安定な数値シミュレーションを行うためのEinstein方程式の定式化とその検証

真貝寿明　@　理化学研究所　計算科学技術推進室

hshinkai@postman.riken.go.jp　　　http://atlas.riken.go.jp/~shinkai

Outline:

- **数値相対論に適したEinstein方程式の定式化が，さまざまに提案されてきたが，それらを統一的に理解する方法を提唱．**
- **「時間発展と共に系が拘束面に漸近的に収束してゆく」発展システムが，既知の発展方程式に拘束条件を用いてLagrange乗数補正することに得られることを示す．**
- **アイデアの中核となるのは，拘束条件式の発展を事前に計算し，Fourierモードに直して，固有値解析を行うこと．**
- **これまでに広く使われているADMや，Nakamura et alの方程式が，更に良い安定性を持つことを，数値的に実証．**

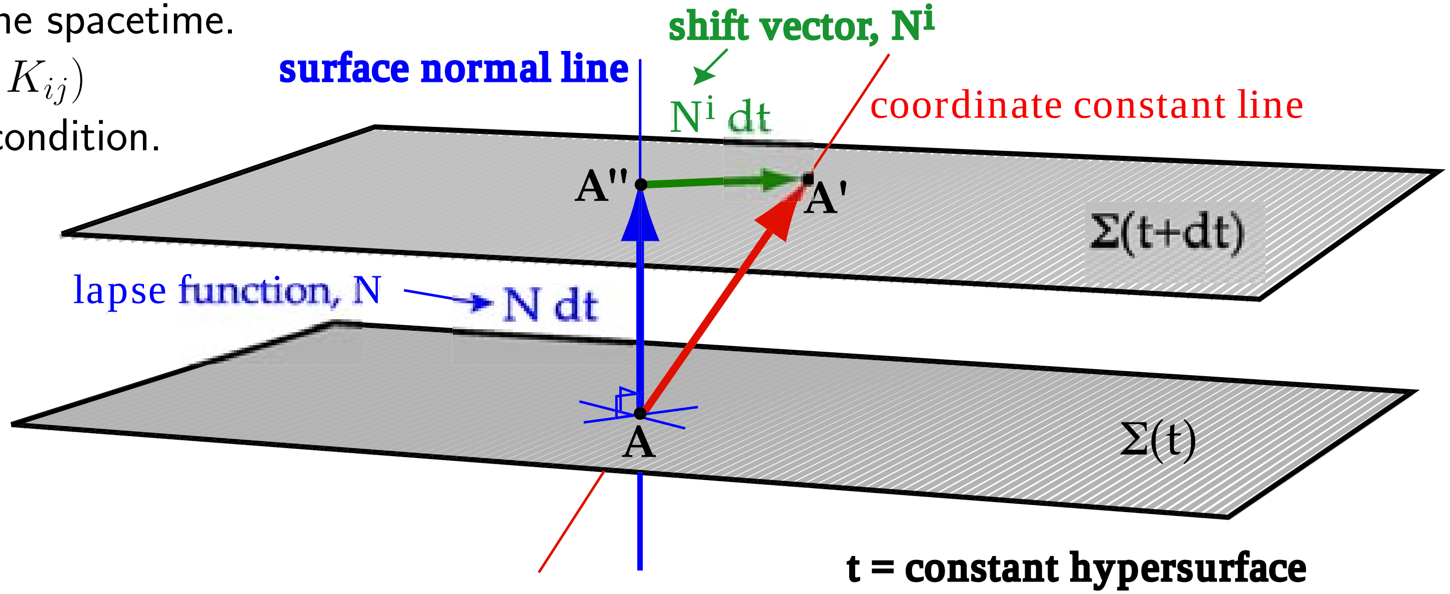
Refs:

Ashtekar variables	PRL **82** (1999) 263, PRD **60** (1999) 101502, IJMPD **9** (2000) 13, CQG **17** (2000) 4799, CQG **18** (2001) 441
ADM variables	PRD **63** (2001) 120419, CQG **19** (2002) 1027
BSSN variables	gr-qc/0204002 (PRD in print)
general	gr-qc/0209106
general	gr-qc/0209111 (review article, to be published from Nove Science Publ.)

早稲田大数理　米田元　との共同研究，科研費 No. 14740179 (2002-2005) + **理研基礎科学特別研究員研究費**

strategy 0 The standard approach :: Arnowitt-Deser-Misner (ADM) formulation (1962)

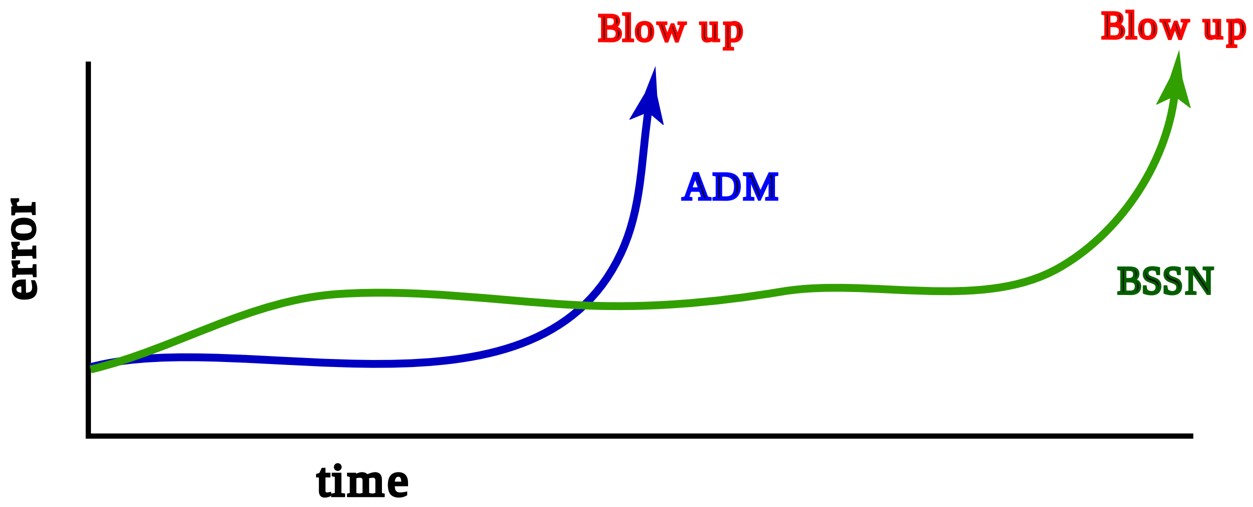
3+1 decomposition of the spacetime.
Evolve 12 variables (γ_{ij}, K_{ij})
with a choice of gauge condition.



