安定な数値シミュレーションを行うためのEinstein方程式の定式化 III

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OUTLINE

安定で長時間シミュレーションできる定式化の方法論

- 混沌とした現状，さまざまな方向 ·········· 双曲型発展方程式，日本型，漸近的拘束型
- 数値的に安定な定式化構築へのレシピの提案······ 拘束条件の発展方程式を固有値解析
- Adjusted ADM/BSSN system を提案
- 数値的検証，今後の展望

Refs:

HS and G Yoneda, gr-qc/0209111 (review article)
G Yoneda and HS, Phys Rev D **66** (2002) 124003
G Yoneda and HS, Class. Quant. Grav. **20** (2003) L31
Mexico Workshop 2002 Participants, in preparation.

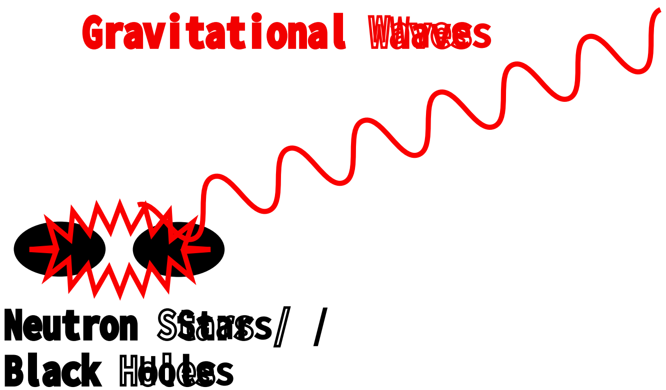
物理学会　2003/3　東北学院大

数値相対論　Numerical Relativity

＝「Einstein方程式を数値シミュレーションする」研究分野

強い重力場の解明に必須

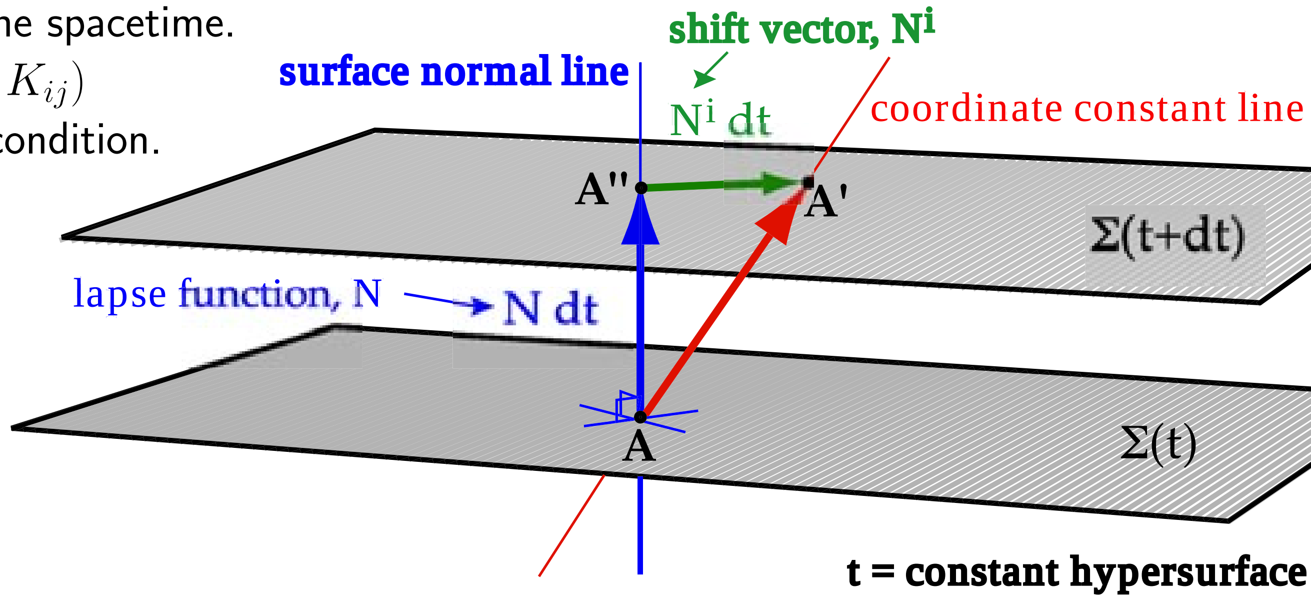
– 重力波の放出メカニズム（ブラックホール，中性子星，超新星爆発，...）
– 相対論的動力学の解明　（宇宙論，銀河中心核，...）
– 数学的問題への傍証　（特異点形成，厳密解，カオス現象，...）
– 重力理論の検証と実験　（拡張された一般相対論，高次元時空モデル，...）



LIGO/VIRGO/GEO/TAMA, ...

strategy 0 The standard approach :: Arnowitt-Deser-Misner (ADM) formulation (1962)

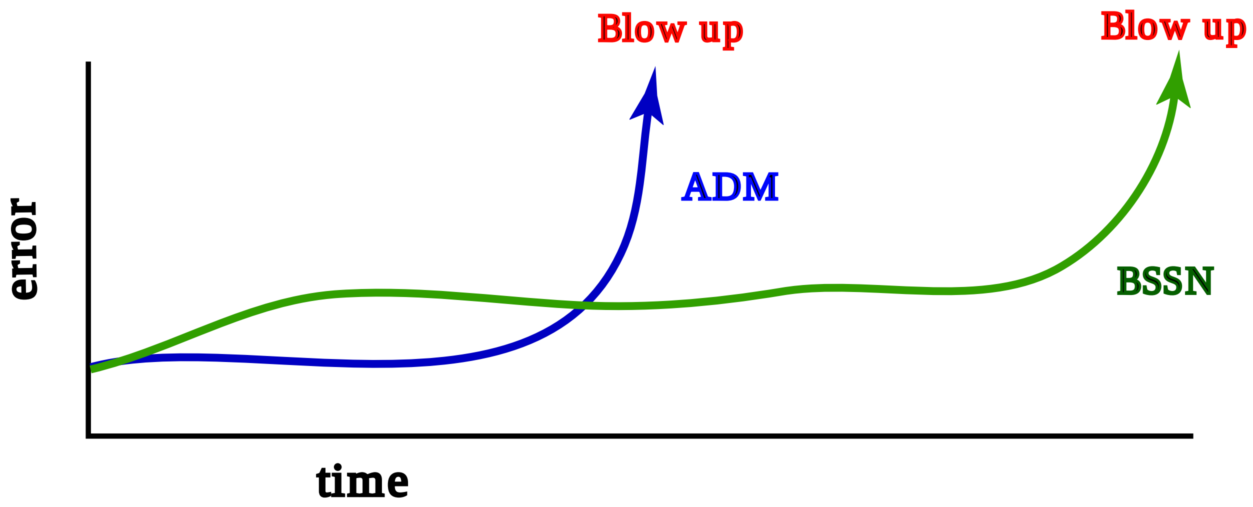
3+1 decomposition of the spacetime.
Evolve 12 variables (γ_{ij}, K_{ij})
with a choice of gauge condition.



