

# Comparison of various methods to extract ringdown frequency from gravitational wave data

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The ringdown part of gravitational waves in the final stage of merger of compact objects tells us the nature of strong gravity which can be used for testing the theories of gravity. The ringdown waveform, however, fades out in a very short time with a few cycles, and hence it is challenging for gravitational wave data analysis to extract the ringdown frequency and its damping time scale. We here propose to build up a suite of mock data of gravitational waves to compare the performance of various approaches developed to detect quasi-normal modes from a black hole. In this paper we present our initial results of comparisons of the following five methods; (1) plain matched filtering with ringdown part (MF-R) method, (2) matched filtering with both merger and ringdown parts (MF-MR) method, (3) Hilbert-Huang transformation (HHT) method, (4) autoregressive modeling (AR) method, and (5) neural network (NN) method. After comparing their performance, we discuss our future projects.

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## I. INTRODUCTION

Both in the year 2016 and 2017, physics community was really got excited by the reports of direct detections of gravitational waves (GWs) by LIGO/Virgo collaborations. The direct detection definitely proves the correctness of the fundamental idea of general relativity (GR), together with that of the directions of the efforts of theoretical and experimental researches of gravity.

Up to now, six events have been reported; five events and one candidate of the mergers of binary black holes (BHs) (GW150914 [1], LVT151012 [1], GW151226 [2], GW170104 [3], GW170608 [4], GW170814 [5]), and one event of the merger of binary neutron stars (GW170817 [6]). Both types of sources gave us certain advances not only to physics, but also to astrophysics.

In 2019, another ground-based GW detector, KAGRA,

will join the network of GW observation, which will make the source localization more precise, and we also expect to detect the polarization of GWs for each event. By accumulating observations, we will be able to investigate new aspects of physics and astronomy, such as the distribution of binary parameters, formation history of binaries, equation of state of the nuclear matter, and cosmological parameters.

Among such possibilities, the test of gravity theories is in our future list to do. GR has passed all the tests in the past century, and nobody doubts gravity is basically described by GR. However, all the tests so far have been performed in the weak gravity regime (tests around the Earth, in the Solar System, or using binary pulsars), and we still require tests in the strong gravity regime, which is relevant to describe, say, BH mergers. Hence, observations of GWs from binary BHs will finally enable us to test the gravity theories in this regime.

The previous detections of BH mergers showed that the inspiral phase (pre-merger phase) is well described by post-Newtonian approximation, while the ringdown phase (post-merger phase), which is supposed to be well described by BH perturbation theory, is not reported to be clearly detected in the GW data. This is because identifying the ringdown mode of a BH from noisy real data is a quite challenging task for data analysis. Ringdown modes decay quite rapidly for a typical BH described by

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GR. For example, for a typical BH formed after merger with mass  $M = 60M_\odot$  and angular momentum (normalized Kerr parameter)  $\chi = 0.75$ , the ringdown mode ( $\ell = m = 2$ ) has frequency  $f_R = 300$  Hz and damping time  $\tau = 3.7$  msec, which indicate that the amplitude is reduced to about 40% after one-cycle of oscillation.

One way to give a clear evidence of the detection of the ringdown mode would be just to improve the detector sensitivity. However, it will also give a similar impact if we can improve the significance of detection by implementing an optimized data analysis method. There have been already several proposals of method to identify the ringdown mode, but we think that fair comparison of the performance of different methods has not been organized. To find out the optimal method developing new techniques, we organize mock-data tests. The idea is to find out ringdown part [its frequency  $f_R$  and damping time  $\tau$  (imaginary part of frequency  $f_I$  or quality factor  $Q$ )] independently from the other part of waveform. In order to include future possibility for testing gravity theories, and in order not to be able to identify the properties of mergers from the inspiral waveform, we prepare a set of blind data which has  $f_R$  and  $f_I$  different from the predicted values in GR (see Sec. II A).

We present our first comparison of the performance as for the following five different methods; (1) matched filtering with ringdown part (MF-R) method, (2) matched filtering with both merger and ringdown parts (MF-MR) method, (3) Hilbert-Huang transformation (HHT) method, (4) autoregressive modeling (AR) method, and (5) neural network (NN) method. Each method will be explained separately in Sec. III.

In Sec. IV, we compare the results together with future directions for improvement and the issues for the application to the real data. The mock data we used in this article are available from our webpage [7], in order for readers' convenience.

## II. BUILDING MOCK DATA

### A. Quasi-Normal Modes

The waveform of the ringdown gravitational waves emitted from an excited BH is modeled as

$$h(t) = Ae^{-(t-t_0)/\tau} \cos(2\pi f_R(t-t_0) - \phi_0), \quad (2.1)$$

where  $f_R$  is the oscillation frequency, and  $\tau$  is the damping time, and  $t_0$  and  $\phi_0$  are the initial time and its phase, respectively. The waveform, Eq. (2.1), is then written as

$$h(t) = \Re[Ae^{-2\pi i f_{\text{qnm}}(t-t_0)}], \quad (2.2)$$

where we call  $f_{\text{qnm}} = f_R - if_I$  the quasi-normal mode (QNM, or ringdown) frequency ( $f_I > 0$  means decaying mode). (Nakano et al. [8] uses different signature on  $f_I$ .) The parameter  $\tau$  is also expressed using a quality factor

$Q$ , or  $f_I$

$$Q = \pi f_R \tau, \quad \text{or} \quad f_I = \frac{1}{2\pi\tau} = \frac{f_R}{2Q}. \quad (2.3)$$

In GR, the set of  $(f_R, f_I)$  is determined by the mass,  $M$ , and angular momentum,  $M^2\chi$ , of the black hole. QNM is obtained from the perturbation analysis of BHs, and its fitting formulas are given by [9]

$$f_R = f_1 + f_2(1-\chi)^{f_3}, \quad (2.4)$$

$$Q = \frac{f_R}{2f_I} = q_1 + q_2(1-\chi)^{q_3}, \quad (2.5)$$

where  $f_i, q_i$  are the fitting coefficients. For the most fundamental mode, which is of the spherical harmonic index  $\ell = 2, m = 2$ , the fitting parameters are  $f_1 = 1.5251$ ,  $f_2 = -1.1568$ ,  $f_3 = 0.1292$ ,  $q_1 = 0.7000$ ,  $q_2 = 1.4187$ , and  $q_3 = -0.4990$ .

If we recover the units,

$$f_R(M, \chi) [\text{Hz}] = \frac{c^3}{2\pi GM} \{f_1 + f_2(1-\chi)^{f_3}\}, \quad (2.6)$$

where  $c$  and  $G$  are the speed of light and the gravitational constant, respectively. From this equation, at linear order, the uncertainties in  $f_R$  and  $Q$  are related to those in mass and angular momentum as

$$\frac{\Delta f_R}{f_R} = -\frac{\Delta M}{M} + \frac{f_2 f_3 (1-\chi)^{f_3-1}}{f_1 + f_2(1-\chi)^{f_3}} \Delta\chi. \quad (2.7)$$

Similarly, from Eq. (2.5), we get

$$\Delta Q = \left( \frac{\Delta f_R}{f_R} - \frac{\Delta f_I}{f_I} \right) Q \quad (2.8)$$

$$= q_2 q_3 (1-\chi)^{q_3-1} \Delta\chi. \quad (2.9)$$

In modified gravity theories the final fate of binary mergers may not be a black hole. There are various possibilities of modification of gravity. The most generic test for the deviation from general relativity would be just checking the consistency between the observed data and the predicted waveform based on general relativity. However, such a generic test will not be very sensitive. If we focus on some class of modification, we would be able to perform a much better test. Here, we assume that even if the gravity is modified the ringdown waveform is characterized by the set of  $(f_R, f_I)$ . For the same inspiral/merger waveform, which predicts the formation of a black hole with  $M$  and  $\chi$  in GR, however, the values of  $(f_R, f_I)$  may be different. Under this assumption, one can test GR by comparing  $(f_R, f_I)$  predicted from the data in the inspiral/merger phase in GR with those directly extracted from the data in the ringdown phase. For this purpose, we wish to minimize the error in the determination of  $(f_R, f_I)$  from the data in the ringdown phase independently of the information contained in the inspiral/merger phase.

