Apparent horizon formation in higher dimensional spacetime

Yuta Yamada (Osaka Institute of Technology, Japan)
山田　祐太 (大阪工業大学　情報科学部)
Hisa-aki Shinkai (Osaka Institute of Technology, Japan)
真貝　寿明 (大阪工業大学　情報科学部)

We numerically investigate the formation of an apparent horizon in 5 dimensional spacetime in the context of the cosmic censorship hypothesis. We model the matter by distributing collisionless particle both in a spheroidal and toroidal configurations. We prepare the sequence of initial data by solving the Hamiltonian constraint equation and search S^4 apparent horizon.

Motivation

☆ Verification of cosmic censorship hypothesis and hoop conjecture in higher dimensional spacetime.

☆ Matter distribution with general configurations.

☆ Apparent Horizon search in numerically constructed spacetime.

Solving the Hamiltonian constraint in 5 demensional spacetime

・For five dimensional spacetime, the metric on spacelike hyper-surface can be written as following by introducing the Cartesian coordinates

$$ds^2 = \psi^2(dx^2 + dy^2 + dz^2 + dw^2)$$

・Conformal transformation

$$\gamma_{ij} = \psi^2 \hat{\gamma}_{ij}$$

・Assuming conformally flat and a moment of time symmetry, the Hamiltonian constraint equation is written as

$$6\hat{D}_a\hat{D}^a\psi = -2\kappa^2\rho$$

where κ is gravitational constant.

Common Apparent horizon formation in 5 dimensional spacetime

・For axisymmetric spacetime, the apparent horizon is a closed marginal surface only if r(ξ) satisfies

$$\ddot{r}_m - \frac{4\dot{r}_m^2}{r_m} - 3r_m + \frac{r_m^2+\dot{r}_m^2}{r_m} \times \left[\frac{2\dot{r}_m}{r_m}\cot\xi - \frac{3}{\psi}(\dot{r}_m\sin\xi + r_m\cos\xi)\frac{\partial\psi}{\partial z} + \frac{3}{\psi}(\dot{r}_m\cos\xi - r_m\sin\xi)\left(\frac{\partial\psi}{\partial x}\sin\xi\cos\theta + \frac{\partial\psi}{\partial y}\sin\xi\sin\theta + \frac{\partial\psi}{\partial w}\cos\xi\right)\right] = 0$$

where

$$\xi = \arctan\frac{\sqrt{x^2+y^2+w^2}}{z}$$

$$\theta = \arctan\frac{\sqrt{x^2+y^2}}{w}$$

・To see whether naked singular is formed or not, We evaluate the Kretchmann Invariant

$$I = R^{abcd}R_{abcd}$$

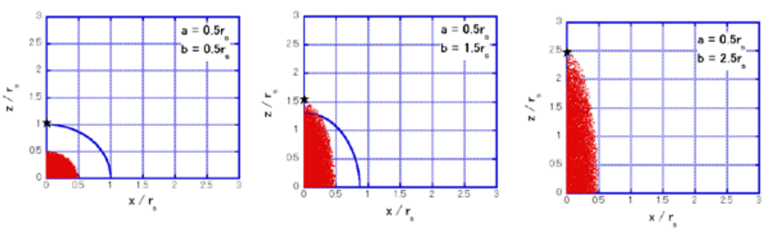
Results

CASE1 : Spheroidal configrations

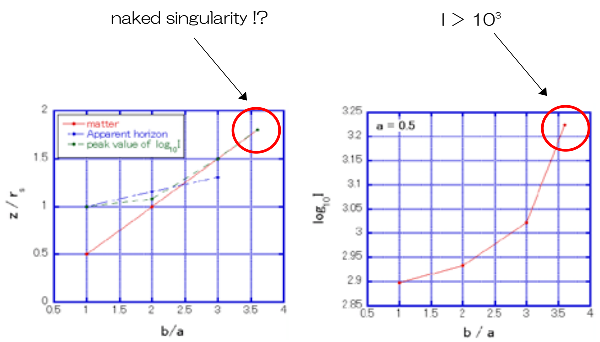
・Spheroidal configrations

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} + \frac{w^2}{a^2} \le 1$$

・We numerically confirmed the analytical work by Yoo et al.(Yoo, Nakao, Ida, Phys. Rev. D. 71, 104014)



・The behaviors are similar to the Shapiro-Teukolsky's 4-D case.

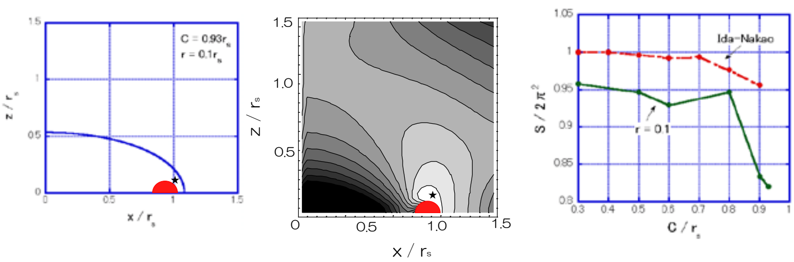


CASE2 : Toroidal configrations

・Toroidal configrations

$$\left(\sqrt{x^2+y^2+w^2} - C\right)^2 + z^2 \le r^2$$

・We assumed homogeneous toroidal, and searched for S^4-horizon.(not a ring horizon)



・Our limiting case coinsides with the analytical work by Ida & Nakao.
(Ida, Nakao, Phys. Rev. D. 66, 064026)

Conclusion

Spheroidal Cases

☆ No AH is formed for highly prolate case.

☆ Large $R_{abcd}R^{abcd}$ suggests the appearance of naked singularity

Toroidal Cases

☆ S^4-AH formed { for C < 0.93 (when r = 0.1),
for C < 0.51 (when r = 0.2).

☆ Largest $R_{abcd}R^{abcd}$ exists at top/down side of the ring(Not on the equatorial plane).

☆ "Naked Ring" might be formed.

☆ With matter distribution, the area of S^4 horizon become smaller than that of the limiting case.(δ-function ring).

Future works

☆ Examine the validity of the Hyper-Hoop Conjecture for general cases.

☆ Find a ring horizon($S^3 \times S^1$).

☆ Proceed time evolution, and study the dynamical process.

e-mail : m1m08a26@info.oit.ac.jp

18th JGRG 2008/11