	Maxwell eqs.	ADM Einstein eq.
constraints	div $\mathbf{E} = 4\pi\rho$ div $\mathbf{B} = 0$	${}^{(3)}R + (\mathrm{tr}K)^2 - K_{ij}K^{ij} = 2\kappa\rho_H + 2\Lambda$ $D_j K^j{}_i - D_i \mathrm{tr}K = \kappa J_i$
evolution eqs.	$\frac{1}{c}\partial_t \mathbf{E} = \text{rot } \mathbf{B} - \frac{4\pi}{c}\mathbf{j}$ $\frac{1}{c}\partial_t \mathbf{B} = -\text{rot } \mathbf{E}$	$\partial_t \gamma_{ij} = -2NK_{ij} + D_j N_i + D_i N_j,$ $\partial_t K_{ij} = N({}^{(3)}R_{ij} + \mathrm{tr}K K_{ij}) - 2NK_{il}K^l{}_j - D_i D_j N$ $+ (D_j N^m)K_{mi} + (D_i N^m)K_{mj} + N^m D_m K_{ij} - N\gamma_{ij}\Lambda$ $- \kappa\alpha\{S_{ij} + \frac{1}{2}\gamma_{ij}(\rho_H - \mathrm{tr}S)\}$

Best formulation of the Einstein eqs. for long-term stable & accurate simulation?

Many (too many) trials and errors, not yet a definit recipe.



Mathematically equivalent formulations, but differ in its stability!

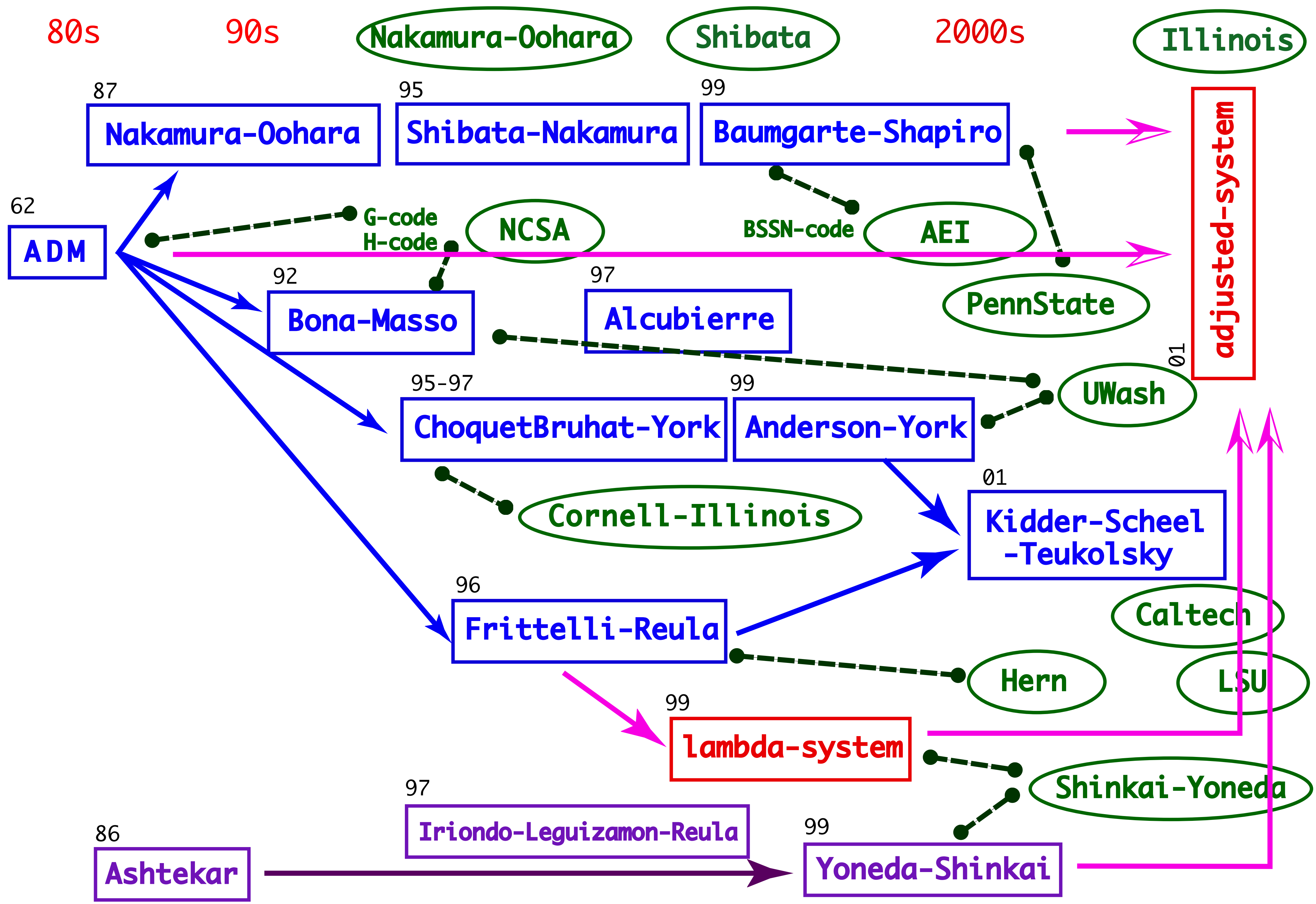
strategy 0: Arnowitt-Deser-Misner formulation

strategy 1: Shibata-Nakamura's (Baumgarte-Shapiro's) modifications to the standard ADM

strategy 2: Apply a formulation which reveals a hyperbolicity explicitly

strategy 3: Formulate a system which is "asymptotically constrained" against a violation of constraints

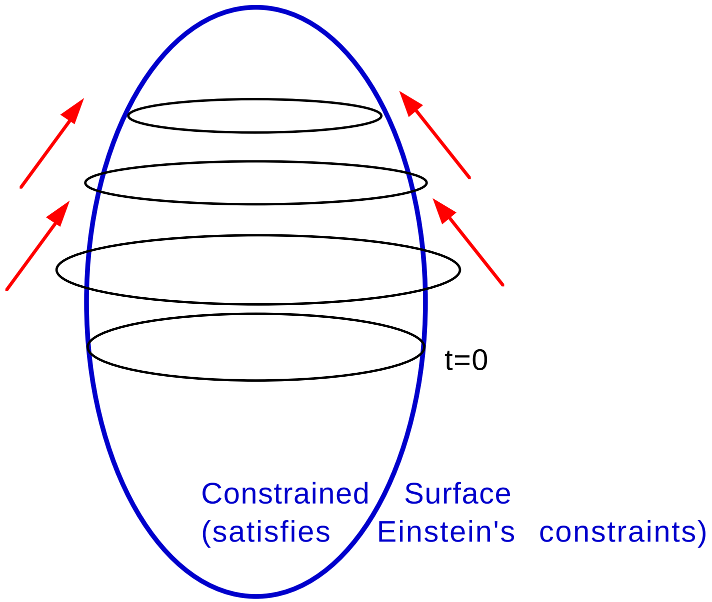
By adding constraints in RHS, we can kill error-growing modes
$\Rightarrow$ How can we understand the features systematically?