	Maxwell eqs.	ADM Einstein eq.
constraints	div $\mathbf{E} = 4\pi\rho$ div $\mathbf{B} = 0$	${}^{(3)}R + (\mathrm{tr}K)^2 - K_{ij}K^{ij} = 2\kappa\rho_H + 2\Lambda$ $D_j K^j{}_i - D_i \mathrm{tr}K = \kappa J_i$
evolution eqs.	$\frac{1}{c}\partial_t \mathbf{E} = \text{rot } \mathbf{B} - \frac{4\pi}{c}\mathbf{j}$ $\frac{1}{c}\partial_t \mathbf{B} = -\text{rot } \mathbf{E}$	$\partial_t \gamma_{ij} = -2NK_{ij} + D_j N_i + D_i N_j,$ $\partial_t K_{ij} = N({}^{(3)}R_{ij} + \mathrm{tr}K K_{ij}) - 2NK_{il}K^l{}_j - D_i D_j N$ $+ (D_j N^m)K_{mi} + (D_i N^m)K_{mj} + N^m D_m K_{ij} - N\gamma_{ij}\Lambda$ $- \kappa\alpha\{S_{ij} + \frac{1}{2}\gamma_{ij}(\rho_H - \mathrm{tr}S)\}$

Best formulation of the Einstein eqs. for long-term stable & accurate simulation?

Many (too many) trials and errors, not yet a definit recipe.



Mathematically equivalent formulations, but differ in its stability!

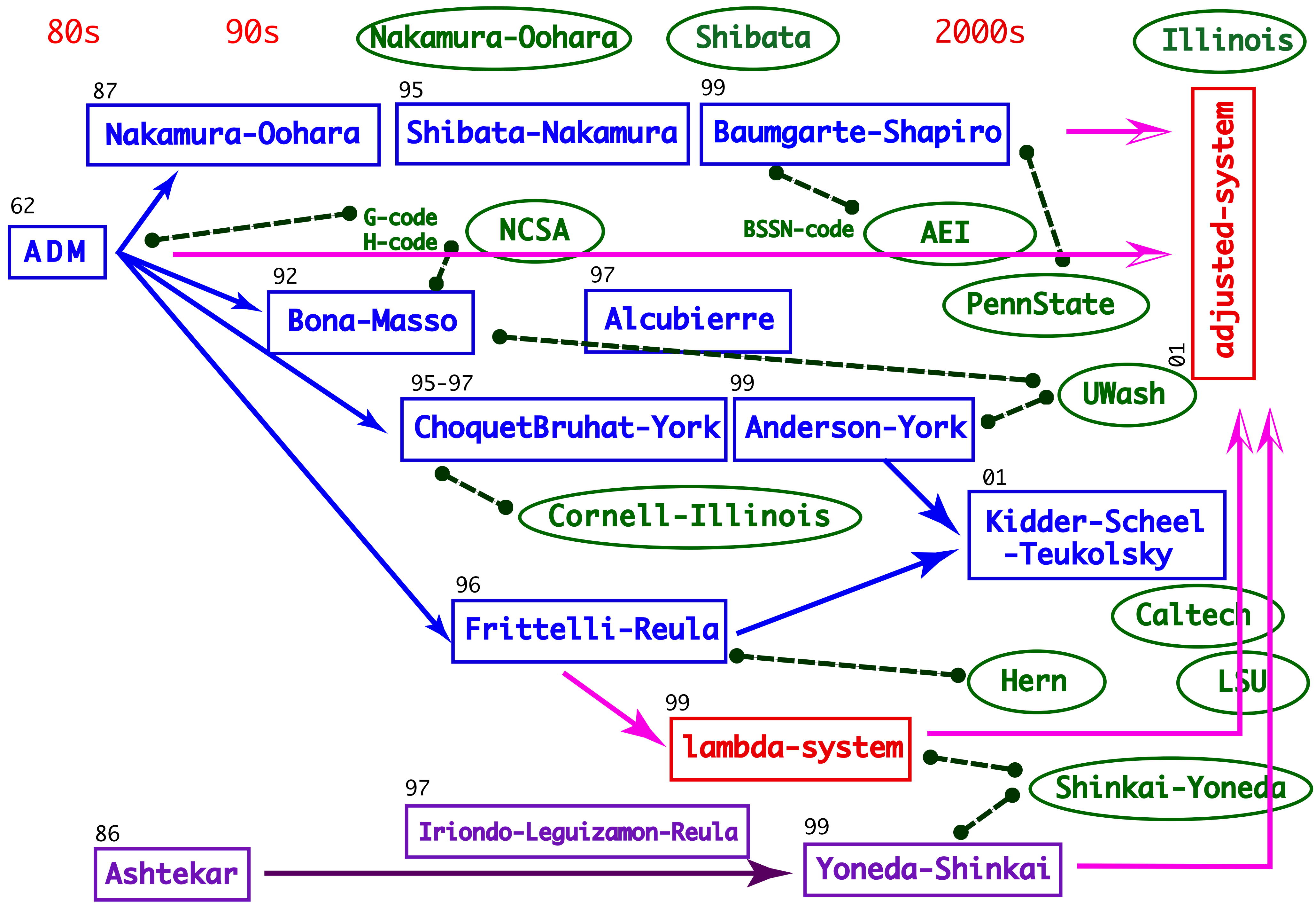
strategy 0: Arnowitt-Deser-Misner formulation

strategy 1: Shibata-Nakamura's (Baumgarte-Shapiro's) modifications to the standard ADM

strategy 2: Apply a formulation which reveals a hyperbolicity explicitly

strategy 3: Formulate a system which is "asymptotically constrained" against a violation of constraints

