Wormholes Dynamics

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Part I: 4-dimensional numerical simulations

with Sean A. Hayward

HS and S.A. Hayward, Phys. Rev. D. 66 (2002) 044005

- "Dynamical Wormhole"
- A numerical approach, dual-null formulation
- Black-Hole Collapse or Inflationary Expansion

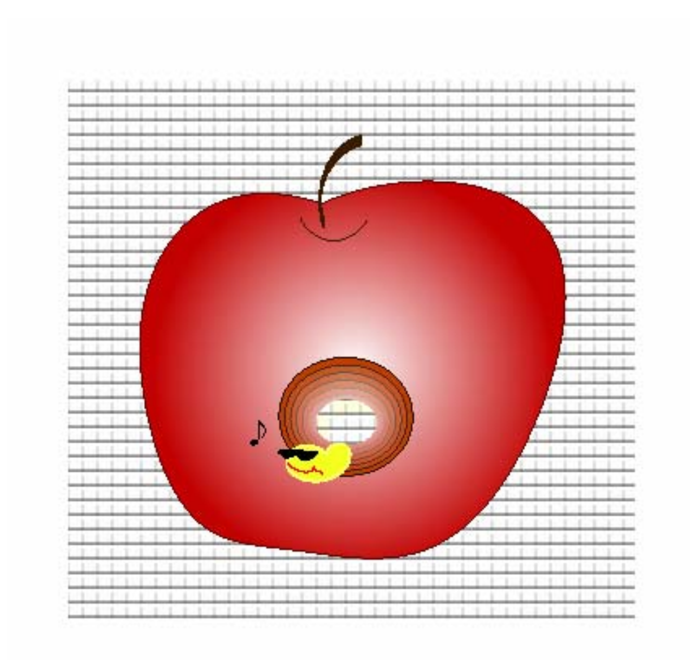
Part II: 5-dimensional numerical simulations

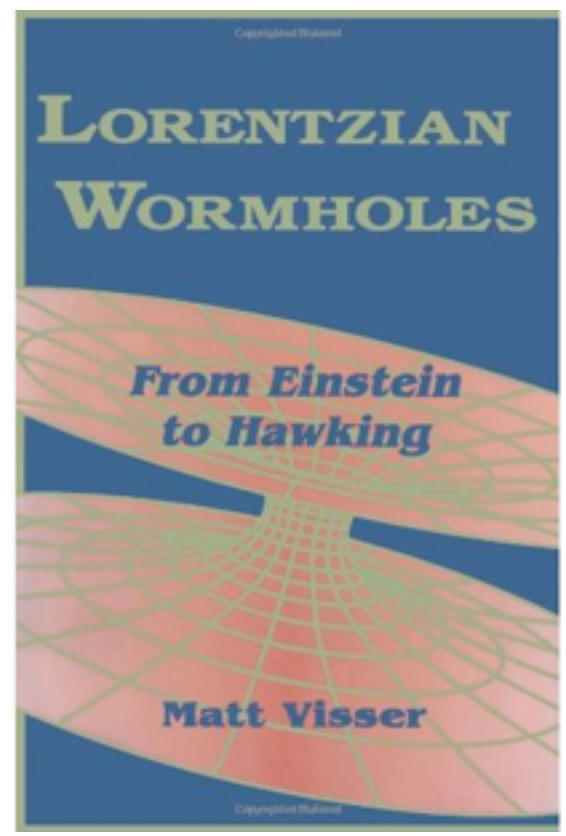
with Takashi Torii (OIT)

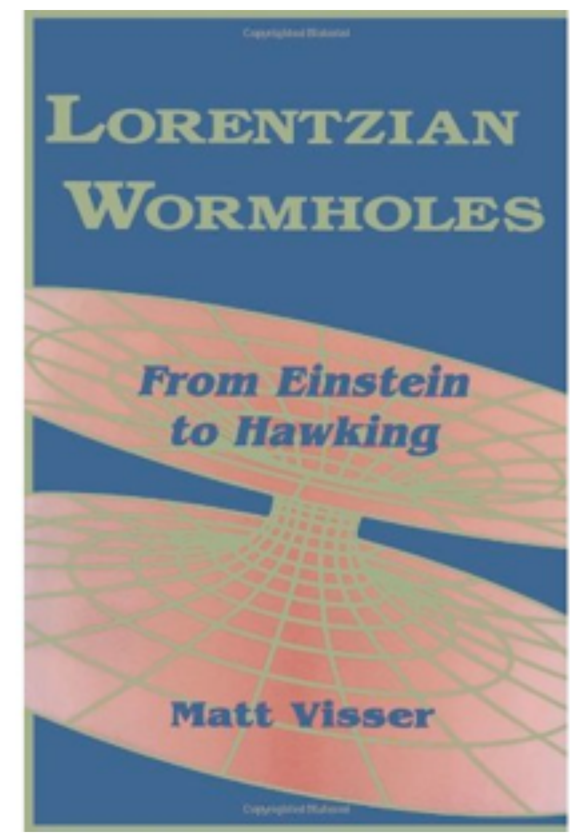
- Wormholes in 5-dim. GR
- Wormholes in 5-dim. Gauss-Bonnet gravity

First of all,

Wormholes are attractive,
but dangerous objects.