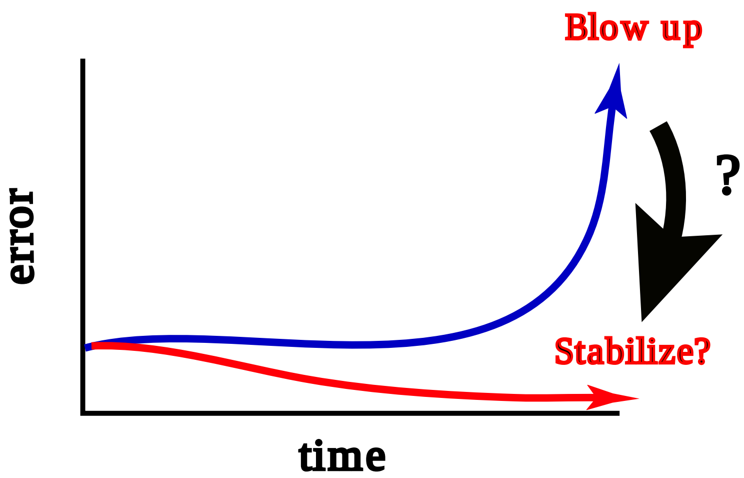


80s
90s
Nakamura-Oohara
Shibata
2000s
Illinois
87
Nakamura-Oohara
95
Shibata-Nakamura
99
Baumgarte-Shapiro
adjusted-system
01
62
ADM
G-code
H-code
NCSA
BSSN-code
AEI
92
Bona-Masso
97
Alcubierre
PennState
95-97
ChoquetBruhat-York
99
Anderson-York
UWash
01
Kidder-Scheel
-Teukolsky
Cornell-Illinois
96
Frittelli-Reula
Caltech
Hern
LSU
99
lambda-system
Shinkai-Yoneda
97
Iriondo-Leguizamon-Reula
86
Ashtekar
99
Yoneda-Shinkai

strategy 3 Formulate a system which is "asymptotically constrained" against a violation of constraints

"Asymptotically Constrained System"– Constraint Surface as an Attractor





method 1: λ-system (Brodbeck et al, 2000)

– Add aritificial force to reduce the violation of constraints

– To be guaranteed if we apply the idea to a symmetric hyperbolic system.

method 2: Adjusted system (Yoneda HS, 2000, 2001)

– We can control the violation of constraints by adjusting constraints to EoM.

– Eigenvalue analysis of constraint propagation equations may prodict the violation of error.

– This idea is applicable even if the system is not symmetric hyperbolic. $\Rightarrow$

for the ADM/BSSN formulation, too!!

Idea of "Adjusted system" and Our Conjecture

CQG18 (2001) 441, PRD 63 (2001) 120419, CQG 19 (2002) 1027

General Procedure

1. prepare a set of evolution eqs. $\partial_t u^a = f(u^a, \partial_b u^a, \cdots)$
2. add constraints in RHS $\partial_t u^a = f(u^a, \partial_b u^a, \cdots) \underbrace{+F(C^a, \partial_b C^a, \cdots)}$
3. choose appropriate $F(C^a, \partial_b C^a, \cdots)$
   to make the system stable evolution

How to specify $F(C^a, \partial_b C^a, \cdots)$?

4. prepare constraint propagation eqs. $\partial_t C^a = g(C^a, \partial_b C^a, \cdots)$
5. and its adjusted version $\partial_t C^a = g(C^a, \partial_b C^a, \cdots) \underbrace{+G(C^a, \partial_b C^a, \cdots)}$
6. Fourier transform and evaluate eigenvalues $\partial_t \hat{C}^k = \underbrace{A(\hat{C}^a)}\ \hat{C}^k$

Conjecture: Evaluate eigenvalues of (Fourier-transformed) constraint propagation eqs.
If their (1) real part is non-positive, or (2) imaginary part is non-zero, then the system is more stable.

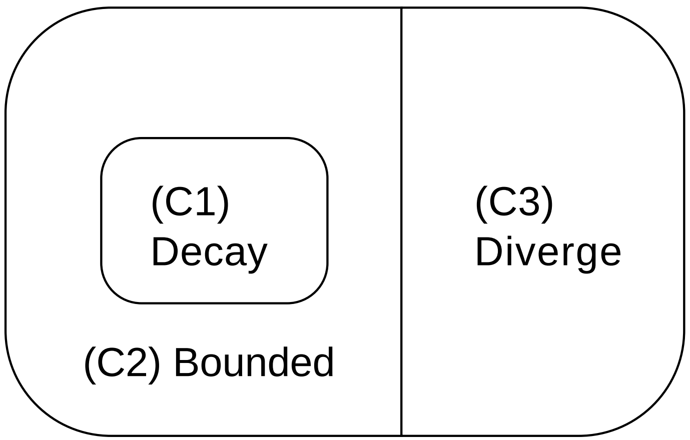
A Classification of Constraint Propagations

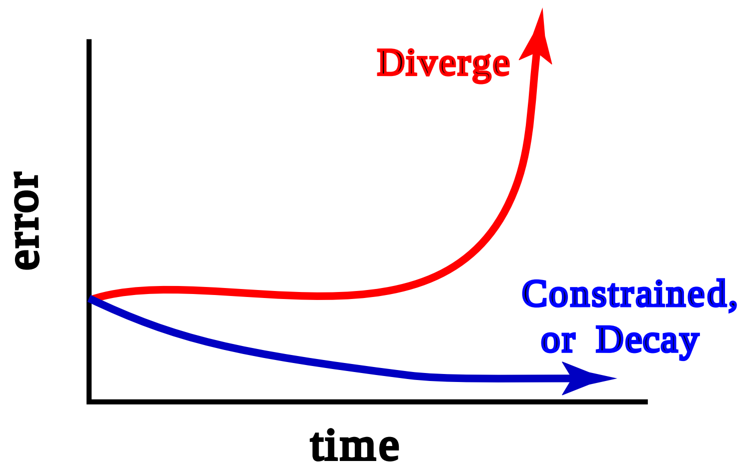
(C1) Asymptotically constrained :
Violation of constraints decays (converges to zero).

(C2) Asymptotically bounded :
Violation of constraints is bounded at a certain value.

(C3) Diverge :
At least one constraint will diverge.

Note that (C1) $\subset$ (C2).





A Classification of Constraint Propagations (cont.)

gr-qc/0209106

(C1) Asymptotically constrained :

Violation of constraints decays (converges to zero).

$\Leftrightarrow$ All the real parts of CAFs are negative.

(C2) Asymptotically bounded :

Violation of constraints is bounded at a certain value.

$\Leftrightarrow$

(a) All the real parts of CAFs are not positive, and

(b1) the CP matrix $M^{\alpha}{}_{\beta}$ is diagonalizable, or

(b2) the real part of the degenerated CAFs is not zero.

(C3) Diverge :

At least one constraint will diverge.

A flowchart to classify the fate of constraint propagation.

Q1: Is there a CAF which real part is positive?

NO / YES → **Diverge**

↓

Q2: Are all the real parts of CAFs negative?

NO / YES → **Asymptotically Constrained**

↓

Q3: Is the constraint propagation matrix diagonalizable?

NO / YES → **Asymptotically Bounded**

↓

Q4: Is a real part of the degenerated CAFs is zero?

YES / NO → **Asymptotically Bounded**

↓

Q5: Is the associated Jordan matrix diagonal?