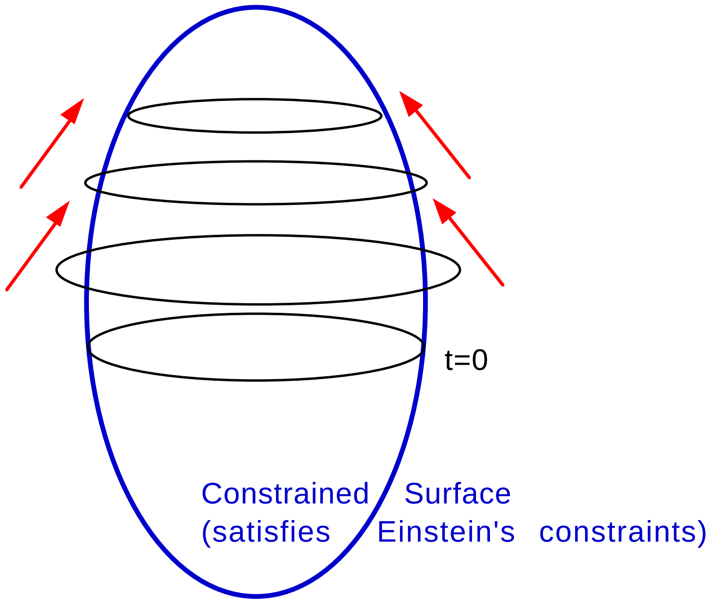
By adding constraints in RHS, we can kill error-growing modes
$\Rightarrow$ How can we understand the features systematically?

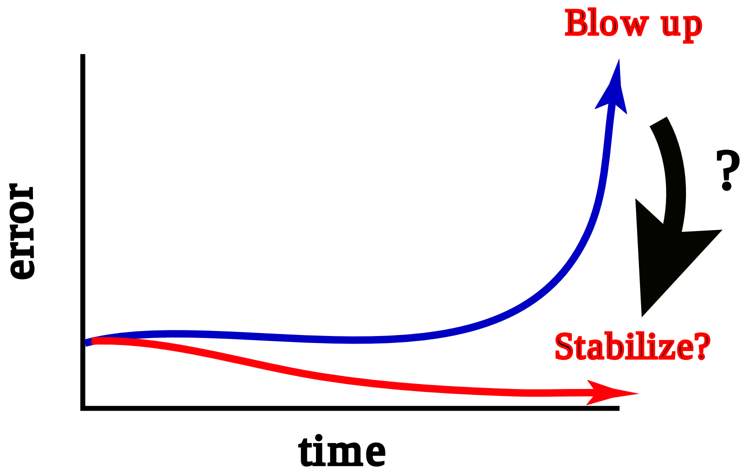


80s
90s
Nakamura-Oohara
Shibata
2000s
Illinois
87
Nakamura-Oohara
95
Shibata-Nakamura
99
Baumgarte-Shapiro
adjusted-system
01
62
ADM
G-code
H-code
NCSA
BSSN-code
AEI
92
Bona-Masso
97
Alcubierre
PennState
95-97
ChoquetBruhat-York
99
Anderson-York
UWash
Cornell-Illinois
01
Kidder-Scheel
-Teukolsky
96
Frittelli-Reula
Caltech
Hern
LSU
99
lambda-system
Shinkai-Yoneda
97
Iriondo-Leguizamon-Reula
86
Ashtekar
99
Yoneda-Shinkai

strategy 3 Formulate a system which is "asymptotically constrained" against a violation of constraints

"Asymptotically Constrained System"– Constraint Surface as an Attractor





method 1: λ-system (Brodbeck et al, 2000)

– Add aritificial force to reduce the violation of constraints

– To be guaranteed if we apply the idea to a symmetric hyperbolic system.

method 2: Adjusted system (Yoneda HS, 2000, 2001)

– We can control the violation of constraints by adjusting constraints to EoM.

– Eigenvalue analysis of constraint propagation equations may prodict the violation of error.

– This idea is applicable even if the system is not symmetric hyperbolic. ⇒ for the ADM/BSSN formulation, too!!

Idea of "Adjusted system" and Our Conjecture

CQG18 (2001) 441, PRD 63 (2001) 120419, CQG 19 (2002) 1027

General Procedure

1. prepare a set of evolution eqs. $\partial_t u^a = f(u^a, \partial_b u^a, \cdots)$
2. add constraints in RHS $\partial_t u^a = f(u^a, \partial_b u^a, \cdots) \underbrace{+F(C^a, \partial_b C^a, \cdots)}$
3. choose appropriate $F(C^a, \partial_b C^a, \cdots)$
   to make the system stable evolution

How to specify $F(C^a, \partial_b C^a, \cdots)$?

4. prepare constraint propagation eqs. $\partial_t C^a = g(C^a, \partial_b C^a, \cdots)$
5. and its adjusted version $\partial_t C^a = g(C^a, \partial_b C^a, \cdots) \underbrace{+G(C^a, \partial_b C^a, \cdots)}$
6. Fourier transform and evaluate eigenvalues $\partial_t \hat{C}^k = \underbrace{A(\hat{C}^a)}\ \hat{C}^k$

Conjecture: Evaluate eigenvalues of (Fourier-transformed) constraint propagation eqs.
If their (1) real part is non-positive, or (2) imaginary part is non-zero, then the system is more stable.