## B. Mock data

The fundamental question we raise here is whether or not one can detect the deviation from the GR prediction, in case only the ringdown frequency is modified. If the breakdown of GR occurs only in the extremely strong gravity regime such as the region close to the BH event horizon, modification of gravity might be completely irrelevant to the evolution during the inspiral/merger phases. Even in such cases, the deviation from the GR prediction may show up in the ringdown waveform. This gives a good motivation to develop a method to identify the ringdown frequency without referring to the information from the inspiral/merger phases.

There are many proposals for the method to extract the ringdown frequency and its damping time scale. In order to compare the performance of various methods by a blind test, we construct some test data which have artificially modified ringdown frequency.

We adopt the following strategy for preparing the data. We take the inspiral/merger waveform from SXS Gravitational Waveform Database [11, 12] (there are also available catalogs for BBH GWs in Refs. [13–16]), and the ringdown waveform modified from GR case is merged. Then, noise is added so as to reproduce the ideal LIGO noise curve (with the signal-to-noise ratio (SNR) = 20, 30 or 60). In doing so, we focus on the fact that the time evolution of the amplitude and the frequency of the GR waveform is rather smooth if the spin precession can be neglected. Our basic assumption is that this smoothness is maintained even if we consider modification of the complex QNM frequencies. Then, the modified waveform cannot have a large variety.

We define a normalized time coordinate  $x = (t - t_p)/M$ , where  $t_p$  denotes the time that the GW amplitude has its peak, and just modify the GW strain after the peak time. This is a reasonable assumption because, if the inspiral/merger parts are also modified, we can detect the deviation from GR even in case when we cannot extract the QNM frequency from the gravitational wave data. Note that  $M$  is the initial total mass of the binary, and we will specify it later to generate the mock data set. In the following, we take the simplification of considering only the ( $\ell = 2, m = 2$ ) GW mode.

To create the mock data, the total mass  $M$  is randomly selected from the range between  $50M_\odot - 70M_\odot$  with uniform probability. The parameters characterizing the ringdown waveform,  $f_R$  and  $f_I$ , are modified from the GR value within  $\pm 30\%$  and  $\pm 50\%$ , respectively. Uniform probability distribution is assumed for both parameters. We present two independent ways to generate the mock data below.

### 1. Set A

In the case of set A, the modification is strictly limited to the time domain after the peak of the GW amplitude.

First, as for the GW amplitude  $A_{22} = r|h_{22}|/M$ , we introduce the following fitting function

$$A_{22}(t) = \frac{A_{22}^{\text{GR}}(t_p) + a_0 x + a_1 x^2}{1 + (M\omega_I + a_0/A_{22}^{\text{GR}}(t_p))x + a_2 x^2} \times \exp(M\omega_I x), \quad (2.10)$$

where we have three fitting parameters  $a_0, a_1$  and  $a_2$ , which are chosen to reproduce the amplitude of the SXS waveform  $A_{22}^{\text{GR}}(t)$  when the adjustable parameter  $\omega_I = f_I/(2\pi)$  is set to the GR value  $\omega_I^{\text{GR}}$  calculated from the remnant BH mass and spin by using Refs. [9, 10]. For example, for SXS:BBH:0174 [12], we have

$$a_0 = 0.0183650, \quad a_1 = 0.000998244, \\ a_2 = 0.00184509, \quad (2.11)$$

with  $A_{22}^{\text{GR}}(t_p) = 0.286987$  and  $M\omega_I^{\text{GR}} = -0.0815196$ . The above fitting function is chosen as such that the first derivative of the amplitude is zero at the peak ( $x = 0$ ). By changing  $\omega_I$ , we can create a mock data.

Second, as for the GW frequency  $\omega_{22}(t)$ , which is defined by the time derivative of the GW phase and supposed to be positive, we use the fitting function

$$M\omega_{22}(t) = (M\omega_{22}^{\text{GR}}(t_p) - M\omega_R + b_0 x + b_1 x^2 + b_2 x^3) \times \exp\left[\frac{(b_0 - M d\omega_{22}^{\text{GR}}(t)/dx|_{t=t_p})x}{M(\omega_R - \omega_{22}(t_p))}\right] + M\omega_R, \quad (2.12)$$

where  $\omega^{\text{GR}}(t)$  is the frequency extracted from the phase of the numerically determined GR template. With this fitting function, the smoothness of GW phase is  $C^2$  at the peak time. The three fitting parameters are, again for SXS:BBH:0174,

$$b_0 = -0.0507805, \quad b_1 = -0.00276104, \\ b_2 = -0.000479913, \quad (2.13)$$

with  $M\omega_{22}^{\text{GR}}(t_p) = 0.375598$  and  $Md\omega_{22}^{\text{GR}}(t)/dx|_{t=t_p} = 0.00492361$ . The mock data is created by changing the input  $\omega_R = f_R/(2\pi)$  from the GR value calculated from the remnant BH mass and spin, *e.g.*,  $M\omega_R^{\text{GR}} = 0.582652$  for SXS:BBH:0174.

### 2. Set B

The modification of the second set is not strictly restricted to the time period after the peak of the amplitude. As another smooth interpolation, we adopt the following modified amplitude

$$A_{22}(t) = \frac{A_{22}^{\text{GR}}(t)}{1 + e^{4M\omega_I^{\text{GR}}x}} + \frac{A_{22}^{\text{RD}}(t)}{1 + e^{-4M\omega_I^{\text{GR}}x}}, \quad (2.14)$$

with

$$A_{22}^{\text{RD}}(t) = \frac{A_{22}}{1 + e^{-M\omega_I^{\text{GR}}x} + e^{M\omega_I x}}, \quad (2.15)$$

and the overall amplitude  $\mathcal{A}_{22}$  determined so that the GR case fits well.

The frequency is also given in a similar simple manner by

$$\omega_{22}(t) = \frac{\omega_{22}^{\text{GR}}(t)}{1 + e^{4M\omega_1^{\text{GR}}x}} + \frac{\omega_{22}^{\text{RD}}(t)}{1 + e^{-4M\omega_1^{\text{GR}}x}}, \quad (2.16)$$

with the GR frequency  $\omega_{22}^{\text{GR}}(t)$  and

$$\omega_{22}^{\text{RD}}(t) = \omega^{\text{GR}}(t_p) + \frac{\omega_{\text{R}} - \omega^{\text{GR}}(t_p)}{1 + e^{-2M\omega_1 x}}. \quad (2.17)$$

For the generation of the set B mock data, we used SXS:BBH:0002, 0004 and 0007.

### III. VARIOUS METHODS FOR IDENTIFYING RINGDOWN MODE

#### A. Matched filtering with ringdown part (MF-R)

We perform the matched filtering analysis using simple damped sinusoidal templates, which are given by

$$\hat{h}(t) = \begin{cases} 0 & (t < t_0) \\ \frac{1}{N} e^{-\omega_1(t-t_0)/Q} \cos[\omega_{\text{R}}(t-t_0) - \phi_0] & (t \geq t_0), \end{cases} \quad (3.1)$$

where  $t_0$  and  $\phi_0$  are the starting time and the initial phase of the template, respectively. The normalization constant  $N$  is chosen so as to satisfy  $(\hat{h}|\hat{h}) = 1$ , where the inner product is defined by

$$(h_1|h_2) = 2 \int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df. \quad (3.2)$$

Here,  $S_n(f)$  is the noise spectral density and the Fourier transform of  $h(t)$  is defined by  $\tilde{h}(f) = \int dt e^{2\pi i f t} h(t)$ , and  $*$  denotes the complex conjugate. We consider to maximize the inner product between the GW data  $s(t)$ , which contains the signal and the noise, and the template  $h(t)$  over the parameters  $\omega_{\text{R}}$ ,  $\omega_1$ ,  $t_0$  and  $\phi_0$ . The SNR against the initial phase of the template can be maximized by rewriting the template in the following form,

$$h(t) = \frac{1}{N} (h_c \cos \phi_0 + h_s \sin \phi_0), \quad (3.3)$$

where

$$\begin{aligned} h_c &= e^{-\omega_1(t-t_0)} \cos[\omega_{\text{R}}(t-t_0)], \\ h_s &= e^{-\omega_1(t-t_0)} \sin[\omega_{\text{R}}(t-t_0)]. \end{aligned} \quad (3.4)$$

