We numerically investigate black-hole (and black-ring) formation in five-dimensional spacetime. We model the initial matter distribution in non-rotating homogeneous spheroidal and toroidal configurations under the momentarily static assumption, and express the matter with collisionless particles. We evolve the spacetime using the maximal time slicing condition for the lapse function and the minimal strain condition for shift vector. We search apparent horizons both of $S^3$ and $S^1 \times S^2$ topology during time evolutions. For toroidal matter cases, we observe the topology of apparent horizons depends on the ring radius, and also we find the topology change of horizon from ring to spherical shape in time evolution. For spheroidal configurations, we cannot find apparent horizon for highly prolate matter, which indicates also the formation of naked singularity. We discuss when black-hole or black-ring is formed and also report the possibility of a naked singularity formation.

1. Motivation

Formation of naked singularities

- Gravitational collapse of collisionless particles with spheroidal configuration. (Shapiro, Teukolsky, PRL66, 994, 1991)

posibility of naked singularities in 5D

- Suggest the formation of naked singularities by spindle collapse. (Yoo, Ida, Nakao, PRD71, 104014, 2005)

Dynamics of Black-Objects

- Formation process ?
- Dynamical features ?
- Search apparent horizons.

2. Our Numerical Approach

- Evolution of two kinds of non-rotating matter configurations.
- Using the $(4+1)$ ADM formalism.
- Express the matter with collisionless particles.
- Search apparent horizons.

3. Initial data (Yamada, Shinkai, CQG27, 045012)

We construct sequences of initial data with

- conformally flat, moment of time symmetry, asymptotically flat

- Conformal transformation $\gamma_{ij} = e^{2\nu} g_{ij}$
- The Hamiltonian constraint equation $\Delta \psi = -16\pi G \rho$
- boundary condition $\psi = 1 + \frac{M_{\text{ADM}}}{\rho}$

4. Evolution

- Evolution equations

$$\frac{\partial \rho_i}{\partial t} = -2\kappa K_{ij} + D_i \beta_j + D_j \beta_i$$

$$\frac{\partial K_{ij}}{\partial t} = \alpha \left( 6K_{ij} + K_{ij} \right) - 2\kappa K_i \kappa_j - 12\pi \rho (S_{ij} + \frac{1}{3} \delta \rho = S)$$

- Maximal time slicing condition for lapse function $\Delta \alpha = \alpha (K_{ij} K^{ij} + 2 \rho \theta^2 + 3\pi \rho n^2)$

- Minimal strain condition for shift vector $\Delta \beta^i + D^i \beta^j + K_{ij} = 2 \rho \beta^j (K_{ij})$

- We assume axi-symmetric space-time using the Cartoon method.

5. Time evolution of spheroidal configuration

Case 1 horizon forms

Case 2 No horizon (Naked singularity?)

6. Time evolution of ring configuration

- We find three different cases for apparent horizon formation depending the ring radius $R_c$ at $t = 0$.

Case 1 common horizon (small radius)

Case 2 ring horizon $\rightarrow$ common horizon

Case 3 ring horizon (large radius)

Lapse function on x-axis for large radius case