...danger than blackholes.

Part I

1 Why Wormhole?

- They make great science fiction – short cuts between otherwise distant regions. Morris & Thorne 1988, Sagan "Contact" etc
- They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.
- They are very similar to black holes –both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole $\equiv$ Hypersurface foliated by marginally trapped surfaces

- BH and WH are interconvertible?
  New duality?

Morris-Thorne's "Traversable" wormhole

M.S. Morris and K.S. Thorne, Am. J. Phys. 56 (1988) 395
M.S. Morris, K.S. Thorne, and U. Yurtsever, PRL 61 (1988) 3182
H.G. Ellis, J. Math. Phys. 14 (1973) 104
(G. Clément, Am. J. Phys. 57 (1989) 967)

Desired properties of traversable WHs

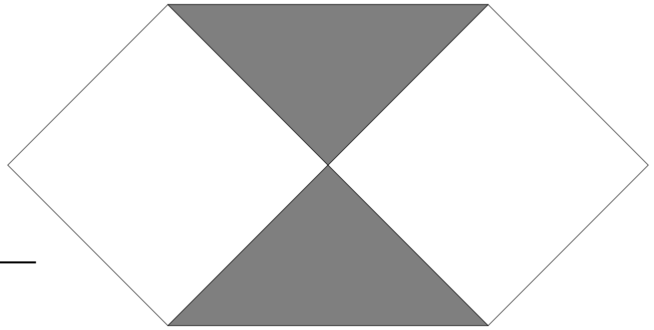
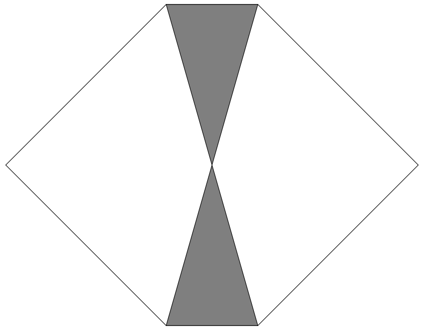
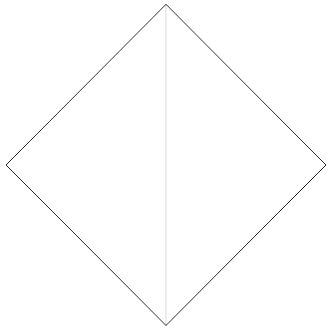
1. Spherically symmetric and Static $\Rightarrow$ M. Visser, PRD 39(89) 3182 & NPB 328 (89) 203
2. Einstein gravity
3. Asymptotically flat
4. No horizon for travel through
5. Tidal gravitational forces should be small for traveler
6. Traveler should cross it in a finite and reasonably small proper time
7. Must have a physically reasonable stress-energy tensor
   $\Rightarrow$ Weak Energy Condition is violated at the WH throat.
   $\Rightarrow$ (Null EC is also violated in general cases.)
8. Should be perturbatively stable
9. Should be possible to assemble

BH and WH are interconvertible ? (New Duality?)

