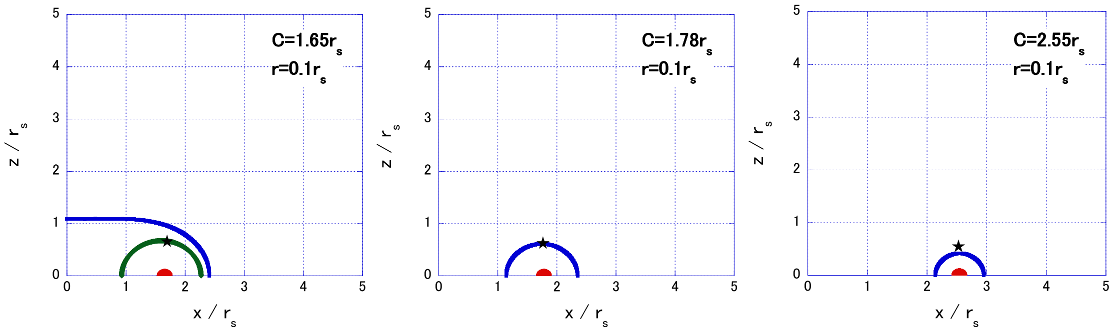

Figure 4: Matter distributions and apparent horizons for toroidal matter distributions.

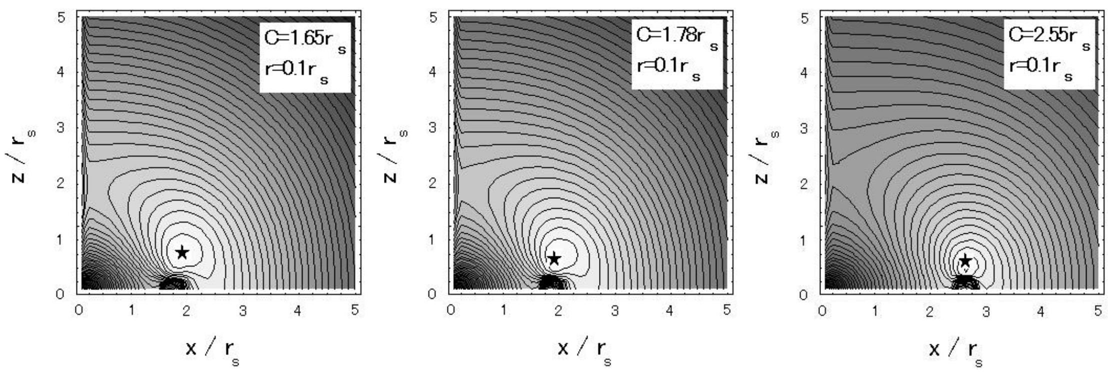

Figure 5: Contours of $I = R_{abcd}R^{abcd}$, corresponding to fig.4.

4 Future works

We showed our preliminary results of constructing initial-data in five-dimensional spacetime. We developed our code for solving the Hamiltonian constraint equation and searching for apparent horizons. Sequences of initial data both for spherical and toroidal matter configurations are obtained, and we searched when apparent horizons are formed.

We are now examining the validity of the hyper-hoop conjecture, and also preparing the configurations with rotations. In the future, we plan to report the fully general relativistic dynamical process in five-dimensional space-time.

References

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