Numerical Study of Five-dimensional Gravitational Collapses

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Five-dimensional(5D) black-hole and black-ring formation are investigated numerically. We express the matter with collisionless particles, and prepare these distribution in homogeneous spheroidal and toroidal configurations under the momentarily static assumption. Evolutions are followed by using ADM formalism $(4+1$ decomposition) and solving the geodesic equation. For spheroidal configurations, we repeat the 4D simulations performed by Shapiro and Teukolsky (1991) that announced an appearance of a naked singularity, and discuss the differences in the 5D version. For toroidal configurations, we consider the rotating collisionless particles which consist of equal numbers co-rotating and counter-rotating under a certain rotational law. We show topology change of apparent horizon from ring-shaped to spherical shape during its evolution.

Keywords: Higher-dimensional black-hole; Gravitational collapse; Numerical relativity.

1. Introduction

For a decade, higher-dimensional black-holes are extensively studied linked with the so-called "large extra-dimensional models". The discovery of black-ring solution[1] with horizon of $S^2 \times S^1$ topology in U(1)× U(1) space, and its associated Saturn-type "black objects" reveal new features of higher-dimensional space-time. However, non-linear dynamical properties, such as formation process, generality, and stability of black-objects, are not yet known. We plan to investigate these outstanding problems using numerical simulations. In this article, we report two models.

First topic is on the naked singularity formation. Shapiro and Teukolsky[2] (ST91) numerically showed that in four-dimensional(4D) space-time, collisionless matter particles in spheroidal distribution will collapse to singularity, without forming apparent horizon(AH) when the spheroids are highly prolate. We repeat their simulations and compare them with those in 5D(§3).[3]

Second topic is the formation of black-ring in 5D(§4). We distributed collisionless matter particles in toroidal configuration, and evolve them with/without their rotations. As the first step, we assume particles are counter-rotating and the system has no net angular momentum.

2. Our numerical code

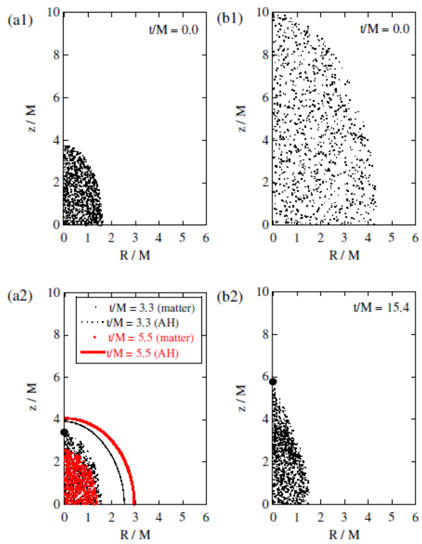
We developed the code for both 4/5D axisymmetric [symmetric on z-axis, SO(3)] or doubly-axisymmetric [symmetric both on x and z-axes, U(1)×U(1)], asymptotically flat space-time.

We start our simulation from time symmetric and conformally-flat initial data, which are obtained by solving the Hamiltonian constraint equations.[4] The matter is described with 5000 collisionless particles, which move along the geodesic equations.

We construct our numerical grids with the Cartesian coordinate (x, z), and apply the so-called Cartoon method to recover the symmetry of space-time. The results shown in this report are obtained with numerical grids, $129\times129\times2\times2$. The space-time is evolved using the Arnowitt-Deser-Misner (ADM) evolution equations. We use the maximal slicing condition for determining the lapse function α, and the minimal strain condition (§3) or zero shift condition (§4) for the shift vectors β^i.

3. Naked Singularity Formation

We first distribute matter particles in spheroidal configurations, and follow their gravitational collapse. We search the location of AH, calculate the Kretchmann invariant ($\mathcal{I} = R_{abcd}R^{abcd}$) on the spacial hypersurface during time evolution. We confirmed the results of ST91 in 4D. Figure 1 shows snapshots of the results in 5D. We see that when the initial configuration is sufficiently large, AH is not formed (Fig.1(b2)) and the maximum of the Kretchmann invariant $\mathcal{I}_{\max}$ becomes $O(1000)$ on z-axis at $t/M = 15.4$ (see Fig.2). This behavior is similar to 4D cases[2] and support the hoop conjecture. The absence of AH with diverging $\mathcal{I}$ suggests a formation of naked singularity in 5D. We observed that 5D collapse proceeds faster then 4D, and also that collapse proceeds more spherical in 5D.