Example: the Maxwell equations

Yoneda HS, CQG 18 (2001) 441

Maxwell evolution equations.

$$\begin{aligned}\partial_t E_i &= c\epsilon_i{}^{jk}\partial_j B_k + P_i C_E + Q_i C_B,\\ \partial_t B_i &= -c\epsilon_i{}^{jk}\partial_j E_k + R_i C_E + S_i C_B,\\ &C_E = \partial_i E^i \approx 0, \quad C_B = \partial_i B^i \approx 0,\end{aligned}$$

$$\begin{cases}\text{sym. hyp} & \Leftrightarrow \quad P_i = Q_i = R_i = S_i = 0,\\ \text{strongly hyp} & \Leftrightarrow \quad (P_i - S_i)^2 + 4R_iQ_i > 0,\\ \text{weakly hyp} & \Leftrightarrow \quad (P_i - S_i)^2 + 4R_iQ_i \geq 0\end{cases}$$

Constraint propagation equations

$$\begin{aligned}\partial_t C_E &= (\partial_i P^i)C_E + P^i(\partial_i C_E) + (\partial_i Q^i)C_B + Q^i(\partial_i C_B),\\ \partial_t C_B &= (\partial_i R^i)C_E + R^i(\partial_i C_E) + (\partial_i S^i)C_B + S^i(\partial_i C_B),\end{aligned}$$

$$\begin{cases}\text{sym. hyp} & \Leftrightarrow \quad Q_i = R_i,\\ \text{strongly hyp} & \Leftrightarrow \quad (P_i - S_i)^2 + 4R_iQ_i > 0,\\ \text{weakly hyp} & \Leftrightarrow \quad (P_i - S_i)^2 + 4R_iQ_i \geq 0\end{cases}$$

CAFs?

$$\partial_t \begin{pmatrix}\hat{C}_E\\ \hat{C}_B\end{pmatrix} = \begin{pmatrix}\partial_i P^i + P^i k_i & \partial_i Q^i + Q^i k_i\\ \partial_i R^i + R^i k_i & \partial_i S^i + S^i k_i\end{pmatrix} \partial_l \begin{pmatrix}\hat{C}_E\\ \hat{C}_B\end{pmatrix} \approx \begin{pmatrix}P^i k_i & Q^i k_i\\ R^i k_i & S^i k_i\end{pmatrix}\begin{pmatrix}\hat{C}_E\\ \hat{C}_B\end{pmatrix} =: T\begin{pmatrix}\hat{C}_E\\ \hat{C}_B\end{pmatrix}$$

$$\Rightarrow \text{CAFs} = (P^i k_i + S^i k_i \pm \sqrt{(P^i k_i + S^i k_i)^2 + 4(Q^i k_i R^j k_j - P^i k_i S^j k_j)})/2$$

Therefore CAFs become negative-real when

$$P^i k_i + S^i k_i < 0, \qquad \text{and} \qquad Q^i k_i R^j k_j - P^i k_i S^j k_j < 0$$

HS original GR code **(2002/October)**

Use the Cactus-code base structure (`http://cactuscode.org`)

-- parallelize, parameter control, I/O, PUGH, elliptic solvers, ...

-- original module for all GR part (initial data/ADMevolution)

`Cactus/arrangements/GR/`

PC cluster with 4 **(2002/September)**

Pentium 4, 2.53GHz, 2GB each, 80 GHD each, gigabit ether

-- about 1.2 x 10^6 yen for total parts

-- TurboLinux 7, Intel Fortran compiler, MPICH, ... all free

-- possible up to 120^3 grid full GR simulation

Grant-in-Aid for Scientific Research Fund of JSPS, No.14740179 (2002-2005)

3 Adjusted ADM systems

We adjust the standard ADM system using constraints as:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \quad (1)$$

$$+P_{ij}\mathcal{H} + Q^k{}_{ij}\mathcal{M}_k + p^k{}_{ij}(\nabla_k \mathcal{H}) + q^{kl}{}_{ij}(\nabla_k \mathcal{M}_l), \quad (2)$$

$$\partial_t K_{ij} = \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij}, \quad (3)$$

$$+R_{ij}\mathcal{H} + S^k{}_{ij}\mathcal{M}_k + r^k{}_{ij}(\nabla_k \mathcal{H}) + s^{kl}{}_{ij}(\nabla_k \mathcal{M}_l), \quad (4)$$

with constraint equations

$$\mathcal{H} := R^{(3)} + K^2 - K_{ij}K^{ij}, \quad (5)$$

$$\mathcal{M}_i := \nabla_j K^j{}_i - \nabla_i K. \quad (6)$$

We can write the adjusted constraint propagation equations as

$$\partial_t \mathcal{H} = \text{(original terms)} + H_1^{mn}[(2)] + H_2^{imn}\partial_i[(2)] + H_3^{ijmn}\partial_i\partial_j[(2)] + H_4^{mn}[(4)], \quad (7)$$

$$\partial_t \mathcal{M}_i = \text{(original terms)} + M_{1i}{}^{mn}[(2)] + M_{2i}{}^{jmn}\partial_j[(2)] + M_{3i}{}^{mn}[(4)] + M_{4i}{}^{jmn}\partial_j[(4)]. \quad (8)$$

Example 1: standard ADM vs original ADM (in Schwarzschild coordinate)