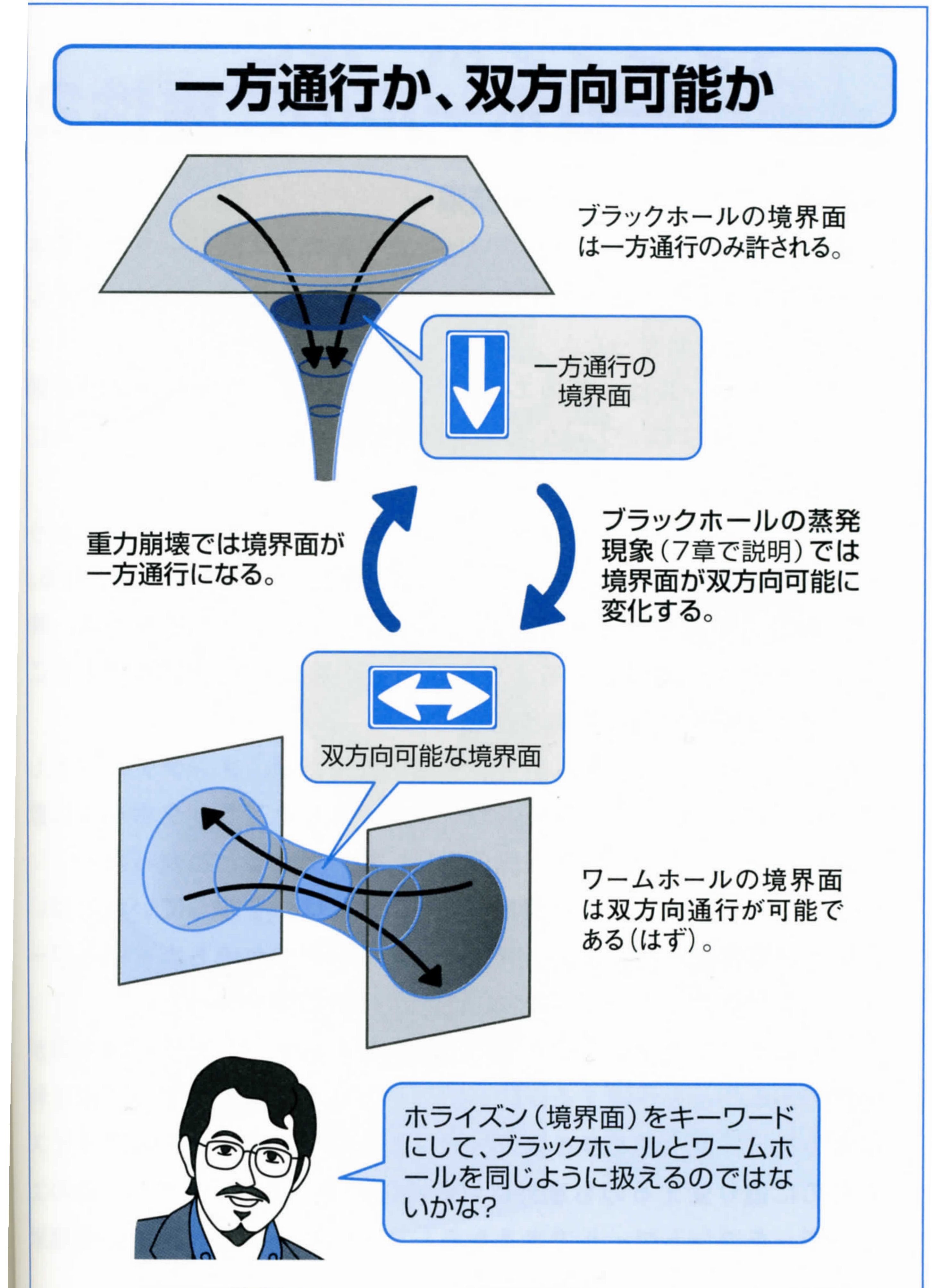
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

- They are very similar – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)
- Only the causal nature of the THs differs, whether THs evolve in plus / minus density.

	Black Hole	Wormhole
Locally defined by	Achronal(spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally. but constructible ???

Time Machine &
Science of Space-time
(HS, 2011)