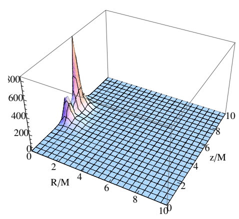


Fig. 1. (left) Snapshots of 5D axisymmetric evolution with the initial matter distribution. Fig.(a1) and (b1) are configurations with eccentricity 0.9 but polar radius is not the same. The big circle indicates the location of $\mathcal{I}_{\max}$.

Fig. 2. (above) Kretchmann invariant $\mathcal{I}$ for Fig.1(b2) at $t/M = 15.4$. The maximum is $O(1000)$, and its location is on z-axis, just outside of the matter.

4. Black-Ring Formation

We next follow the 5D gravitational collapses of toroidal objects. We assume the matter consists of equal number of corotating and counter-rotating particles which obey Kepler-like law $V(|\mathbf{r}-\mathbf{R_c}|) \propto \xi/|\mathbf{r}-\mathbf{R_c}|$ where R_c is the radius of configuration and ξ is a parameter. Figure 3 shows snapshots of non-rotating toroidal matter. We observe topological transition of AHs from ring-shape to spherical (Fig.3(b)) when R_c is large. We see that area of AH increase gradually during time evolution (Fig.4). Adding rotations of particles delays the gravitational collapse, which makes the appearance of ring AH later. Details will be reported elsewhere.[5]

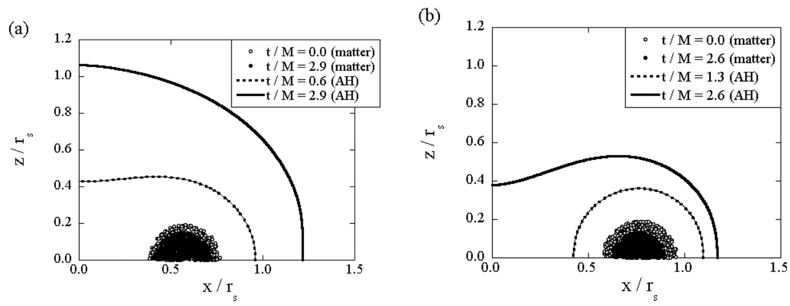


Fig. 3. Snapshots of time evolutions of toroidal matter collapses. Particles and location of AHs are drawn. We set initial ring radius $R_c/r_s = 0.6$ (left) and 0.8 (right), and set rotating parameter $\xi = 0$.

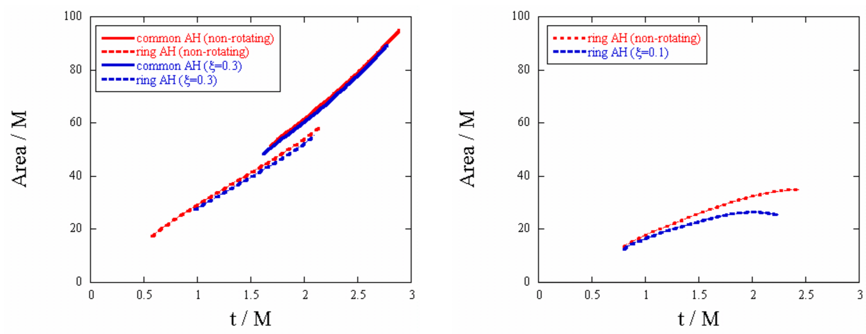


Fig. 4. Area of common and ring-shaped horizons as a function of time. Figure show results of time evolution for initial ring radius $0.8r_s$ and $1.5r_s$, respectively. $\xi = 0.3$ corresponds to the rotation energy 18% in the total energy.

References

1. R. Emparan and H. S. Reall, Phys. Rev. Lett. **88**, 101101 (2002).
2. S. L. Shapiro and S. A. Teukolsky, Phys. Rev. Lett. **66**, 994 (1991).
3. Y. Yamada and H. Shinkai, Phys. Rev. D **83**, 064006 (2011).
4. Y. Yamada and H. Shinkai, Class. Quant. Grav. **27**, 045012 (2010).
5. Y. Yamada and H. Shinkai, in preparation.