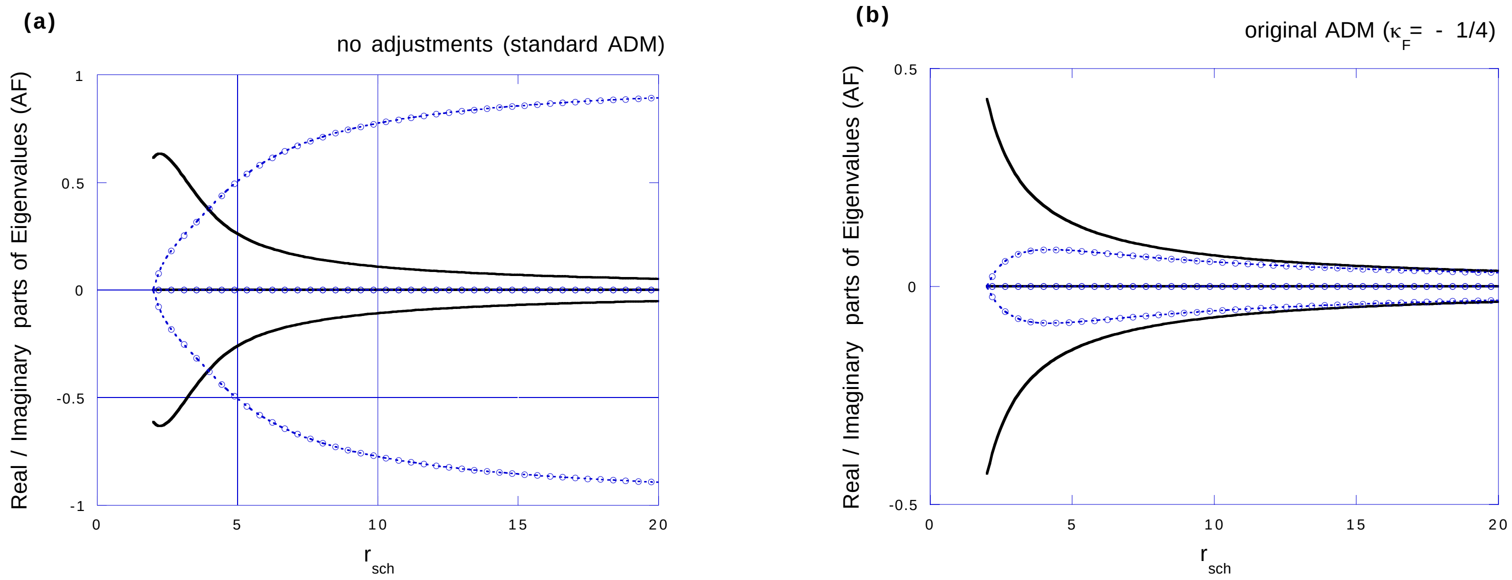


Figure 1: Amplification factors (AFs, eigenvalues of homogenized constraint propagation equations) are shown for the standard Schwarzschild coordinate, with (a) no adjustments, i.e., standard ADM, (b) original ADM ($\kappa_F = -1/4$). The solid lines and the dotted lines with circles are real parts and imaginary parts, respectively. They are four lines each, but actually the two eigenvalues are zero for all cases. Plotting range is $2 < r \leq 20$ using Schwarzschild radial coordinate. We set $k = 1, l = 2$, and $m = 2$ throughout the article.

$$\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \\
\partial_t K_{ij} &= \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij} + \kappa_F \alpha \gamma_{ij} \mathcal{H},
\end{aligned}$$

Example 2: Detweiler-type adjusted (in Schwarzschild coord.)

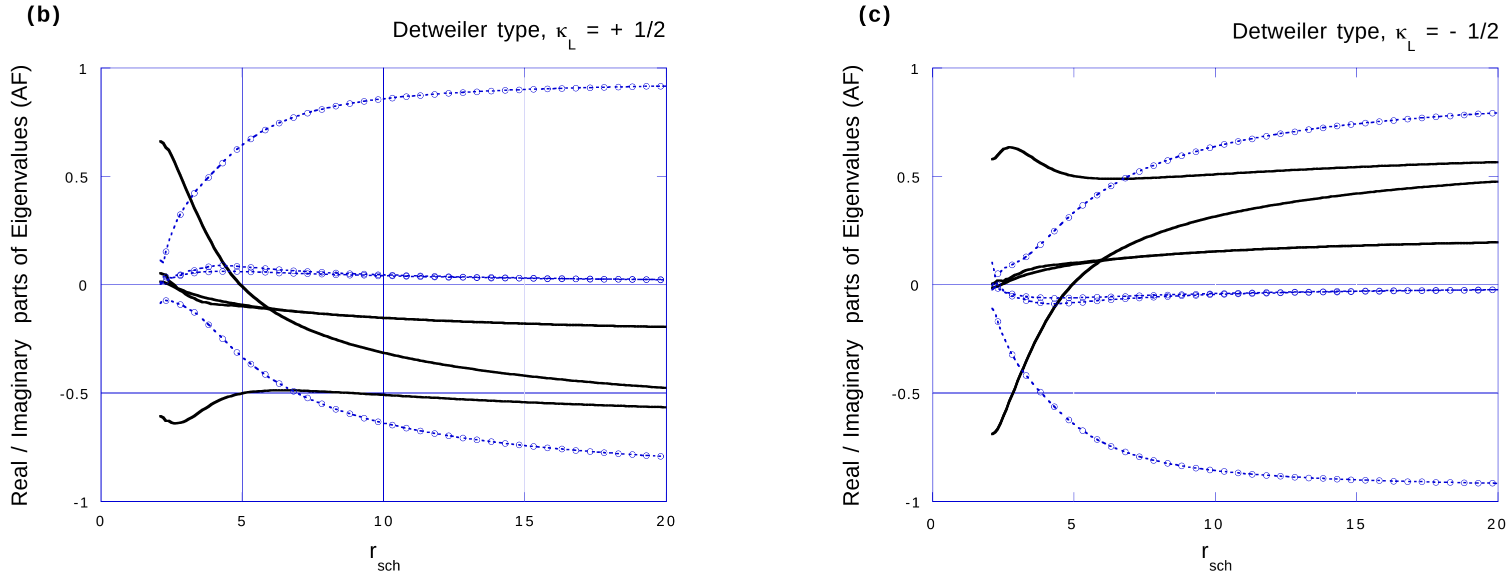


Figure 2: Amplification factors of the standard Schwarzschild coordinate, with Detweiler type adjustments. Multipliers used in the plot are (b) $\kappa_L = +1/2$, and (c) $\kappa_L = -1/2$.

$$\partial_t \gamma_{ij} = \text{(original terms)} + P_{ij}\mathcal{H},$$

$$\partial_t K_{ij} = \text{(original terms)} + R_{ij}\mathcal{H} + S^k{}_{ij}\mathcal{M}_k + s^{kl}{}_{ij}(\nabla_k \mathcal{M}_l),$$

$$\text{where} \quad P_{ij} = -\kappa_L \alpha^3 \gamma_{ij}, \quad R_{ij} = \kappa_L \alpha^3 (K_{ij} - (1/3)K\gamma_{ij}),$$

$$S^k{}_{ij} = \kappa_L \alpha^2 [3(\partial_{(i}\alpha)\delta^k_{j)} - (\partial_l \alpha)\gamma_{ij}\gamma^{kl}], \quad s^{kl}{}_{ij} = \kappa_L \alpha^3 [\delta^k_{(i}\delta^l_{j)} - (1/3)\gamma_{ij}\gamma^{kl}],$$

Comparisons of Adjusted ADM systems (Teukolsky wave)