NO / YES → **Asymptotically Bounded**

↓

Diverge

3 Adjusted ADM systems

We adjust the standard ADM system using constraints as:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \quad (1)$$

$$+P_{ij}\mathcal{H} + Q^k{}_{ij}\mathcal{M}_k + p^k{}_{ij}(\nabla_k \mathcal{H}) + q^{kl}{}_{ij}(\nabla_k \mathcal{M}_l), \quad (2)$$

$$\partial_t K_{ij} = \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij}, \quad (3)$$

$$+R_{ij}\mathcal{H} + S^k{}_{ij}\mathcal{M}_k + r^k{}_{ij}(\nabla_k \mathcal{H}) + s^{kl}{}_{ij}(\nabla_k \mathcal{M}_l), \quad (4)$$

with constraint equations

$$\mathcal{H} := R^{(3)} + K^2 - K_{ij}K^{ij}, \quad (5)$$

$$\mathcal{M}_i := \nabla_j K^j{}_i - \nabla_i K. \quad (6)$$

We can write the adjusted constraint propagation equations as

$$\partial_t \mathcal{H} = \text{(original terms)} + H_1^{mn}[(2)] + H_2^{imn}\partial_i[(2)] + H_3^{ijmn}\partial_i\partial_j[(2)] + H_4^{mn}[(4)], \quad (7)$$

$$\partial_t \mathcal{M}_i = \text{(original terms)} + M_{1i}{}^{mn}[(2)] + M_{2i}{}^{jmn}\partial_j[(2)] + M_{3i}{}^{mn}[(4)] + M_{4i}{}^{jmn}\partial_j[(4)]. \quad (8)$$

Example 1: standard ADM vs original ADM (in Schwarzschild coordinate)

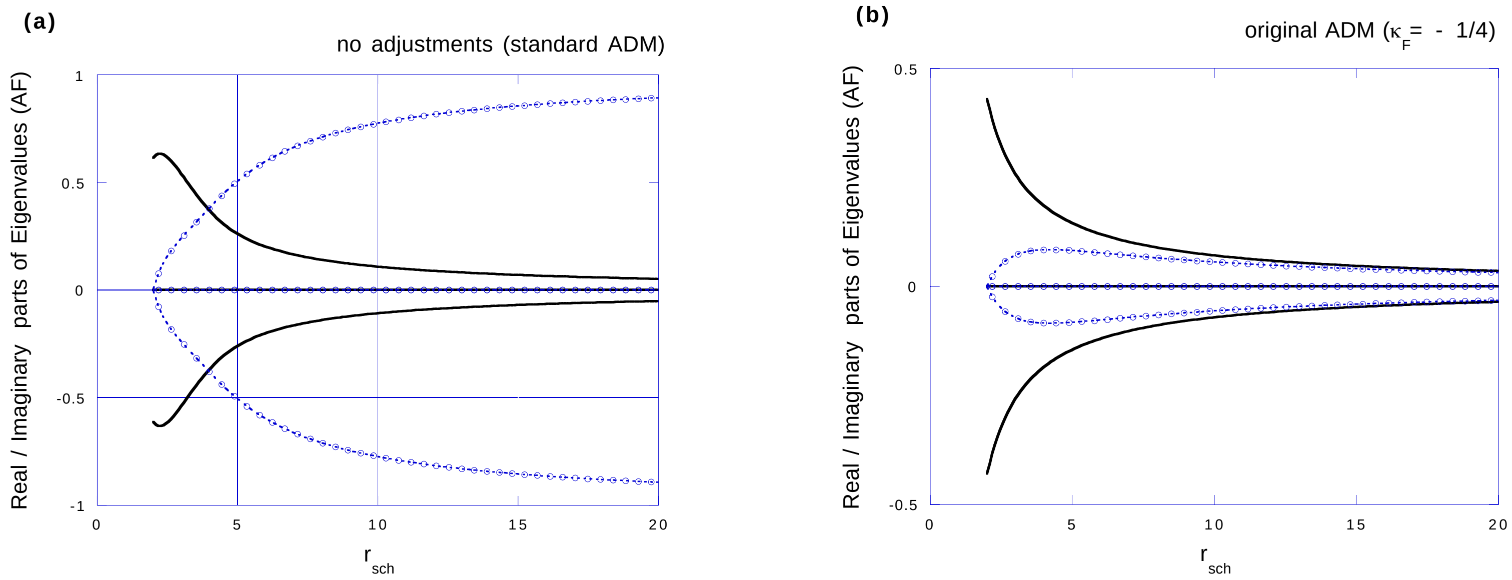


Figure 1: Amplification factors (AFs, eigenvalues of homogenized constraint propagation equations) are shown for the standard Schwarzschild coordinate, with (a) no adjustments, i.e., standard ADM, (b) original ADM ($\kappa_F = -1/4$). The solid lines and the dotted lines with circles are real parts and imaginary parts, respectively. They are four lines each, but actually the two eigenvalues are zero for all cases. Plotting range is $2 < r \leq 20$ using Schwarzschild radial coordinate. We set $k = 1, l = 2$, and $m = 2$ throughout the article.

$$
\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \\
\partial_t K_{ij} &= \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij} + \kappa_F \alpha \gamma_{ij} \mathcal{H},
\end{aligned}
$$

Example 2: Detweiler-type adjusted (in Schwarzschild coord.)

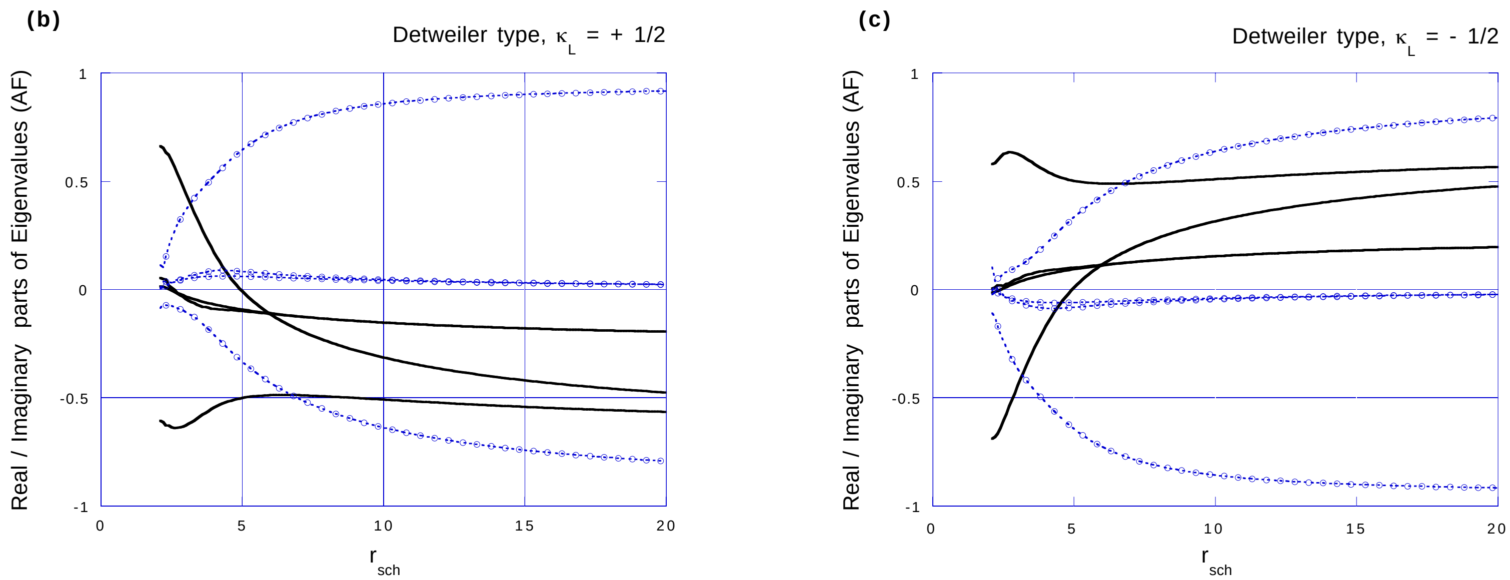


Figure 2: Amplification factors of the standard Schwarzschild coordinate, with Detweiler type adjustments. Multipliers used in the plot are (b) $\kappa_L = +1/2$, and (c) $\kappa_L = -1/2$.