Then, the maximum of the SNR against the initial phase  $\phi_0$  can be given as [17],

$$\rho^2|_{\max \phi_0} = \frac{(s|\hat{h}_c)^2 + (s|\hat{h}_s)^2 - 2(s|\hat{h}_c)(s|\hat{h}_s)(\hat{h}_c|\hat{h}_s)}{1 - (\hat{h}_c|\hat{h}_s)^2}, \quad (3.5)$$

where

$$\hat{h}_c = \frac{h_c}{\sqrt{(h_c|h_c)}}, \quad \hat{h}_s = \frac{h_s}{\sqrt{(h_s|h_s)}}. \quad (3.6)$$

The phase  $\phi_0$  that gives the maximum  $\rho$  is given by

$$\tan \phi_0 = \frac{(h_c|h_s)(h|h_c) - (h_c|h_c)(h|h_s)}{(h_c|h_s)(h|h_s) - (h_s|h_s)(h|h_c)}. \quad (3.7)$$

Then, we are left with three parameters to explore. Since the best choice of the starting time of QNM is unknown, we vary  $t_0$  from the merger time  $t_c$  to  $t_{c+}$  something. Then, we search for the best fit values of the parameters  $(f, Q)$  for each  $t_0$ . Finally, we calculate median values of  $\{f(t_0), Q(t_0)\}$  which we regard as our estimate of the QNM frequency.

#### B. Matched filtering with both merger and ringdown parts (MF-MR)

The plain matched filtering using the damped sinusoidal waveform, which was introduced in the preceding subsection, has the difficulty in choosing the appropriate starting time,  $t_0$ . On one hand, if  $t_0$  is chosen to be too early, we pick up lower frequency oscillations before the QNM starts to dominate. On the other hand, if  $t_0$  is chosen to be too late, the signal has already become very faint and is buried in noise. Therefore, it is likely that this method is not the optimal method to determine the QNM frequency from the data.

The basic idea for the improvement of matched filtering is as discussed in Ref. [18]. If we know the modified ringdown waveform in advance, we can construct the best linear filter that produces the largest SNR by using it. As in the construction of our mock data in Sec. II A, here we also assume that this smoothness is maintained even for the modified waveform. Then, the variety of the possible waveforms would be effectively limited well.

To obtain a better fit, one may think it would be necessary to introduce, at least, two more parameters in addition to  $(\omega_{\text{R}}, \omega_1)$ , i.e., the amplitude of QNM relative to that of the inspiral/merger phases, and the transition rapidity to reach the final QNM frequency. It might be reasonable to perform the matched filtering using this generalized waveform including the inspiral/merger phases. To find the best fit parameter values in the four parameter space is doable. However, we adopt the simplifications of neglecting these additional parameters here, in order to reduce the computational cost in the present analysis, leaving this possible extension to our future work. To reduce the impact of neglecting the relative amplitude of the QNM, we make use of the fact that the inspiral/merger parts are basically unchanged for all modified templates. Namely, we introduce the following sharp window function

$$W(t) = \frac{1}{1 + e^{-50(t-t_c)\omega_1^{\text{GR}}}}, \quad (3.8)$$

to make the relative amplitude between the inspiral-merger phase and the ringdown phase almost irrelevant, instead of introducing one additional model parameter.

The procedure is summarized as follows. We first calculate the whitened signal and template multiplied by the window function, which are more explicitly defined by

$$\hat{s}(t) = W(t) \int df e^{-2\pi i f t} \frac{\tilde{s}(f)}{\sqrt{S_n(f)}}, \quad (3.9)$$

and

$$\hat{h}(t) = NW(t) \int df e^{-2\pi i f t} \frac{\tilde{h}(f)}{\sqrt{S_n(f)}}, \quad (3.10)$$

where  $N$  is the normalization factor that is determined so as to satisfy  $(\hat{h}, \hat{h})^{(w)} = 1$ , where  $(*, *)^{(w)}$  is the inner product in Eq. (3.2) with  $S_n(f)$  replaced with unity. After this preprocessing, the correlations between the data and templates are calculated as in the case of the standard matched filtering, besides the point that the inner product  $(*, *)^{(w)}$  is used instead of  $(*, *)$ . Here, we simply choose the time shift and the phase such that maximize the signal to noise ratio  $\rho$  for each template, instead of marginalizing over these parameters. To obtain the distribution of the parameters  $\omega_R$  and  $\omega_I$ , we simply used the probability given by  $\propto \exp(\rho^2/2)$ , which corresponds to the posterior distribution for the flat prior ansatz.

To perform the correlation analysis, we also need to specify a template waveform which includes the real and imaginary part of the QNM frequency as free parameters. To obtain the necessary template waveform, we shall use the same prescription as set B that is used to generate a half of the mock data. We understand that this makes the comparison for set B unfair, but one purpose of testing the improved matched filtering method arranged in this manner is to give a relevant standard to evaluate the efficiency of the other methods. The standard matched filtering using the damped sinusoidal wave as templates might be too naive to use it as the standard reference to assess the performance of the other methods. This improved matched filtering method is actually guaranteed to give the best linear filtering. Therefore, we think that the results obtained by this method offers a good reference to evaluate the performance of the other methods.

## C. Hilbert-Huang transformation (HHT) method

### 1. Basic idea

The Hilbert-Huang transform (HHT) is a time-frequency analysis method [19], which is constructed under the aim to manipulate non-stationary and/or non-linear system. Some applications of the HHT to the data analysis of gravitational waves have been proposed [20–24]. The HHT is based on a signal analysis by the Hilbert transform. We describe the concept of the signal analysis

by the Hilbert transform and its difficulty to be applied to real-world signals, and then explain how the HHT overcomes the difficulty.

Letting  $\check{s}(t)$  be the Hilbert transform of a signal  $s(t)$ , it is defined by

$$\check{s}(t) = \frac{1}{\pi} \text{PV} \int dt' \frac{s(t')}{t-t'}, \quad (3.11)$$

where PV denotes the Cauchy principal value. The complex signal  $z(t)$ , which is defined by  $z(t) = s(t) + i\check{s}(t)$ , can be represented by the exponential form:

$$z(t) = a(t)e^{i\phi(t)}, \quad (3.12)$$

where  $a(t)$  and  $\phi(t)$  are defined by

$$a(t) = \sqrt{s(t)^2 + \check{s}(t)^2}, \quad (3.13)$$

$$\phi(t) = \arctan\left(\frac{\check{s}(t)}{s(t)}\right). \quad (3.14)$$

Therefore,

$$s(t) = a(t) \cos \phi(t) \quad (3.15)$$

is established. Only when the signal  $s(t)$  is monochromatic over short periods of time,  $z(t)$  is an analytic signal of  $s(t)$ , in other words, the Fourier components of  $z(t)$  are the same as  $s(t)$  in the positive frequency range, and zero in the negative frequency range [25], and then  $a(t)$  and  $\phi(t)$  are called instantaneous amplitude (IA) and instantaneous phase (IP) of  $s(t)$ , respectively. The monochromaticity of  $s(t)$  over short periods of time means that  $a(t)$  has only lower frequency components than  $\cos \phi(t)$ , or  $a(t)$  and  $\cos \phi(t)$  are the modulator and the carrier of the signal  $s(t)$ , respectively. In that case, the local mean  $m(t)$  of  $s(t)$ , which is defined by

$$m(t) = \frac{u(t) + l(t)}{2}, \quad (3.16)$$

where  $u(t)$  and  $l(t)$  are the upper and lower envelopes of  $s(t)$ , respectively, is zero at any point. We call this feature *zero-mean*. An instantaneous frequency (IF) of  $s(t)$  is defined by

$$f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}. \quad (3.17)$$

This analysis to estimate the IA and IF from a signal is called the Hilbert spectral analysis (HSA). The HSA has an advantage of higher resolution than the other time-frequency analysis such as the short-time Fourier transform and the Wavelet transform. However, it cannot be applied to most real-world signals, because they are basically composites of some components and are not monochromatic. Huang *et al.* [19] overcame the difficulty by combining a mode decomposition part with the HSA, and the method of combination of them is the HHT.