3-dim, harmonic slice, periodic BC   HS original Cactus/GR code

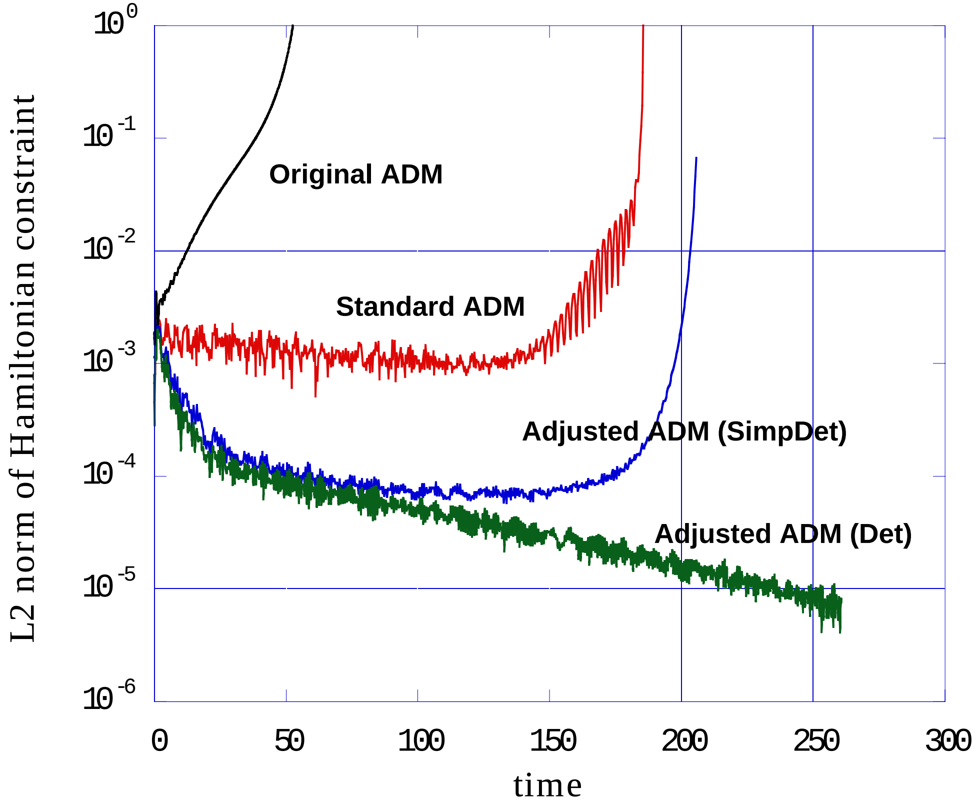


Figure 1: Violation of Hamiltonian constraints versus time: Adjusted ADM systems applied for Teukolsky wave initial data evolution with harmonic slicing, and with periodic boundary condition. Cactus/GR/evolveADMeq code was used. Grid = 24^3, $\Delta x = 0.25$, iterative Crank-Nicholson method.

strategy 1 Shibata-Nakamura's (Baumgarte-Shapiro's) modifications to the standard ADM

– define new variables ($\phi, \tilde{\gamma}_{ij}, K, \tilde{A}_{ij}, \tilde{\Gamma}^i$), instead of the ADM's (γ_{ij}, K_{ij}) where

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}, \qquad \tilde{A}_{ij} \equiv e^{-4\phi}(K_{ij} - (1/3)\gamma_{ij}K), \qquad \tilde{\Gamma}^i \equiv \tilde{\Gamma}^i_{jk}\tilde{\gamma}^{jk},$$

use momentum constraint in Γ^i-eq., and impose $\det\tilde{\gamma}_{ij} = 1$ during the evolutions.

– The set of evolution equations become

$$\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\phi &= -(1/6)\alpha K, \\
(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} &= -2\alpha\tilde{A}_{ij}, \\
(\partial_t - \mathcal{L}_\beta)K &= \alpha\tilde{A}_{ij}\tilde{A}^{ij} + (1/3)\alpha K^2 - \gamma^{ij}(\nabla_i\nabla_j\alpha), \\
(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} &= -e^{-4\phi}(\nabla_i\nabla_j\alpha)^{TF} + e^{-4\phi}\alpha R^{(3)}_{ij} - e^{-4\phi}\alpha(1/3)\gamma_{ij}R^{(3)} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k{}_j) \\
\partial_t\tilde{\Gamma}^i &= -2(\partial_j\alpha)\tilde{A}^{ij} - (4/3)\alpha(\partial_j K)\tilde{\gamma}^{ij} + 12\alpha\tilde{A}^{ji}(\partial_j\phi) - 2\alpha\tilde{A}_k{}^j(\partial_j\tilde{\gamma}^{ik}) - 2\alpha\tilde{\Gamma}^k{}_{lj}\tilde{A}^j{}_k\tilde{\gamma}^{il} \\
&\quad -\partial_j\left(\beta^k\partial_k\tilde{\gamma}^{ij} - \tilde{\gamma}^{kj}(\partial_k\beta^i) - \tilde{\gamma}^{ki}(\partial_k\beta^j) + (2/3)\tilde{\gamma}^{ij}(\partial_k\beta^k)\right)
\end{aligned}$$

$$\begin{aligned}
R_{ij} &= \partial_k\Gamma^k_{ij} - \partial_i\Gamma^k_{kj} + \Gamma^m_{ij}\Gamma^k_{mk} - \Gamma^m_{kj}\Gamma^k_{mi} =: \tilde{R}_{ij} + R^\phi_{ij} \\
R^\phi_{ij} &= -2\tilde{D}_i\tilde{D}_j\phi - 2\tilde{g}_{ij}\tilde{D}^l\tilde{D}_l\phi + 4(\tilde{D}_i\phi)(\tilde{D}_j\phi) - 4\tilde{g}_{ij}(\tilde{D}^l\phi)(\tilde{D}_l\phi) \\
\tilde{R}_{ij} &= -(1/2)\tilde{g}^{lm}\partial_{lm}\tilde{g}_{ij} + \tilde{g}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \tilde{\Gamma}^k\tilde{\Gamma}_{(ij)k} + 2\tilde{g}^{lm}\tilde{\Gamma}^k_{l(i}\tilde{\Gamma}_{j)km} + \tilde{g}lm\tilde{\Gamma}^k_{im}\tilde{\Gamma}_{klj}
\end{aligned}$$

– No explicit explanations why this formulation works better.
AEI group (2000): the replacement by momentum constraint is essential.