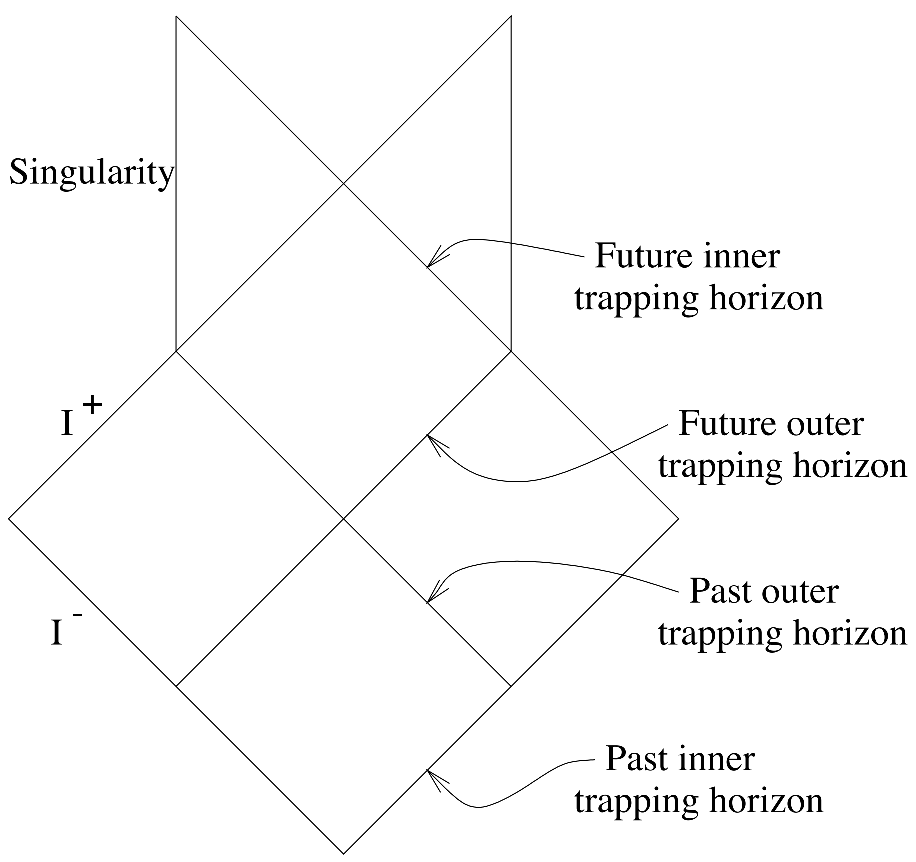


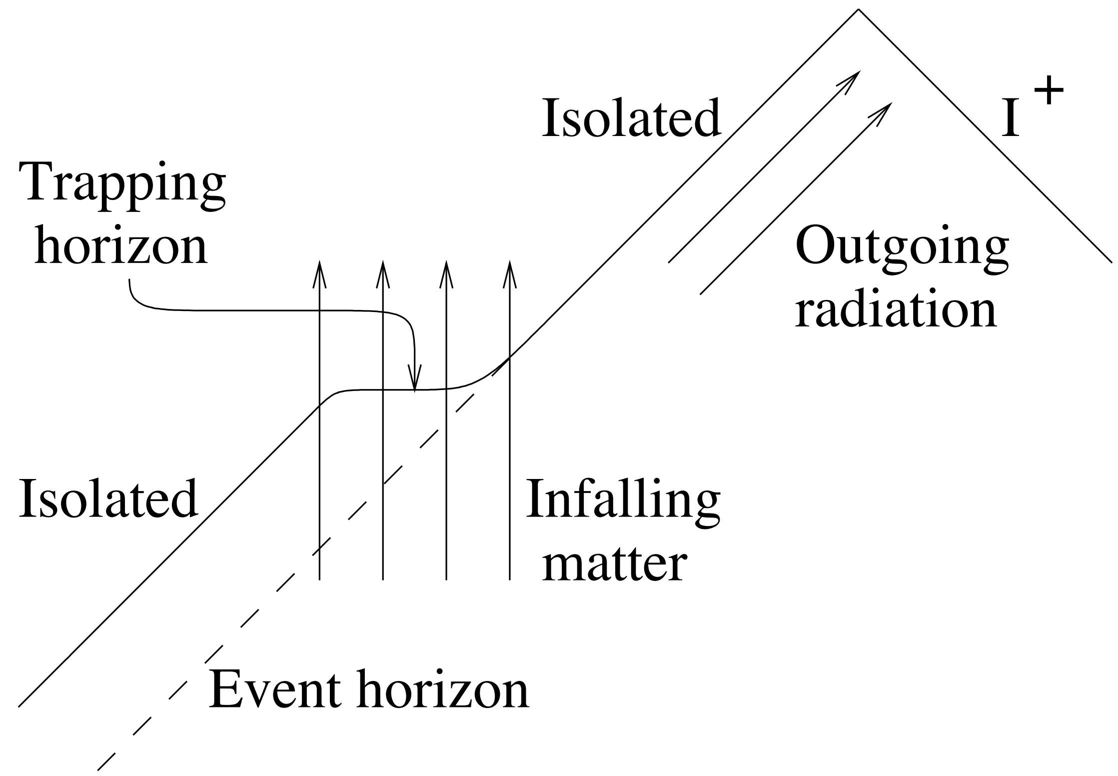
Trapping Horizon (TH) (Hayward, PRD49 (1994) 6467 & gr-qc/0008071)

Suppose $\theta_+ = 0$ expresses a marginal surface.

$$\begin{cases} \text{future horizon... if } \theta_- < 0 \\ \text{past horizon... if } \theta_- > 0 \end{cases}$$

$$\begin{cases} \text{inner horizon... if } \partial_-\theta_+ > 0 \\ \text{outer horizon... if } \partial_-\theta_+ < 0 \end{cases}$$





Part I

2 Fate of Morris-Thorne (Ellis) wormhole?

- **"Dynamical wormhole" defined by local trapping horizon**
- **spherically symmetric, both normal/ghost KG field**
- **apply dual-null formulation in order to seek horizons**
- **Numerical simulation**