Huang *et al.* developed a method to decompose an input data  $x(t)$  into zero-mean components and a non-oscillatory component. They named the method empirical mode decomposition (EMD) and also named the decomposed zero-mean components intrinsic mode functions (IMFs) of the input data. Algorithm 1 shows the procedure of the EMD, where  $c_i(t)$  and  $r(t)$  are the  $i$ th IMF and a non-oscillatory component of  $x(t)$ , respectively. The first step is forming the upper envelope  $u_{i,j}(t)$  and the lower envelope  $l_{i,j}(t)$ , connecting the maxima and the minima of the data by cubic splines. Then, the mean  $m_{i,j}(t)$  of these envelope is subtracted from the input data to obtain the residual  $x_{i,(j+1)}$ . When the mean  $m_{i,j}(t)$  becomes approximately zero after several iterations,  $x_{i,j}$  is adopted as the IMF  $c_i(t)$ , since it can be considered to be zero-mean. This criteria  $\epsilon_e$  is a parameter of the EMD. After all oscillatory components are extracted, the residual  $r(t)$  is a non-oscillatory component of  $x(t)$ . Letting  $N_{\text{IMF}}$  be the number of IMFs of  $x(t)$ ,  $x(t)$  is recovered by

$$x(t) = \sum_{n=1}^{N_{\text{IMF}}} c_n(t) + r(t). \quad (3.18)$$

IMFs,  $c_1(t), c_2(t), \dots, c_{N_{\text{IMF}}}(t)$ , are in order from the highest to the lowest frequency components. After the above decomposition, the IA and IP of each IMF can be estimated by the HSA. Consequently, letting  $a_n(t)$  and  $\phi_n(t)$  be the IA and IP of  $n$ th IMF, the data can be expressed as

$$x(t) = \sum_{n=1}^{N_{\text{IMF}}} a_n(t) \cos \phi_n(t) + r(t). \quad (3.19)$$

---

### Algorithm 1 Empirical mode decomposition

---

```

1:  $i = 1, x_1(t) = x(t)$ .
2: while  $x_i(t)$  contains oscillatory components do
3:    $j = 1, x_{i,1}(t) = x_i(t)$ 
4:   while the local mean of  $x_{i,j}(t)$  is not zero do
5:      $u_{i,j}(t) =$  (the upper envelope of  $x_{i,j}(t)$ ).
6:      $l_{i,j}(t) =$  (the lower envelope of  $x_{i,j}(t)$ ).
7:      $m_{i,j}(t) = (u_{i,j}(t) + l_{i,j}(t))/2$ .
8:      $x_{i,(j+1)}(t) = x_{i,j}(t) - m_{i,j}(t)$ .
9:      $j = j + 1$ 
10:  end while
11:   $c_i(t) = x_{i,j}(t)$ 
12:   $x_{i+1}(t) = x_i(t) - c_i(t)$ .
13:   $i = i + 1$ .
14: end while
15:  $r(t) = x_i(t)$ 

```

---

In this study, we used the ensemble EMD (EEMD) as the mode decomposition method. In the beginning of the EEMD,  $N_e$  white noises  $\{w^{(m)}(t)\}$  with the standard deviation being  $\sigma_e$  are created, and then the IMFs of the noise-added data  $x^{(m)}(t) = x(t) + w^{(m)}(t)$  are calculated

by the EMD:

$$x^{(m)}(t) = x(t) + w^{(m)}(t) \quad (3.20)$$

$$= \sum_{n=1}^{N_{\text{IMF}}} c_n^{(m)}(t) + r^{(m)}. \quad (3.21)$$

The IMFs of an input data  $x(t)$  are estimated as the mean of the corresponding IMFs of  $\{x^{(m)}(t)\}$ :

$$c_n(t) = \frac{1}{N_e} \sum_{m=1}^{N_e} c_n^{(m)}(t). \quad (3.22)$$

The EEMD has two parameters ( $\sigma_e, \epsilon_e$ ).  $\sigma_e$  is a standard deviation of added white noise, and  $\epsilon_e$  is a convergence-condition constant. The details of EEMD is shown in Refs. [23, 26].

The basic concept of HHT for the QNM is as follows. If the QNM is perfectly extracted in the  $j$ th IMF, the IA and IP of the IMF must be expressed by

$$a_j(t) = Ae^{-(t-t_0)/\tau}, \quad (3.23)$$

$$\phi_j(t) = 2\pi f_R(t - t_0) + \phi_0. \quad (3.24)$$

Therefore, we can estimate the QNM frequency by fitting the IA and IP individually.

In reality, the IMF also contains other modes before the QNM starts, and noise components become dominant after the QNM is sufficiently damped. Equations (3.23) and (3.24) do not hold in the merger phase and the noise-dominant segment. Therefore, to estimate the QNM frequency with Eqs. (3.23) and (3.24), we need to estimate the segment where IA and IP most properly fits the equations. We constructed a method to estimate the segment, named *QNM-dominant segment* (QDS), and the QNM frequency [24]. In the method, a bandpass filter, whose higher cutoff frequency is properly configured, will be applied as a preprocessing to extract a QNM into the 1st IMF.

## 2. QDS estimation

Here, we briefly describe how to estimate QDS  $[\hat{n}_0, \hat{n}_0 + \hat{N}]$ . Note that we represent discrete sequences with brackets, such as  $t[n]$  and  $a_1[n]$ . Assuming the QNM is extracted in the 1st IMF, and its merger time  $t[n_m]$  is known, we first search for the longest segment  $[n_b, n_e]$  after  $n_m$  where the  $a_1[n]$  decreases monotonically. For every possible subsegments  $[n_0, n_0 + N]$  of  $[n_b, n_e]$ , where  $N_{\text{min}} \leq N \leq n_e - n_b$ , we calculate root mean squared errors  $\text{RMSE}(n_0, N)$  of fitting  $\ln a_1[n]$  with  $bt[n] + c$ :

$$\text{RMSE}(n_0, N) = \min_{b,c} \sqrt{\frac{1}{N} \sum_{n=n_0}^{n_0+N-1} (\ln a_1[n] - bt[n] - c)^2}. \quad (3.25)$$

We set  $N_{\min}$  to 5, the same configuration with Ref. [24]. The optimal  $n_0$  for each  $N$  is determined by

$$\hat{n}_0(N) = \underset{n_0}{\operatorname{argmin}} \operatorname{RMSE}(n_0, N), \quad (3.26)$$

and we define  $e(N)$  as

$$e(N) = \operatorname{RMSE}(\hat{n}_0(N), N). \quad (3.27)$$

The optimal  $N$  is determined as the transition point of a slope of  $N-e(N)$  plot:

$$\hat{N} = \underset{N}{\operatorname{argmin}} [\operatorname{Err}(N_{\min}, N) + \operatorname{Err}(N+1, n_e - n_b)], \quad (3.28)$$

where  $\operatorname{Err}(N_1, N_2)$  is an error of the fitting  $e(N)$  with  $aN + b$ :

$$\operatorname{Err}(N_1, N_2) = \min_{a,b} \sqrt{\frac{\sum_{N=N_1}^{N_2} (e(N) - aN - b)^2}{N_2 - N_1}}. \quad (3.29)$$

Consequently, by letting  $\hat{n}_0 = \hat{n}_0(\hat{N})$ , the QDS  $[\hat{n}_0, \hat{n}_0 + \hat{N}]$  is estimated. Then, the QNM frequency  $f_{\text{qnm}}$  can be estimated by fitting the IA and IP with Eqs. (3.23) and (3.24) in the QDS.