$$\partial_t \gamma_{ij} = \text{(original terms)} + P_{ij}\mathcal{H},$$

$$\partial_t K_{ij} = \text{(original terms)} + R_{ij}\mathcal{H} + S^k{}_{ij}\mathcal{M}_k + s^{kl}{}_{ij}(\nabla_k \mathcal{M}_l),$$

where $P_{ij} = -\kappa_L \alpha^3 \gamma_{ij}, \quad R_{ij} = \kappa_L \alpha^3 (K_{ij} - (1/3)K\gamma_{ij}),$

$$S^k{}_{ij} = \kappa_L \alpha^2 [3(\partial_{(i}\alpha)\delta^k_{j)} - (\partial_l \alpha)\gamma_{ij}\gamma^{kl}], \quad s^{kl}{}_{ij} = \kappa_L \alpha^3 [\delta^k_{(i}\delta^l_{j)} - (1/3)\gamma_{ij}\gamma^{kl}],$$

HS original GR code **(2002/October)**

Use the Cactus-code base structure (`http://cactuscode.org`)

-- parallelize, parameter control, I/O, PUGH, elliptic solvers, ...

-- original module for all GR part (initial data/ADMevolution)

`Cactus/arrangements/GR/`

PC cluster with 4 **(2002/September)**

Pentium 4, 2.53GHz, 2GB each, 80 GHD each, gigabit ether

-- about 1.2 x 10^6 yen for total parts

-- TurboLinux 7, Intel Fortran compiler, MPICH, ... all free

-- possible up to 120^3 grid full GR simulation

Grant-in-Aid for Scientific Research Fund of JSPS, No.14740179 (2002-2005)

Comparisons of Adjusted ADM systems (Teukolsky wave) :: Detweiler type
3-dim, harmonic slice, periodic BC — HS original Cactus/GR code

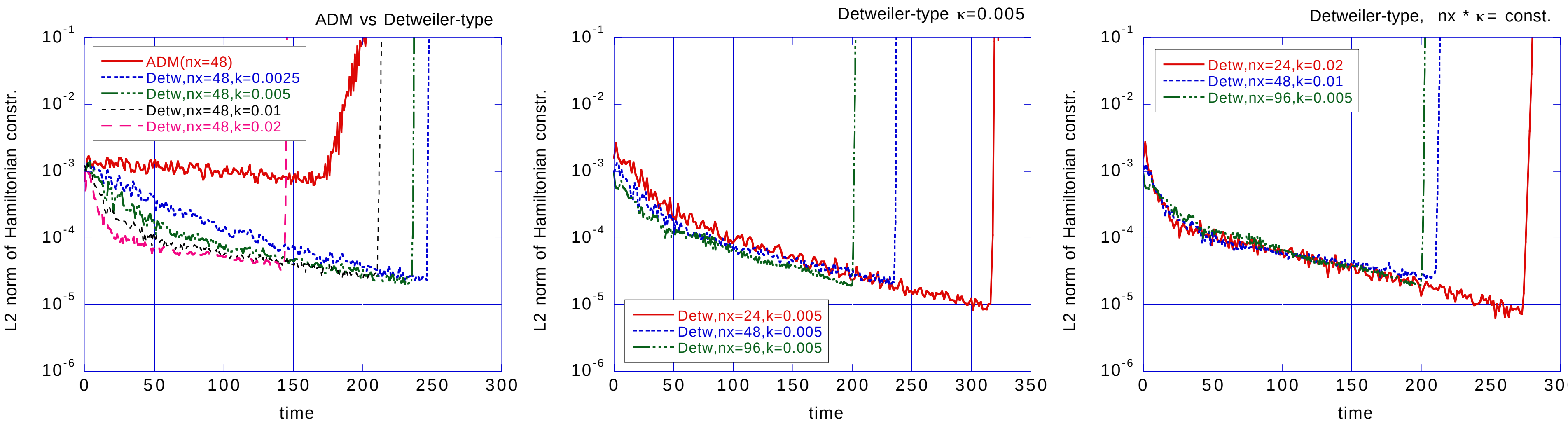


Figure 2: Violation of Hamiltonian constraints versus time: Adjusted ADM (Detweiler-type) system is applied for Teukolsky wave initial data evolution with harmonic slicing, and with periodic boundary condition. Cactus/GR/evolveADMeq code was used. $(x, y, z) = [-3, 3]$, iterative Crank-Nicholson method.

$$
\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i - \kappa_L \alpha^3 \gamma_{ij} \mathcal{H} \\
\partial_t K_{ij} &= \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij} \\
&\quad + \kappa_L \alpha^3 (K_{ij} - (1/3) K \gamma_{ij}) \mathcal{H} + \kappa_L \alpha^2 [3(\partial_{(i}\alpha)\delta^k_{j)} - (\partial_l \alpha)\gamma_{ij}\gamma^{kl}] \mathcal{M}_k \\
&\quad + \kappa_L \alpha^3 [\delta^k_{(i}\delta^l_{j)} - (1/3)\gamma_{ij}\gamma^{kl}] (\nabla_k \mathcal{M}_l)
\end{aligned}
$$

Comparisons of Adjusted ADM systems (Teukolsky wave) :: Simplified-Detweiler type
3-dim, harmonic slice, periodic BC HS original Cactus/GR code