2.1 ghost/normal Klein-Gordon fields

Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{16\pi} - \frac{1}{4\pi}\underbrace{\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)}_{\text{normal}} + \frac{1}{4\pi}\underbrace{\left(\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)}_{\text{ghost}}\right]$$

The field equations

$$G_{\mu\nu} = 2\left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] - 2\left[\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right]$$

$$\Box\psi = \frac{dV_1(\psi)}{d\psi}, \qquad \Box\phi = \frac{dV_2(\phi)}{d\phi}.$$ (Hereafter, we set $V_1(\psi) = 0, V_2(\phi) = 0$)

2.2 dual-null formulation, spherically symmetric spacetime

S A Hayward, CQG 10 (1993) 779, PRD 53 (1996) 1938, CQG 15 (1998) 3147

- The spherically symmetric line-element:

$$ds^2 = r^2 dS^2 - 2e^{-f} dx^+ dx^-,$$

where $r = r(x^+, x^-)$, $f = f(x^+, x^-)$, $\cdots$

- The Einstein equations:

$$\begin{aligned}
\partial_\pm \partial_\pm r + (\partial_\pm f)(\partial_\pm r) &= -r(\partial_\pm \psi)^2 + r(\partial_\pm \phi)^2, \\
r\partial_+ \partial_- r + (\partial_+ r)(\partial_- r) + e^{-f}/2 &= 0, \\
r^2 \partial_+ \partial_- f + 2(\partial_+ r)(\partial_- r) + e^{-f} &= +2r^2(\partial_+ \psi)(\partial_- \psi) - 2r^2(\partial_+ \phi)(\partial_- \phi), \\
r\partial_+ \partial_- \phi + (\partial_+ r)(\partial_- \phi) + (\partial_- r)(\partial_+ \phi) &= 0, \\
r\partial_+ \partial_- \psi + (\partial_+ r)(\partial_- \psi) + (\partial_- r)(\partial_+ \psi) &= 0.
\end{aligned}$$

- To obtain a system accurate near $\Im^\pm$, we introduce the conformal factor $\boxed{\Omega = 1/r}$. We also define first-order variables, the conformally rescaled momenta

expansions	$\vartheta_\pm = 2\partial_\pm r = -2\Omega^{-2}\partial_\pm \Omega \qquad (\theta_\pm = 2r^{-1}\partial_\pm r)$	(1)
inaffinities	$\nu_\pm = \partial_\pm f$	(2)
momenta of ϕ	$\wp_\pm = r\partial_\pm \phi = \Omega^{-1}\partial_\pm \phi$	(3)
momenta of ψ	$\pi_\pm = r\partial_\pm \psi = \Omega^{-1}\partial_\pm \psi$	(4)

The set of equations (cont.):

$$\partial_\pm \vartheta_\pm = -\nu_\pm \vartheta_\pm - 2\Omega\pi_\pm^2 + 2\Omega\wp_\pm^2, \quad (5)$$
$$\partial_\pm \vartheta_\mp = -\Omega(\vartheta_+\vartheta_-/2 + e^{-f}), \quad (6)$$
$$\partial_\pm \nu_\mp = -\Omega^2(\vartheta_+\vartheta_-/2 + e^{-f} - 2\pi_+\pi_- + 2\wp_+\wp_-), \quad (7)$$
$$\partial_\pm \wp_\mp = -\Omega\vartheta_\mp\wp_\pm/2, \quad (8)$$
$$\partial_\pm \pi_\mp = -\Omega\vartheta_\mp\pi_\pm/2. \quad (9)$$

and remember the identity: $\partial_+\partial_- = \partial_-\partial_+$:

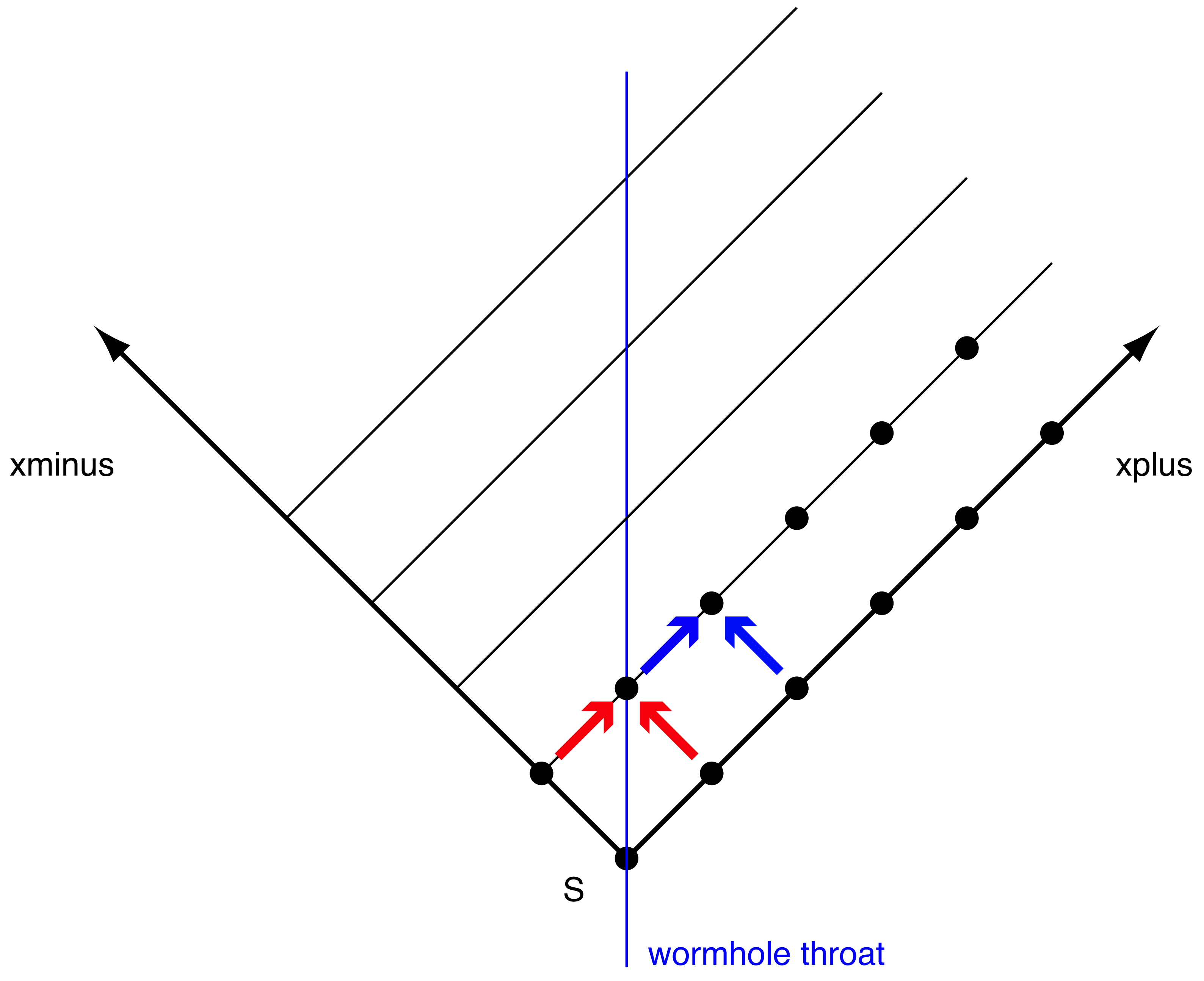
2.3 Initial data on $x^+ = 0$, $x^- = 0$ slices and on S

Generally, we have to set :

$(\Omega, f, \vartheta_\pm, \phi, \psi)$ on S: $x^+ = x^- = 0$

$(\nu_\pm, \wp_\pm, \pi_\pm)$ on $\Sigma_\pm$: $x^\mp = 0$, $x^\pm \geq 0$

Grid Structure for Numerical Evolution



2.4 Morris-Thorne (Ellis) wormhole as the initial data

	on Σ_+ ($x^- = 0$ surface)	on Σ_- ($x^+ = 0$ surface)
Ω	$1/\sqrt{a^2+z^2}$	$1/\sqrt{a^2+z^2}$
f	0	0
$\vartheta_\pm$	$\pm\sqrt{2}z/\sqrt{a^2+z^2}$	$\mp\sqrt{2}z/\sqrt{a^2+z^2}$
ν_+	0	
ν_-		0
ϕ	$\tan^{-1}(z/a)$	$-\tan^{-1}(z/a)$
$\wp_+$	$+a/\sqrt{2}\sqrt{a^2+z^2}$	
$\wp_-$		$-a/\sqrt{2}\sqrt{a^2+z^2}$
ψ	0	0
π_+	0	
π_-		0

where $z = (x^+ - x^-)/\sqrt{2}$.

We put the perturbation in $\wp_+$:
$\delta\wp_+ = c_a \exp(-c_b(z-c_c)^2)$
where c_a, c_b, c_c are parameters.