### 3. Method

Here, we briefly explain the whole method to estimate QNM frequency from observed strain data  $h[n]$ . The outline of the method is described in Algorithm 2.

First, we have to determine a candidate sets  $F$ ,  $E$ ,  $\Sigma$ , which are sets of a higher cutoff frequency  $f_H$  of a bandpass filter, a convergence criteria  $\epsilon_e$  of the EEMD, and a standard deviation  $\sigma_e$  of the added-noise in the EEMD, respectively. In this study, we determined the sets as follows:

$$F = \{220, 225, 230, \dots, 500\} \text{ Hz}, \quad (3.30)$$

$$E = \{1 \times 10^{-1}, 4 \times 10^{-2}, 2 \times 10^{-2}, \dots, 1 \times 10^{-3}\}. \quad (3.31)$$

$$\Sigma = \{1 \times 10^{-3}, 4 \times 10^{-4}, 2 \times 10^{-4}, \dots, 1 \times 10^{-5}\}, \quad (3.32)$$

and  $f_L$  is set to 20 Hz. For each parameter candidates  $(f_H, \epsilon_e, \sigma_e)$ , a series of processing, including a bandpass filter with cutoff frequency  $(f_L, f_H)$ , the HHT and the search of QDS, is applied to the input strain data  $h[n]$ . After that, the optimal set of the parameters will be selected under an objective function  $O$ . We used the slope of the linear function obtained by fitting  $f_1[n]$  in the range of the searched QDS as the objective function  $O$ , since the IF must be flat in the QDS if a QNM is properly extracted.

---

### Algorithm 2 Estimation method of the QNM frequency with the HHT

---

**Require:** Strain  $h[n]$  contains a BBH signal and merger time  $t_m$  is known

- 1: **for all**  $(f_H, \epsilon_e, \sigma_e) \in F \otimes E \otimes \Sigma$  **do**
  - 2:      $h[n] \rightarrow h_{\text{filtered}}[n]$ :  
          apply a bandpass filter with cutoff freq.  $(f_L, f_H)$
  - 3:      $h_{\text{filtered}}[n] \rightarrow a_1[n], \phi_1[n]$ :  
          apply the HHT with parameters  $(\epsilon_e, \sigma_e)$ .
  - 4:      $[\hat{n}_0, \hat{n}_0 + \hat{N}]$ :  
          search the QNM dominant segment (QDS) in  $a_1[n]$
  - 5:      $\hat{f}_{\text{qnm}}(f_H, \hat{\epsilon}_e, \hat{\sigma}_e)$ :  
          estimate the QNM freq. by fitting  $a_1[n]$ ,  $\phi_1[n]$  in the QDS
  - 6: **end for**
  - 7:  $(\hat{f}_H, \hat{\epsilon}_e, \hat{\sigma}_e) = \underset{f_H, \epsilon_e, \sigma_e}{\operatorname{argmin}} O(f_H, \hat{\epsilon}_e, \hat{\sigma}_e)$ :  
          select the set of the parameters that optimizes an objective function  $O$ .
  - 8:  $\hat{f}_{\text{qnm}} = \hat{f}_{\text{qnm}}(\hat{f}_H, \hat{\epsilon}_e, \hat{\sigma}_e)$ :  
          the value of the selected combination is the estimated value of this method.
- 

### D. Autoregressive modeling (AR) method

#### 1. Basic idea

Autoregressive (AR) method is well-known time-sequence analysis method which are used in, e.g., acoustic signal processing [27]. Suppose we have the signal data of a segment,  $x_n = x(n\Delta t)$ , ( $n = 1, 2, \dots, N$ ). The main idea is to express the signal  $x_n$  with its previous  $M (< N)$  data,

$$x_n = \sum_{j=1}^M a_j x_{n-j} + \varepsilon, \quad (3.33)$$

where  $a_j$  and  $M$  are the coefficients and the order of AR model, respectively, and  $\varepsilon$  is the residual treated as white-noise in this modeling. If the data  $x_n$  is damped sinusoidal wave without noise, then we analytically can express  $x_n$  with  $M = 2$ . Even when the data includes noise, we expect to extract the actual signals by tuning  $N$  and  $M$ . There are various methods proposed to determine  $a_j$  and  $M$ . In this article, we present the results using Burg method for  $a_j$  and final prediction error (FPE) method for  $M$ . The details and other trials are in Ref. [28].

Once the model (3.33) is fixed, we then reconstruct wave signal from Eq. (3.33) and analyze it. By setting  $z(f) = e^{2\pi i f \Delta t}$ , the power spectrum of the wave signal can be expressed as

$$p(f) = \sigma^2 \left| 1 - \sum_{j=1}^M a_j z^{-j} \right|^{-2}, \quad (3.34)$$

where  $\sigma$  is the variance of  $\varepsilon$ . The resolution of frequency in Eq. (3.34) is not limited by the length of the original data set, so that AR method is expected to identify signal frequency more precisely than the standard (short-) Fourier decomposition.

From Eq. (3.33), the (local) maximums of the spectrum,  $p(f)$ , are given at

$$F(z) = 1 - \sum_{j=1}^M a_j z^{-j} \approx 0. \quad (3.35)$$

This is a  $M$ -th order polynomial equation. The solutions of the characteristic equation,  $F(z) = 0$ , also express the fundamental modes which consist the data segment. By interpreting the  $M$  solutions as  $z_k = e^{2\pi i f_k \Delta t}$  ( $k = 1, \dots, M$ ), we get the fundamental frequencies from the real part of  $f_k$ , and damping rates from the imaginary part of  $f_k$ . (Actually,  $|z_k| \leq 1$  is expected for the expression (3.33) to be stable.) Therefore, AR method can determine the frequencies and damping rates of quasi-normal modes from the data themselves.

## 2. Method

We divided the given mock data into segments of the length of  $\Delta T = 1/128$  sec ( $N = 32$ ). The neighboring segments are largely overlapping shifted only by 4 points. For each segment, we modeled the data with Eq. (3.33) with Burg and FPE methods. Normally  $M$  falls into the range 2–5.

We then get the power-spectrum  $p(f)$  from Eq. (3.34) at each segment, and locate its local maximums  $f_1, f_2, \dots$  with their one-sigma widths. We also solve Eq. (3.35) at each segment (which is at most 5-th order polynomial equation), and identify the solution  $z_k$  of which real part of frequency is the closest to  $f_1, f_2, \dots$ .

We list these solutions  $z_k$  of each segment, and check whether they remain almost unchanged over several segments. If the successive segments has a common frequency mode within one-sigma width, then we make a short list as the candidates for ring-down modes.

We see sometimes a segment is full of noises and shows quite different numbers from neighboring segments. Most cases, however, after the time of black hole merger, we can identify one common frequency which overlaps within one-sigma width for several data segments.

## E. Neural network (NN) method

### 1. Convolutional Neural Network

In this challenge, we use a ‘‘convolutional neural network’’ (CNN) which can extract local patterns of the data. CNNs are often used for the image recognition and we expect CNNs can be applied to the GW data

analysis [29]. We try various CNNs which have different structure, layers, neurons and filters. The final configuration of the CNN which is used here is shown in Table I.

In general, the input and output data of a convolutional layer have multi channel. The data with multi channel have multi values in each pixel (e.g., RGB components of images). In a convolutional layer, convolutions of the data containing  $L$  channels and the  $L'$  filters  $h$ ,

$$z'_{i,l'} = \sum_{l=1}^L \sum_{p=1}^H z_{i+p,l} h_{p,l'} + b_{i,l'}, \quad (3.36)$$

are calculated to extract the local patterns. Here,  $z$  and  $z'$  are the input and output vectors of the layer, respectively. The number and the length of the filters,  $L'$  and  $H$ , are fixed before training and the coefficients  $h$  and biases  $b$  are optimized in the training procedure. In this work, we use four convolutional layers. The lengths of the filters are 32, 16, 8, 8, and the numbers of the filters are 64, 128, 256, 512, respectively.