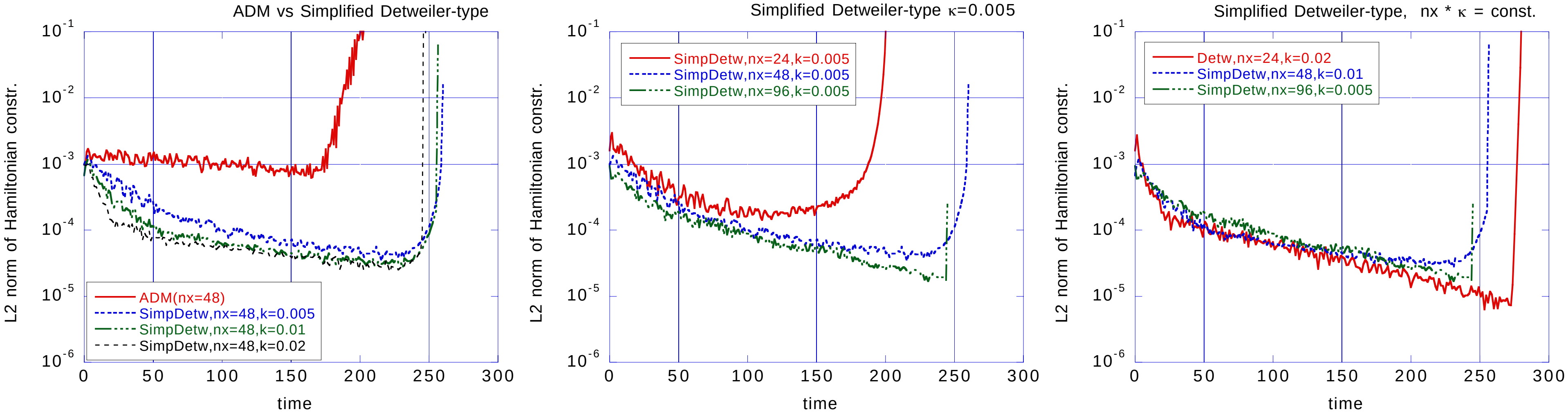


Figure 3: Violation of Hamiltonian constraints versus time: Adjusted ADM (Simplified Detweiler-type) system is applied for Teukolsky wave initial data evolution with harmonic slicing, and with periodic boundary condition. Cactus/GR/evolveADMeq code was used. $(x, y, z) = [-3, 3]$, iterative Crank-Nicholson method.

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i - \kappa_L \alpha \gamma_{ij} \mathcal{H}$$

$$\partial_t K_{ij} = \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij}$$

Comparisons of Adjusted ADM systems (Teukolsky wave)
3-dim, harmonic slice, periodic BC

HS original Cactus/GR code

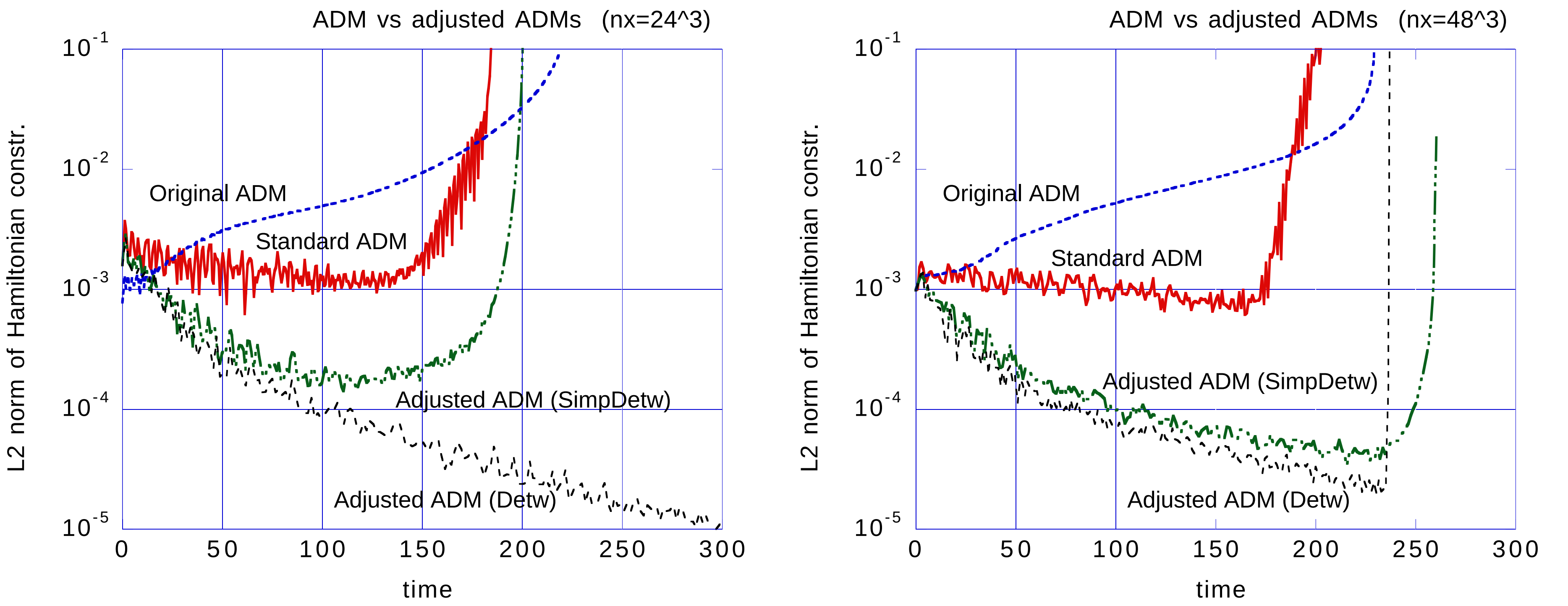


Figure 1: Violation of Hamiltonian constraints versus time: Adjusted ADM systems are applied for Teukolsky wave initial data evolution with harmonic slicing, and with periodic boundary condition. Cactus/GR/evolveADMeq code was used. Grid $= 24^3$ and 48^3, with $(x, y, z) = [-3, 3]$, iterative Crank-Nicholson method. Adjusted parameters $\kappa = 0.005$.

strategy 1 Shibata-Nakamura's (Baumgarte-Shapiro's) modifications to the standard ADM

– define new variables ($\phi, \tilde\gamma_{ij}, K, \tilde A_{ij}, \tilde\Gamma^i$), instead of the ADM's (γ_{ij}, K_{ij}) where

$$\tilde\gamma_{ij} \equiv e^{-4\phi}\gamma_{ij}, \qquad \tilde A_{ij} \equiv e^{-4\phi}(K_{ij} - (1/3)\gamma_{ij}K), \qquad \tilde\Gamma^i \equiv \tilde\Gamma^i_{jk}\tilde\gamma^{jk},$$

use momentum constraint in Γ^i-eq., and impose $\det\tilde\gamma_{ij} = 1$ during the evolutions.

– The set of evolution equations become

$$\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\phi &= -(1/6)\alpha K, \\
(\partial_t - \mathcal{L}_\beta)\tilde\gamma_{ij} &= -2\alpha\tilde A_{ij}, \\
(\partial_t - \mathcal{L}_\beta)K &= \alpha\tilde A_{ij}\tilde A^{ij} + (1/3)\alpha K^2 - \gamma^{ij}(\nabla_i\nabla_j\alpha), \\
(\partial_t - \mathcal{L}_\beta)\tilde A_{ij} &= -e^{-4\phi}(\nabla_i\nabla_j\alpha)^{TF} + e^{-4\phi}\alpha R^{(3)}_{ij} - e^{-4\phi}\alpha(1/3)\gamma_{ij}R^{(3)} + \alpha(K\tilde A_{ij} - 2\tilde A_{ik}\tilde A^k{}_j) \\
\partial_t\tilde\Gamma^i &= -2(\partial_j\alpha)\tilde A^{ij} - (4/3)\alpha(\partial_j K)\tilde\gamma^{ij} + 12\alpha\tilde A^{ji}(\partial_j\phi) - 2\alpha\tilde A_k{}^j(\partial_j\tilde\gamma^{ik}) - 2\alpha\tilde\Gamma^k{}_{lj}\tilde A^j{}_k\tilde\gamma^{il} \\
&\quad -\partial_j\left(\beta^k\partial_k\tilde\gamma^{ij} - \tilde\gamma^{kj}(\partial_k\beta^i) - \tilde\gamma^{ki}(\partial_k\beta^j) + (2/3)\tilde\gamma^{ij}(\partial_k\beta^k)\right)
\end{aligned}$$