2.5 Gravitational mass-energy

- Localizing, the local gravitational mass-energy is given by the Misner-Sharp energy E,

$$E = (1/2)r[1 - g^{-1}(dr, dr)] = (1/2)r + e^f r\,(\partial_+ r)(\partial_- r) = \frac{1}{2\Omega}[1 + \frac{1}{2}e^f\,\vartheta_+\vartheta_-]$$

while the (localized Bondi) conformal flux vector components $\varphi^\pm$

$$\varphi^\pm = r^2 T^{\pm\pm}\partial_\pm r = r^2 e^{2f} T_{\mp\mp}\partial_\pm r = e^{2f}(\pi_\mp^2 - \wp_\mp^2)\vartheta_\pm/8\pi.$$

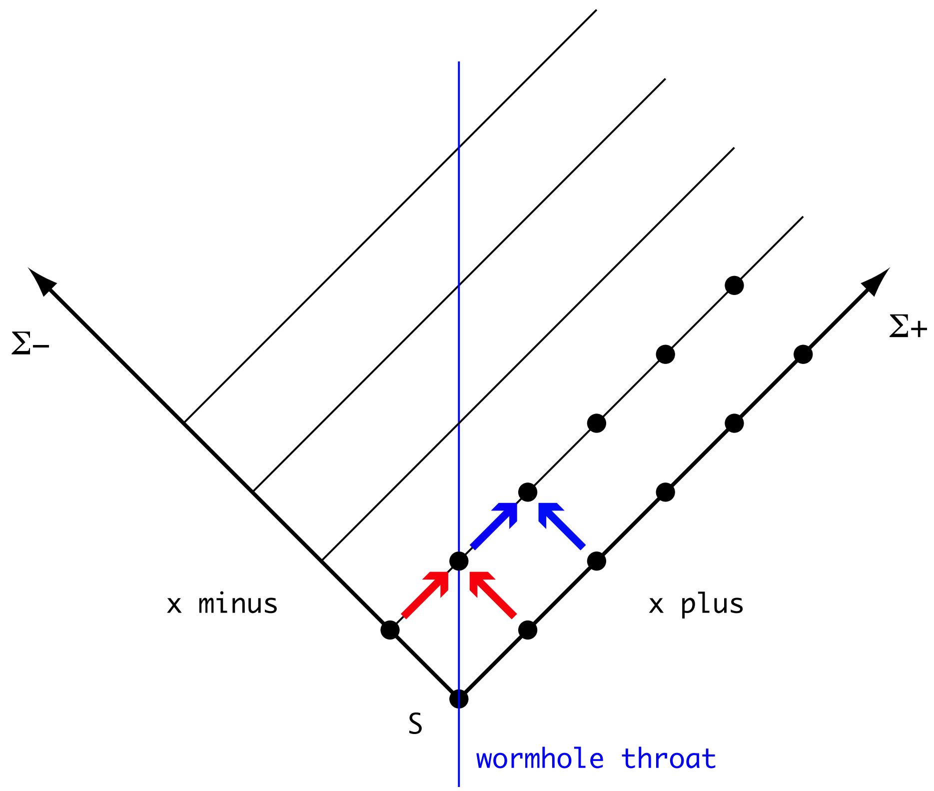
- They are related by the energy propagation equations or unified first law. $\partial_\pm E = 4\pi\varphi_\pm$,

$$E(x^+, x^-) = \frac{a}{2} + 4\pi \int_{(0,0)}^{(x^+,x^-)} (\varphi_+ dx^+ + \varphi_- dx^-),$$

where the integral is independent of path, by conservation of energy.

  - $\lim_{x^+\to\infty} E$ is the Bondi energy
  - $\lim_{x^+\to\infty} \varphi_-$ the Bondi flux for the right-hand universe.
  - For the static wormhole, the energy $E = a^2/2\sqrt{a^2+z^2}$ is everywhere positive, maximal at the throat and zero at infinity, $z \to \pm\infty$, i.e. the Bondi energy is zero.
  - Generally, the Bondi energy-loss property, that it should be non-increasing for matter satisfying the null energy condition, is reversed for the ghost field.