A pooling layer, often placed after a convolutional layer, compresses information, combining a few pixels into one pixel. In this work, we use the max pooling,

$$z'_i = \max_{k=1, \dots, p} z_{si+k}. \quad (3.37)$$

Here,  $s$  is the stride, and  $p$  is the size of the pooling filter. We set  $s = p = 2$ .

In most cases, dense layers, which are linear transformations,

$$z'_i = \sum_{j=1}^N w_{ij} z_j + b_i, \quad (3.38)$$

with  $N$  being the number of input values, are located after convolutional layers. The weights  $w$  and biases  $b$  are optimized in training.

An activation function plays an important role to carry out nonlinear transformations. In this work, we use the rectified linear unit (ReLU),

$$z' = h(z) = \max(z, 0), \quad (3.39)$$

as activation functions.

For accurate estimation, weights and biases in dense layers and filters in convolutional layers need to be optimized using training data. In the case of supervised learning, each training data is a pair of input data and target vector. In our work, the input data is the time series of a gravitational wave signal with noise and the target vector is composed of the QNM frequency,  $(f_R, f_I)$ . For an input, the neural network returns an estimated vector and compares it with a target vector. The loss function is computed to evaluate the error between the estimated and target vectors. In batch learning, a group of data, called ‘‘batch’’ with its size  $M$  fixed before training, is used to define the loss function. As the loss function, we adopt the mean square error,

$$J(y, t) = \frac{1}{M} \sum_{m=1}^M (y_m - t_m)^2, \quad (3.40)$$



TABLE I: The configuration of the CNN we use.  $(x, y)$  means that the data each layer returns has  $x$  points and  $y$  channels. Each input data has 2 channels,  $h_+$  and  $h_\times$ , which are composed of 256 points. A "Flatten" layer reshapes a 2-dimensional data to a 1-dimensional data.

| layer   | dimension       |
|---------|-----------------|
| Input   | (256, 2)        |
| Conv    | (256, 64)       |
| Pooling | (128, 64)       |
| ReLU    | (128, 64)       |
| Conv    | (128, 128)      |
| Pooling | (64, 128)       |
| ReLU    | (64, 128)       |
| Conv    | (64, 256)       |
| Pooling | (32, 256)       |
| ReLU    | (32, 256)       |
| Conv    | (32, 512)       |
| ReLU    | (32, 512)       |
| Flatten | $32 \times 512$ |
| Dense   | 256             |
| ReLU    | 256             |
| Dense   | 2               |
| Output  | 2               |

where  $y$  and  $t$  are the estimated and target vector, respectively. We set  $M = 16$ . As an optimization method, we use the Adam (Adaptive Moment Estimation) [30].

We use the Keras with the TensorFlow backend [31]. For accelerating learning with NVIDIA GPU, we employ the CUDA deep neural network library (cuDNN) [32].

## 2. Training Dataset

First, we construct the template bank for the training using the modified waveform which is based on the same method as Eqs. (2.14) and (2.16). For template bank,  $f_R$  and  $f_I$  are uniformly placed in the range 209–378 Hz and 23–69 Hz. The template bank contains  $21 \times 21$  waveforms.

Next, each waveform is injected to simulated noise data generated based on the aLIGO's design sensitivity (aLIGOZeroDetHighPower). We generate 20 different noise realizations for each waveform, and thus the training dataset contains 8,820 simulated signals including noise. They have the similar overall SNRs as the mock data.

After generating simulated signals, we carry out the following pre-processes. Each signal is bandpass filtered between 20 and 2048 Hz. From each signal, we pick up the segment that consists of 256 points starting from the coalescence time. Each segment is normalized to have a maximum value 1.0. We use these segments as the input data.

## IV. COMPARISON AND SUMMARY

### 1. Overview

We prepare 30 mock data in total by using the method described in Sec. II B. A half of them are generated using Eqs. (2.10) and (2.12), and the others are generated using Eqs. (2.14) and (2.16). We refer to the former as set A and the latter as set B. For both sets, we generated five mock data, respectively, with overall SNR,  $\rho_{\text{all}} = 60, 30$  and 20. The SNR for the ringdown part,  $\rho_{\text{rd}}$ , turned out to be roughly  $1/5 \sim 1/3$  of  $\rho_{\text{all}}$ . We listed the details of the mock data in Table II. We calculated  $\rho_{\text{rd}}$  by the standard inner product for the injected waveform after the peak of the amplitude without noise.

The five challenging groups received 30 data-files of  $h(t)$ , together with rough information of the merger time,  $t_0$ , for each data, but not with the frequency of the injected ringdown waveform,  $(f_R^{(\text{inj})}, f_I^{(\text{inj})})$ . Since  $(f_R^{(\text{inj})}, f_I^{(\text{inj})})$  are randomly shifted from the values in general relativity, the challengers cannot use the information of inspiral part for their estimation of  $(f_R, f_I)$ . Some methods (MF-R/MR, AR) can derive the estimation value  $(f_R, f_I)$  with their error bars, while some (HHT, NN) cannot. Therefore we simply compare the results of the estimated (center) values.

### 2. Results of Each Method

For 30 data, each challenging group handed in  $(f_R, f_I)$  as the result of their blind analyses. The differences  $(f_R - f_R^{(\text{inj})})/f_R^{(\text{inj})}$ , and  $(f_I - f_I^{(\text{inj})})/f_I^{(\text{inj})}$  and those absolute values are shown in Fig. 1 and Fig. 2, respectively.

As expected, the differences are smaller for larger  $\rho_{\text{rd}}$ . The message we can read from Fig. 1 and Fig. 2 is that we are able to estimate  $f_R$  within 10% (20 %) from  $f_R^{(\text{inj})}$  for the data  $\rho_{\text{rd}} > 15$  (8), while some points are exceptional.

The estimations of  $f_I$  are generally one-order worse than those of  $f_R$ . This tells us the difficulty of identifying the damping rate. For all five methods, we see the estimated values of  $f_R$  are roughly distributed around the injected one  $f_R^{(\text{inj})}$ , while there are some tendencies that  $f_I^{(\text{inj})}$  is over or underestimated, depending on the method. These results would be suggestive in the interpretation of the future application of each method to real data.

### 3. Comparison of Each Method

In order to compare five methods in detail, we introduce the logarithmic average and variance defined by

$$\overline{\delta \log Q} = \frac{1}{N} \sum_{n=1}^N \left( \log \frac{Q_n^{(\text{estimate})}}{Q_n^{(\text{inj})}} \right),$$

TABLE II: List of mock data. Set A was generated using Eqs. (2.10) and (2.12), while Set B was from Eqs. (2.14) and (2.16). The overall SNR  $\rho_{\text{all}}$ , SNR of the ringdown part  $\rho_{\text{rd}}$ , and injected value of the ringdown waveform ( $f_{\text{R}}^{(\text{inj})}$ ,  $f_{\text{I}}^{(\text{inj})}$ ) are shown.