$$\begin{aligned}
R_{ij} &= \partial_k\Gamma^k_{ij} - \partial_i\Gamma^k_{kj} + \Gamma^m_{ij}\Gamma^k_{mk} - \Gamma^m_{kj}\Gamma^k_{mi} =: \tilde R_{ij} + R^\phi_{ij} \\
R^\phi_{ij} &= -2\tilde D_i\tilde D_j\phi - 2\tilde g_{ij}\tilde D^l\tilde D_l\phi + 4(\tilde D_i\phi)(\tilde D_j\phi) - 4\tilde g_{ij}(\tilde D^l\phi)(\tilde D_l\phi) \\
\tilde R_{ij} &= -(1/2)\tilde g^{lm}\partial_{lm}\tilde g_{ij} + \tilde g_{k(i}\partial_{j)}\tilde\Gamma^k + \tilde\Gamma^k\tilde\Gamma_{(ij)k} + 2\tilde g^{lm}\tilde\Gamma^k_{l(i}\tilde\Gamma_{j)km} + \tilde g lm\tilde\Gamma^k_{im}\tilde\Gamma_{klj}
\end{aligned}$$

– No explicit explanations why this formulation works better.

AEI group (2000): the replacement by momentum constraint is essential.

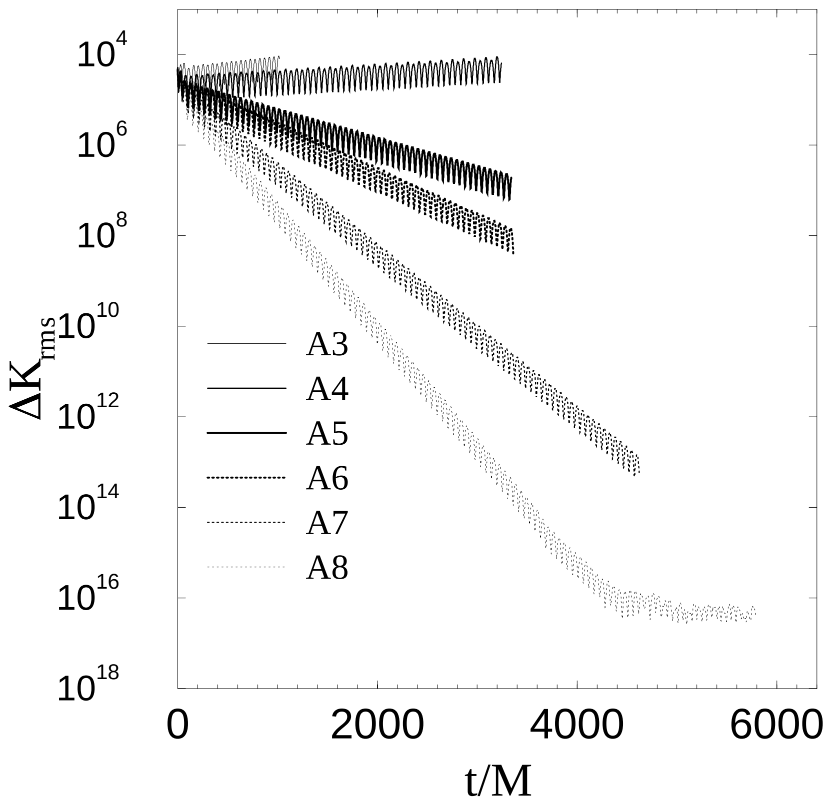
Constraint Amplification Factors with each adjustment