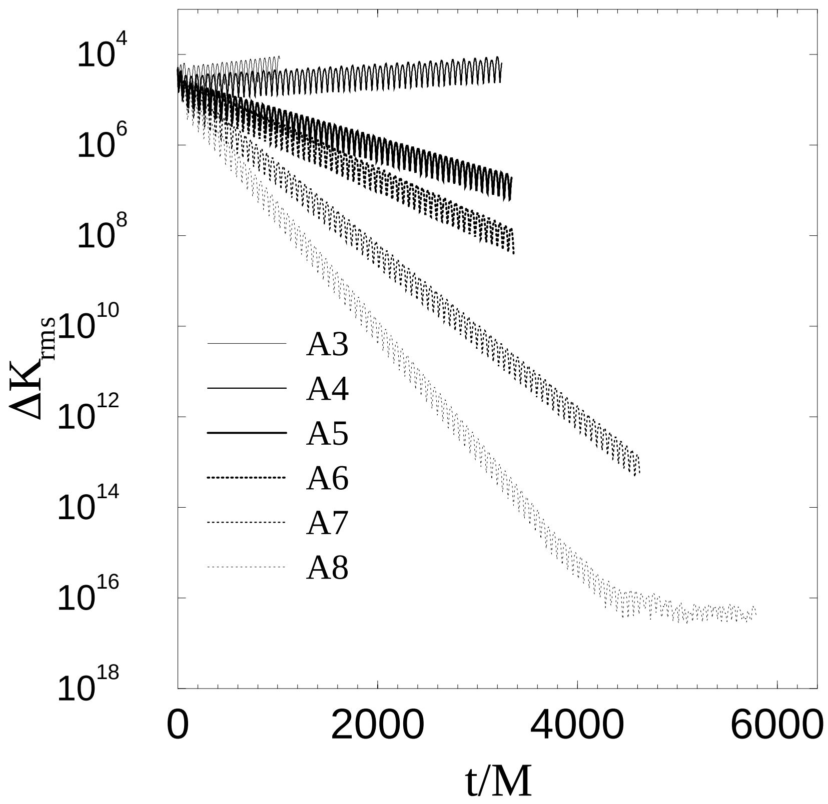
Constraint Amplification Factors with each adjustment