| data | SNR                 |                    | injected                      |                               |
|------|---------------------|--------------------|-------------------------------|-------------------------------|
|      | $\rho_{\text{all}}$ | $\rho_{\text{rd}}$ | $f_{\text{R}}^{(\text{inj})}$ | $f_{\text{I}}^{(\text{inj})}$ |
| A-01 | 60.0                | 13.81              | 260.68                        | 44.58                         |
| A-02 | 60.0                | 12.73              | 345.16                        | 50.49                         |
| A-03 | 60.0                | 13.79              | 382.53                        | 32.58                         |
| A-04 | 60.0                | 11.84              | 284.18                        | 44.73                         |
| A-05 | 60.0                | 16.78              | 346.20                        | 23.07                         |
| A-06 | 30.0                | 5.57               | 272.85                        | 33.40                         |
| A-07 | 30.0                | 6.56               | 272.85                        | 44.54                         |
| A-08 | 30.0                | 7.27               | 301.89                        | 42.24                         |
| A-09 | 30.0                | 6.93               | 324.60                        | 27.25                         |
| A-10 | 30.0                | 7.88               | 282.55                        | 37.45                         |
| A-11 | 20.0                | 6.36               | 314.24                        | 30.58                         |
| A-12 | 20.0                | 3.45               | 382.10                        | 48.60                         |
| A-13 | 20.0                | 4.68               | 249.36                        | 47.97                         |
| A-14 | 20.0                | 4.13               | 299.32                        | 41.88                         |
| A-15 | 20.0                | 4.54               | 319.42                        | 31.55                         |
| B-01 | 60.0                | 17.58              | 352.56                        | 36.20                         |
| B-02 | 60.0                | 14.27              | 210.78                        | 42.77                         |
| B-03 | 60.0                | 13.67              | 258.83                        | 48.42                         |
| B-04 | 60.0                | 20.09              | 271.13                        | 25.40                         |
| B-05 | 60.0                | 17.07              | 291.99                        | 34.20                         |
| B-06 | 30.0                | 9.53               | 411.57                        | 29.48                         |
| B-07 | 30.0                | 6.29               | 295.78                        | 59.38                         |
| B-08 | 30.0                | 6.03               | 312.39                        | 59.24                         |
| B-09 | 30.0                | 6.01               | 198.34                        | 57.91                         |
| B-10 | 30.0                | 8.31               | 323.32                        | 37.86                         |
| B-11 | 20.0                | 5.20               | 208.80                        | 39.75                         |
| B-12 | 20.0                | 6.60               | 246.66                        | 27.85                         |
| B-13 | 20.0                | 4.46               | 323.71                        | 62.51                         |
| B-14 | 20.0                | 6.20               | 215.15                        | 33.15                         |
| B-15 | 20.0                | 5.85               | 335.20                        | 25.11                         |

$$\sigma(Q) = \left[ \frac{1}{N} \sum_{n=1}^N \left( \log \frac{Q_n^{(\text{estimate})}}{Q_n^{(\text{inj})}} \right)^2 \right]^{1/2}, \quad (4.1)$$

as indicators of the bias and the average magnitude of the parameter estimation error, where  $Q_n^{(\text{estimate})}$  is the estimated value of the quantity  $Q$  for the  $n$ th data and  $Q_n^{(\text{inj})}$  is the corresponding injected values. In Table III, we show the values of  $\overline{\delta \log f_{\text{R}}}$ ,  $\sigma(f_{\text{R}})$ ,  $\overline{\delta \log f_{\text{I}}}$  and  $\sigma(f_{\text{I}})$  for the methods we tried. We show the results limited to set A on the first law and those limited to set B on the second law.

We should recall that in the actual implementation of the MF-MR described in Sec. III B we adopt the same modified template that is used to generate the set B mock

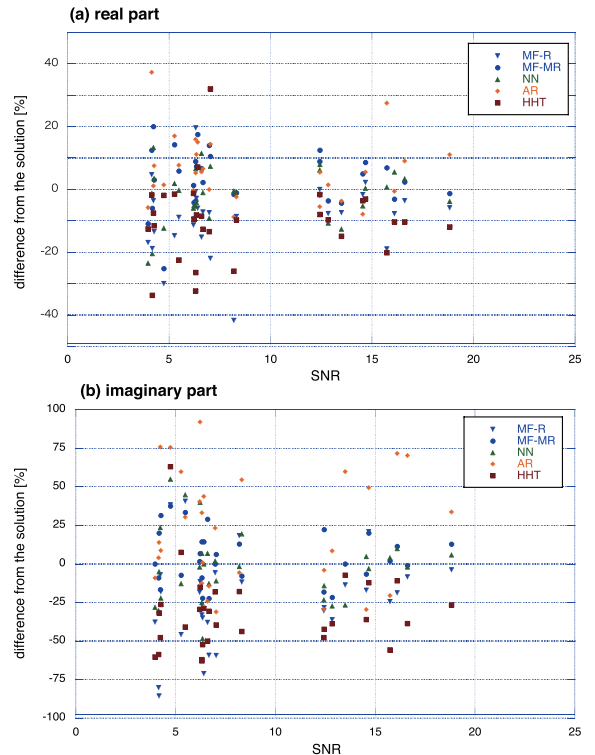


FIG. 1: Differences between  $(f_{\text{R}}, f_{\text{I}})$  and  $(f_{\text{R}}^{(\text{inj})}, f_{\text{I}}^{(\text{inj})})$  for five methods with the horizontal axis of  $\rho_{\text{rd}}$ . The abbreviations are MF-R (matched filtering with ringdown part), MF-MR (matched filtering with both merger and ringdown parts), HHT (Hilbert-Huang transformation), AR (autoregressive modeling), and NN (neural network).

TABLE III: We show the values of  $\overline{\delta \log f_{\text{R}}}$ ,  $\sigma(f_{\text{R}})$ ,  $\overline{\delta \log f_{\text{I}}}$  and  $\sigma(f_{\text{I}})$  for various methods. The results limited to set A are given on the first law of each method while those limited to set B on the second.

|       |   | $\overline{\delta \log f_{\text{R}}}(\%)$ | $\sigma(f_{\text{R}})(\%)$ | $\overline{\delta \log f_{\text{I}}}(\%)$ | $\sigma(f_{\text{I}})(\%)$ |
|-------|---|---|----------------------------|---|----------------------------|
| MF-R  | A | -5.96                                     | 13.27                      | -49.02                                    | 78.67                      |
|       | B | -14.27                                    | 18.79                      | -26.63                                    | 42.05                      |
| MF-MR | A | 2.99                                      | 12.57                      | 0.55                                      | 17.49                      |
|       | B | 2.97                                      | 5.62                       | 5.57                                      | 15.35                      |
| HHT   | A | -12.73                                    | 18.58                      | -49.95                                    | 63.58                      |
|       | B | -9.74                                     | 16.07                      | -38.58                                    | 44.57                      |
| AR    | A | 4.93                                      | 10.66                      | 12.85                                     | 27.44                      |
|       | B | 6.64                                      | 13.58                      | 17.55                                     | 38.07                      |
| NN    | A | -4.50                                     | 12.08                      | -13.48                                    | 29.34                      |
|       | B | 0.69                                      | 4.75                       | 3.36                                      | 16.67                      |

data. Also, the NN method introduced in Sec. III E uses the template bank generated in the same way as the set B mock data to train the network.

The error of MF-R using the simple damped sinusoidal

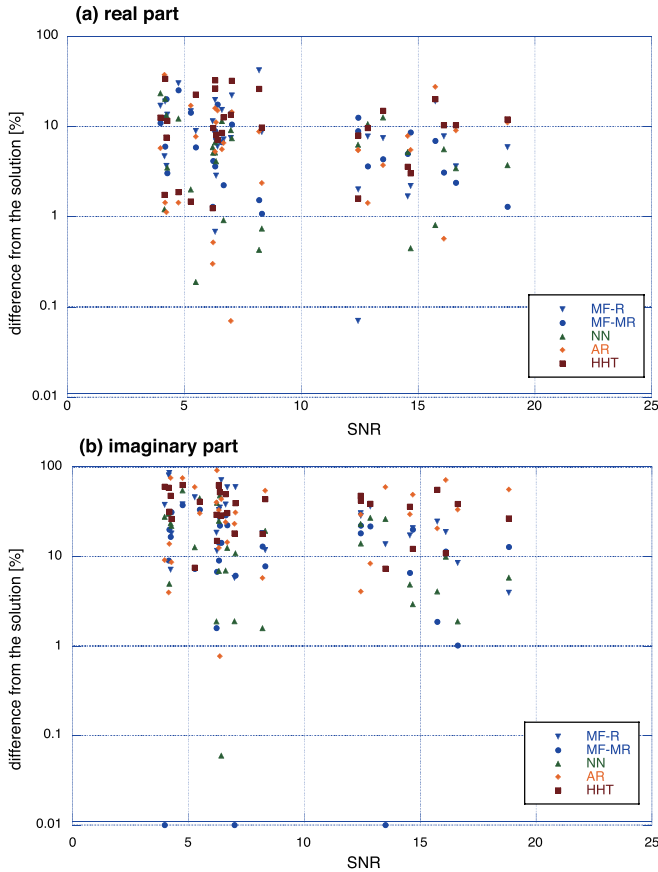


FIG. 2: Absolute differences between  $(f_R, f_I)$  and  $(f_R^{(inj)}, f_I^{(inj)})$  for five methods with the horizontal axis of  $\rho_{rd}$ .

waveform is relatively large as expected. In fact, the error of the estimate of  $f_I$  is the largest among five methods. The results of HHT method are not so good, either. At least, the current way of using MF-R or HHT for the estimate of imaginary part of QNM frequency does not seem to be competitive compared with the other methods. However, as for the real part, we find that MF-R and HHT are not so bad, if we recall that the comparison with MF-MR and NN is not fare in the case of the set B mock data.