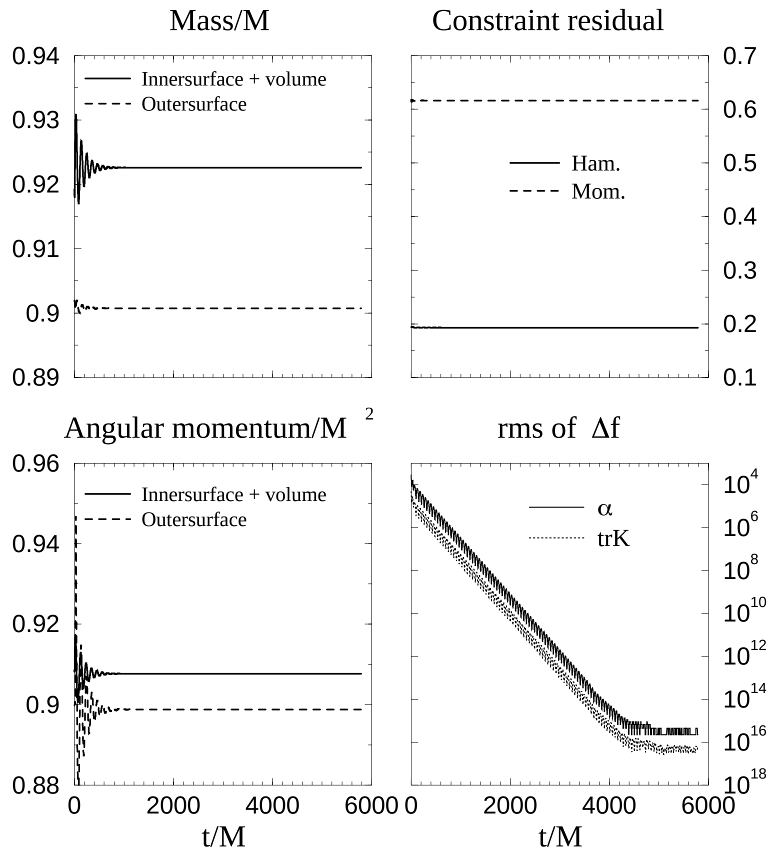
adjustment		CAFs	diag?	effect of the adjustment	
$\partial_t\phi$	$\kappa_{\phi\mathcal{H}}\,\alpha\mathcal{H}$	$(0,0,\pm\sqrt{-k^2}(*3),8\kappa_{\phi\mathcal{H}}k^2)$	no	$\kappa_{\phi\mathcal{H}}<0$ makes 1 Neg.	
$\partial_t\phi$	$\kappa_{\phi\mathcal{G}}\,\alpha\tilde{D}_k\mathcal{G}^k$	$(0,0,\pm\sqrt{-k^2}(*2)$, long expressions)	yes	$\kappa_{\phi\mathcal{G}}<0$ makes 2 Neg. 1 Pos.	
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{SD}\,\alpha\tilde{\gamma}_{ij}\mathcal{H}$	$(0,0,\pm\sqrt{-k^2}(*3),(3/2)\kappa_{SD}k^2)$	yes	$\kappa_{SD}<0$ makes 1 Neg.	Case (B)
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{G}1}\,\alpha\tilde{\gamma}_{ij}\tilde{D}_k\mathcal{G}^k$	$(0,0,\pm\sqrt{-k^2}(*2)$, long expressions)	yes	$\kappa_{\tilde{\gamma}\mathcal{G}1}>0$ makes 1 Neg.	
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{G}2}\,\alpha\tilde{\gamma}_{k(i}\tilde{D}_{j)}\mathcal{G}^k$	(0,0, $(1/4)k^2\kappa_{\tilde{\gamma}\mathcal{G}2}\pm\sqrt{k^2(-1+k^2\kappa_{\tilde{\gamma}\mathcal{G}2}/16)}(*2)$, long expressions)	yes	$\kappa_{\tilde{\gamma}\mathcal{G}2}<0$ makes 6 Neg. 1 Pos.	Case (E1)
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{S}1}\,\alpha\tilde{\gamma}_{ij}\mathcal{S}$	$(0,0,\pm\sqrt{-k^2}(*3),3\kappa_{\tilde{\gamma}\mathcal{S}1})$	no	$\kappa_{\tilde{\gamma}\mathcal{S}1}<0$ makes 1 Neg.	
$\partial_t\tilde{\gamma}_{ij}$	$\kappa_{\tilde{\gamma}\mathcal{S}2}\,\alpha\tilde{D}_i\tilde{D}_j\mathcal{S}$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{\tilde{\gamma}\mathcal{S}2}k^2)$	no	$\kappa_{\tilde{\gamma}\mathcal{S}2}>0$ makes 1 Neg.	
$\partial_t K$	$\kappa_{K\mathcal{M}}\,\alpha\tilde{\gamma}^{jk}(\tilde{D}_j\mathcal{M}_k)$	$(0,0,0,\pm\sqrt{-k^2}(*2)$, $(1/3)\kappa_{K\mathcal{M}}k^2\pm(1/3)\sqrt{k^2(-9+k^2\kappa_{K\mathcal{M}}^2)})$	no	$\kappa_{K\mathcal{M}}<0$ makes 2 Neg.	
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{M}1}\,\alpha\tilde{\gamma}_{ij}(\tilde{D}^k\mathcal{M}_k)$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{A\mathcal{M}1}k^2)$	yes	$\kappa_{A\mathcal{M}1}>0$ makes 1 Neg.	
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{M}2}\,\alpha(\tilde{D}_{(i}\mathcal{M}_{j)})$	(0,0, $-k^2\kappa_{A\mathcal{M}2}/4\pm\sqrt{k^2(-1+k^2\kappa_{A\mathcal{M}2}/16)}(*2)$, long expressions)	yes	$\kappa_{A\mathcal{M}2}>0$ makes 7 Neg	Case (D)
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{A}1}\,\alpha\tilde{\gamma}_{ij}\mathcal{A}$	$(0,0,\pm\sqrt{-k^2}(*3),3\kappa_{A\mathcal{A}1})$	yes	$\kappa_{A\mathcal{A}1}<0$ makes 1 Neg.	
$\partial_t\tilde{A}_{ij}$	$\kappa_{A\mathcal{A}2}\,\alpha\tilde{D}_i\tilde{D}_j\mathcal{A}$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{A\mathcal{A}2}k^2)$	yes	$\kappa_{A\mathcal{A}2}>0$ makes 1 Neg.	
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{H}}\,\alpha\tilde{D}^i\mathcal{H}$	$(0,0,\pm\sqrt{-k^2}(*3),-\kappa_{A\mathcal{A}2}k^2)$	no	$\kappa_{\tilde{\Gamma}\mathcal{H}}>0$ makes 1 Neg.	
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{G}1}\,\alpha\mathcal{G}^i$	$(0,0,(1/2)\kappa_{\tilde{\Gamma}\mathcal{G}1}\pm\sqrt{-k^2+\kappa_{\tilde{\Gamma}\mathcal{G}1}^2}(*2)$, long.)	yes	$\kappa_{\tilde{\Gamma}\mathcal{G}1}<0$ makes 6 Neg. 1 Pos.	Case (E2)
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{G}2}\,\alpha\tilde{D}^j\tilde{D}_j\mathcal{G}^i$	$(0,0,-(1/2)\kappa_{\tilde{\Gamma}\mathcal{G}2}\pm\sqrt{-k^2+\kappa_{\tilde{\Gamma}\mathcal{G}2}^2}(*2)$, long.)	yes	$\kappa_{\tilde{\Gamma}\mathcal{G}2}>0$ makes 2 Neg. 1 Pos.	
$\partial_t\tilde{\Gamma}^i$	$\kappa_{\tilde{\Gamma}\mathcal{G}3}\,\alpha\tilde{D}^i\tilde{D}_j\mathcal{G}^j$	$(0,0,-(1/2)\kappa_{\tilde{\Gamma}\mathcal{G}3}\pm\sqrt{-k^2+\kappa_{\tilde{\Gamma}\mathcal{G}3}^2}(*2)$, long.)	yes	$\kappa_{\tilde{\Gamma}\mathcal{G}3}>0$ makes 2 Neg. 1 Pos.	

Yoneda-HS, PRD66 (2002) 124003

An Evolution of Adjusted BSSN Formulation

by Yo-Baumgarte-Shapiro, PRD 66 (2002) 084026





Kerr-Schild BH (0.9 J/M), excision with cube, $1 + \log$-lapse, Γ-driver shift.

$$\partial_t \tilde{\Gamma}^i = (\cdots) + \frac{2}{3}\tilde{\Gamma}^i \beta^i{}_{,j} - (\chi + \frac{2}{3})\mathcal{G}^i \beta^j{}_{,j} \qquad \chi = 2/3 \text{ for (A4)-(A8)}$$

$$\partial_t \tilde{\gamma}_{ij} = (\cdots) - \kappa \alpha \tilde{\gamma}_{ij} \mathcal{H} \qquad \kappa = 0.1 \sim 0.2 \text{ for (A5), (A6) and (A8)}$$

まとめ

安定な数値計算を可能にするEinstein方程式の定式化は何か？

今まで

- 数値的安定性が，定式化に依存することは事実として報告されている．
- 「初期値に解いた拘束条件は，時間発展でも保存」 ==>> 数値的には必ずしも保証されない．
- ADM形式を改良するさまざまな試みが，過去10年にわたって続けられてきた．

我々の提案　これまでの試みを統一的に説明

- 「拘束条件式の発展方程式解析」 そして 「固有値解析による安定性への十分条件」
- 標準的なADM形式 は，BH近傍では，constraint violation modeが顕著に存在
- 「時間発展と共に，系が漸近的に拘束面へ収束していく」システムの構築がわずかな工夫で可能
- Adjusted ADM形式，Adjusted SN-BS形式を提案

数値的追認

- Adjusted ADM形式：これまで以上の数値的安定性が確認された．by PennState group / myself
- Adjusted SN-BS形式：これまで以上の数値的安定性が確認された．by Illinois/ AEI groups

「双曲型偏微分方程式論」より有効で現実的な安定性予測！！

そのほか　最近の話題から

- adjusted systems 次の一手
  - Lagrange multipliers の自動応答制御　試行錯誤中
- 一般相対論的 MHD の定式化
  - Baumgarte-Shapiro (astro-ph/0211340) がADM変数をベースにして，定式化．
  - Constraint Propagation 解析を進行中
- 「数値相対論における定式化問題　研究会」2002年5月 @ メキシコ
  - 11グループ，24人が2週間参加
    AEI/Pitt/Caltech/PennState/UNAM/RIKEN/...
  - 論文１「比較フォーマット」は，原稿作成進行中．
    `http://www.ApplesWithApples.org/`
    - ゲージ波 (Minkowskii 時空のnon trivialな発展)
    - 線形重力波
    - Gowdy時空　などなど
  - 論文２「比較計算結果１」は，データ回収中．
  - 今年12月「Mexico2　研究会」企画中．