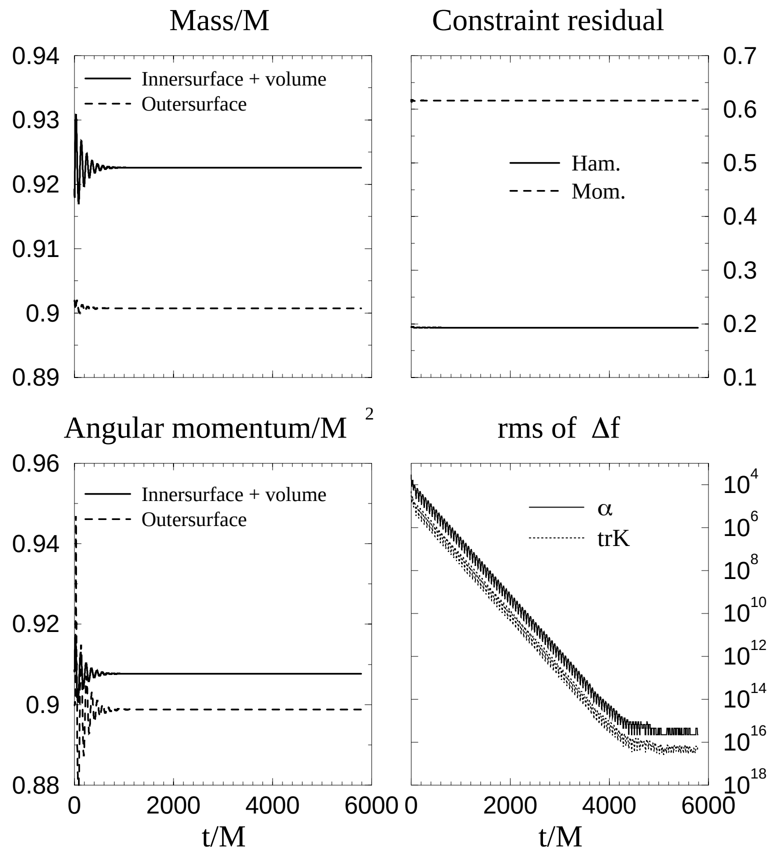
adjustment		CAFs	diag?	effect of the adjustment
$\partial_t\phi$	$\kappa_{\phi\mathcal{H}}\,\alpha\mathcal{H}$	$(0,0,\pm\sqrt{-k^2}(*3), 8\kappa_{\phi\mathcal{H}}k^2)$	no	$\kappa_{\phi\mathcal{H}}<0$ makes 1 Neg.
$\partial_t\phi$	$\kappa_{\phi\mathcal{G}}\,\alpha\tilde{D}_k\mathcal{G}^k$	$(0,0,\pm\sqrt{-k^2}(*2)$, long expressions)	yes	$\kappa_{\phi\mathcal{G}}<0$ makes 2 Neg. 1 Pos.
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{SD}\,\alpha\tilde{\gamma}_{ij}\mathcal{H}$	$(0,0,\pm\sqrt{-k^2}(*3),(3/2)\kappa_{SD}k^2)$	yes	$\kappa_{SD}<0$ makes 1 Neg. Case (B)
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{G}1}\,\alpha\tilde{\gamma}_{ij}\tilde{D}_k\mathcal{G}^k$	$(0,0,\pm\sqrt{-k^2}(*2)$, long expressions)	yes	$\kappa_{\tilde{\gamma}\mathcal{G}1}>0$ makes 1 Neg.
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{G}2}\,\alpha\tilde{\gamma}_{k(i}\tilde{D}_{j)}\mathcal{G}^k$	(0,0, $(1/4)k^2\kappa_{\tilde{\gamma}\mathcal{G}2}\pm\sqrt{k^2(-1+k^2\kappa_{\tilde{\gamma}\mathcal{G}2}/16)}(*2)$, long expressions)	yes	$\kappa_{\tilde{\gamma}\mathcal{G}2}<0$ makes 6 Neg. 1 Pos. Case (E1)
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{S}1}\,\alpha\tilde{\gamma}_{ij}\mathcal{S}$	$(0,0,\pm\sqrt{-k^2}(*3), 3\kappa_{\tilde{\gamma}\mathcal{S}1})$	no	$\kappa_{\tilde{\gamma}\mathcal{S}1}<0$ makes 1 Neg.
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{S}2}\,\alpha\tilde{D}_i\tilde{D}_j\mathcal{S}$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{\tilde{\gamma}\mathcal{S}2}k^2)$	no	$\kappa_{\tilde{\gamma}\mathcal{S}2}>0$ makes 1 Neg.
$\partial_t K$	$\kappa_{K\mathcal{M}}\,\alpha\tilde{\gamma}^{jk}(\tilde{D}_j\mathcal{M}_k)$	$(0,0,0,\pm\sqrt{-k^2}(*2)$, $(1/3)\kappa_{K\mathcal{M}}k^2\pm(1/3)\sqrt{k^2(-9+k^2\kappa_{K\mathcal{M}}^2)})$	no	$\kappa_{K\mathcal{M}}<0$ makes 2 Neg.
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{M}1}\,\alpha\tilde{\gamma}_{ij}(\tilde{D}^k\mathcal{M}_k)$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{A\mathcal{M}1}k^2)$	yes	$\kappa_{A\mathcal{M}1}>0$ makes 1 Neg.
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{M}2}\,\alpha(\tilde{D}_{(i}\mathcal{M}_{j)})$	(0,0, $-k^2\kappa_{A\mathcal{M}2}/4\pm\sqrt{k^2(-1+k^2\kappa_{A\mathcal{M}2}/16)}(*2)$, long expressions)	yes	$\kappa_{A\mathcal{M}2}>0$ makes 7 Neg Case (D)
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{A}1}\,\alpha\tilde{\gamma}_{ij}\mathcal{A}$	$(0,0,\pm\sqrt{-k^2}(*3), 3\kappa_{A\mathcal{A}1})$	yes	$\kappa_{A\mathcal{A}1}<0$ makes 1 Neg.
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{A}2}\,\alpha\tilde{D}_i\tilde{D}_j\mathcal{A}$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{A\mathcal{A}2}k^2)$	yes	$\kappa_{A\mathcal{A}2}>0$ makes 1 Neg.
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{H}}\,\alpha\tilde{D}^i\mathcal{H}$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{A\mathcal{A}2}k^2)$	no	$\kappa_{\tilde{\Gamma}\mathcal{H}}>0$ makes 1 Neg.
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{G}1}\,\alpha\mathcal{G}^i$	$(0,0,(1/2)\kappa_{\tilde{\Gamma}\mathcal{G}1}\pm\sqrt{-k^2+\kappa_{\tilde{\Gamma}\mathcal{G}1}^2}(*2)$, long.)	yes	$\kappa_{\tilde{\Gamma}\mathcal{G}1}<0$ makes 6 Neg. 1 Pos. Case (E2)
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{G}2}\,\alpha\tilde{D}^j\tilde{D}_j\mathcal{G}^i$	$(0,0,-(1/2)\kappa_{\tilde{\Gamma}\mathcal{G}2}\pm\sqrt{-k^2+\kappa_{\tilde{\Gamma}\mathcal{G}2}^2}(*2)$, long.)	yes	$\kappa_{\tilde{\Gamma}\mathcal{G}2}>0$ makes 2 Neg. 1 Pos.
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{G}3}\,\alpha\tilde{D}^i\tilde{D}_j\mathcal{G}^j$	$(0,0,-(1/2)\kappa_{\tilde{\Gamma}\mathcal{G}3}\pm\sqrt{-k^2+\kappa_{\tilde{\Gamma}\mathcal{G}3}^2}(*2)$, long.)	yes	$\kappa_{\tilde{\Gamma}\mathcal{G}3}>0$ makes 2 Neg. 1 Pos.

gr-qc/0204002 (PRD in print)

An Evolution of Adjusted BSSN Formulation

by Yo-Baumgarte-Shapiro, PRD 66 (2002) 084026





Kerr-Schild BH (0.9 J/M), excision with cube, $1+\log$-lapse, Γ-driver shift.

$$\partial_t \tilde{\Gamma}^i = (\cdots) + \frac{2}{3}\tilde{\Gamma}^i \beta^i{}_{,j} - (\chi + \frac{2}{3})\mathcal{G}^i \beta^j{}_{,j} \qquad \chi = 2/3 \text{ for (A4)-(A8)}$$

$$\partial_t \tilde{\gamma}_{ij} = (\cdots) - \kappa \alpha \tilde{\gamma}_{ij} \mathcal{H} \qquad \kappa = 0.1 \sim 0.2 \text{ for (A5), (A6) and (A8)}$$

Summary

Towards a stable and accurate formulation for numerical relativity

- Several reports say numerical stabilities depend on the formulations to apply, although they are mathematically equivalent.
- status = chaotic, many trials and errors
  We tried to understand the background in an unified way.
- Our proposal = "Evaluate eigenvalues of constraint propagation eqns"
  We give satisfactory conditions for stable evolutions.
  Fourier transformation allows to discuss lower-order terms.
- Our Observation = "Stability will change by adding constraints in RHS"
  We named "Adjusted System".
  Numerically confirmed in the Maxwell system and Ashtekar system.
- Our Observation 2= The idea works even for the ADM formulation
  We explain the effective parameter range of Detweiler's system (1987).
  We proposed variety of adjustments. Several numerical confirmations.
- Our Observation 3= The idea works also for the BSSN formulation
  We explain why adjusting momentum constraints improves the stability.
  We proposed variety of adjustments. Several numerical confirmations.

Evaluation of CAFs may be an alternative guideline to hyperbolization of the system.