The performances of the other three methods, i.e., MF-MR, AR and NN methods, are almost comparable. The results of MF-MR and NN are better for set B, as expected. The difference can be clearly seen except for the imaginary part of MF-MR. For AR method the error is smaller for set A, but it is not clear why AR gives a better estimate for set A.

The variance might be determined by a small number of data with a large error. To check if it is the case or not, we give plots of the absolute magnitude of the error  $|\log(Q_n^{(estimate)}/Q_n^{(inj)})|$  sorted in the ascending order for each method in Fig. 3. Although the number of data is small, these figures tell that the tendencies mentioned above are not the ones that hold only for the data with

a large error. We cannot explain the reason why the results are different between set A and set B for the other methods than MF-MR and NN.

#### 4. Error estimate

MF-MR and AR methods give the error estimates as explained in Sec. III. The consistency of these error estimate is briefly checked below.

In the case of MF-MR method for set B data, the expected result is obtained. Namely, there are 30 numbers to guess in the present test (15 real parts and 15 imaginary parts). The 90% confidence interval is given by cutting 5% probability regions on both small and large value sides. This estimate of the confidence interval just take into account the statistical error. The true value fell outside of the confidence interval 1 times out of 30. This is slightly better than the expectation. On the other hand, for set A data, this happened 11 times, which means that the contribution of the systematic bias is significantly large.

For AR method, the true value becomes outside the confidence interval only once for the imaginary part while it happened 14 times for the real part.

## V. CONCLUDING REMARKS

We implemented five methods for extracting ringdown waves solely, and tested them with mock data by a method of “blind analysis”.

Comparison shows that all five methods can pick up the frequency of ringdown wave  $f_R$  within 10% difference from the injected one for the SNR of the ringdown part greater than 15, while the imaginary part of the frequency  $f_I$  (related to the damping period) is rather difficult to determine. However, we believe that the possibility and the limitation of independent estimation of ringdown mode was shown in this paper, and this opens a way of testing gravity theories.

When the error circles derived by using some combination of several methods are overlapping, we might be able to more confidently claim that the QNM frequency is determined by the observational data. However, currently only two of our analysis methods (MF-R/MR, AR) can give the error circle, and the others (HHT, NN) cannot. Once we have various methods whose error estimate is reliable, there might be a possibility to combine the estimates properly.

Through the mock data challenges, we also learned the directions of further improvements of each methods.

- The MF methods do not have much room for further improvement. As for MF-MR, one possible extension is to adopt a little wider class of templates which depend on parameters other than the QNM

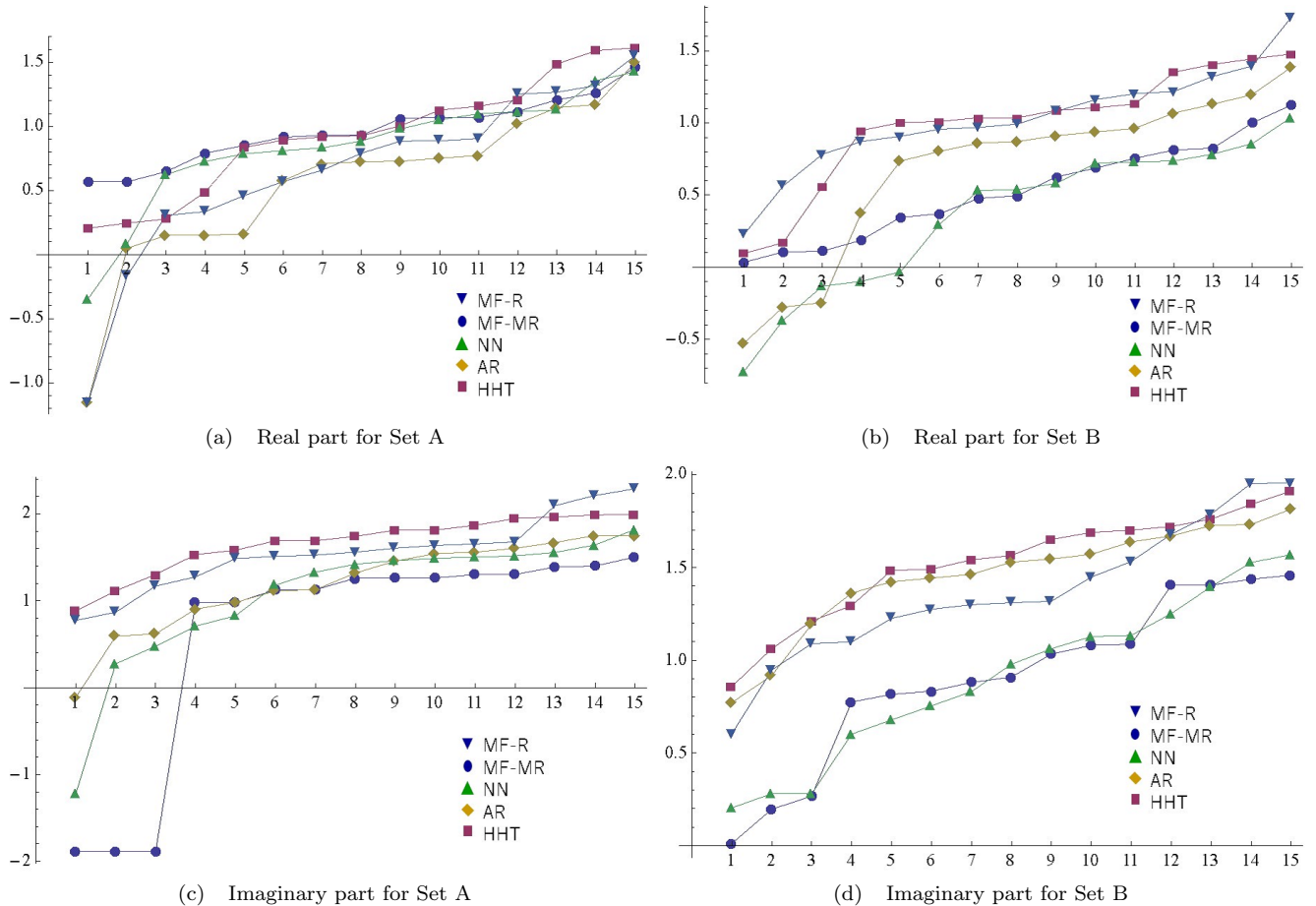


FIG. 3: Plots of the base 10 logarithm of the error in the estimate for each test data. The data number is sorted for each method in the ascending order of the magnitude of the error.

frequency. However, the preliminary trial calculations suggest that the extension in this direction will not be so successful.

- The AR method, presented here, used the Burg method for fitting data and final prediction error method for fixing length of data sequence. We think that it will be interesting to compare with similar but slightly differ approaches, such as those proposed by Berti *et al.* [33].
- In the HHT method, there exists the mode-splitting problem of the EMD [34]. We are planning to resolve the problem by taking sparsity in the frequency domain into account to the EMD. It may improve the accuracy of the extraction of a QNM since the instantaneous frequency of the QNM is constant.
- The NN method can give only single value for estimation. We need to find the method how NN can estimate the prediction error. In Ref. [29], the hierarchical training is proposed. We will implement

this to make it for NN possible to be applied to the signals having any SNR.

After implementations of such improvements, we are planning to apply our methods to the real GW data, to discuss the validity of general relativity.

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