Numerical Grid / Convergence test



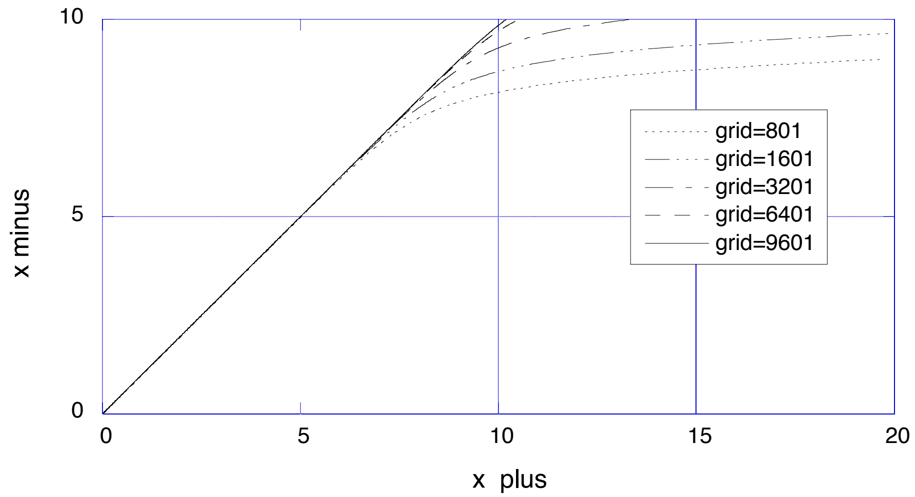


Figure 1: Numerical grid structure. Initial data are given on null hypersurfaces $\Sigma_\pm$ ($x^\mp = 0$, $x^\pm > 0$) and their intersection S.
Figure 2: Convergence behaviour of the code for exact static wormhole initial data. The location of the trapping horizon $\vartheta_- = 0$ is plotted for several resolutions labelled by the number of grid points for $x^+ = [0, 20]$. We see that numerical truncation error eventually destroys the static configuration.

Stationary Configurations

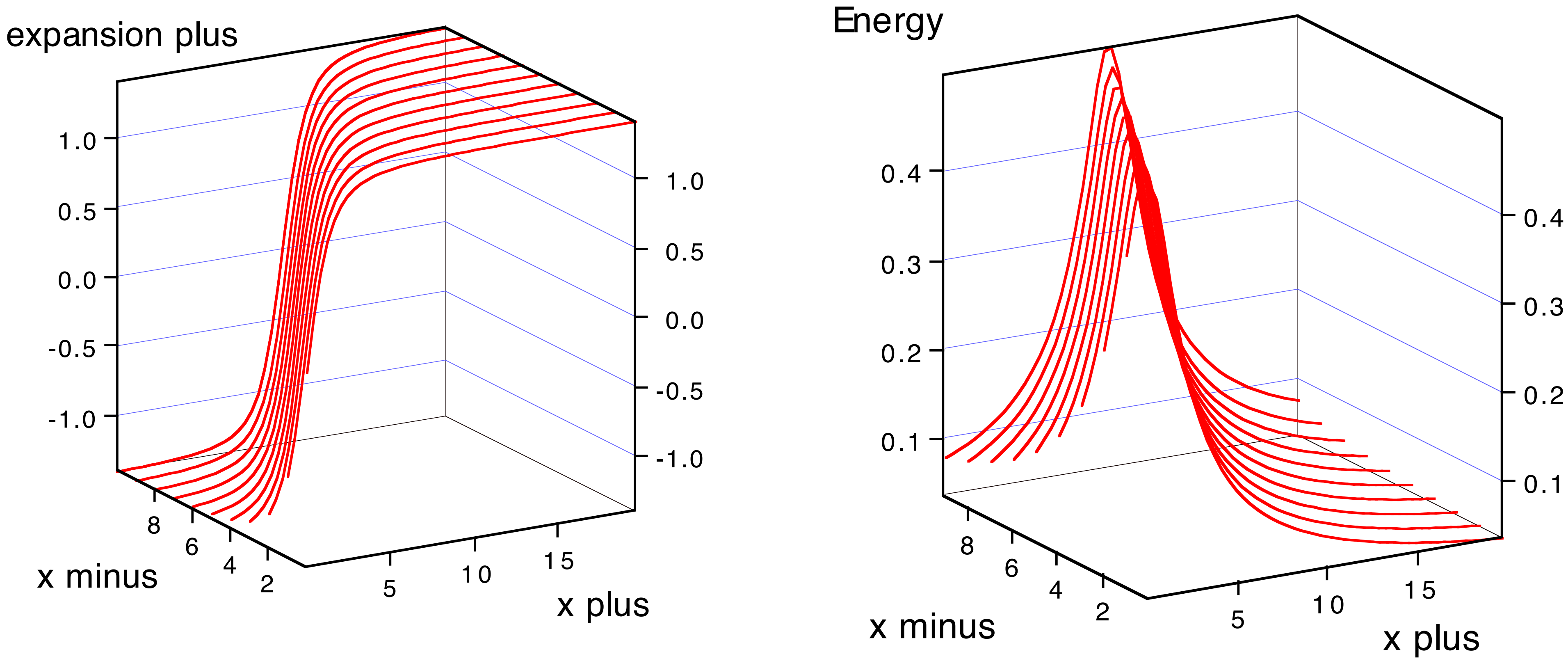


Figure 2: Static wormhole configuration obtained with the highest resolution calculation: (a) expansion ϑ_+ and (b) local gravitational mass-energy E are plotted as functions of (x^+, x^-). Note that the energy is positive and tends to zero at infinity.

Ghost pulse input -- Bifurcation of the horizons

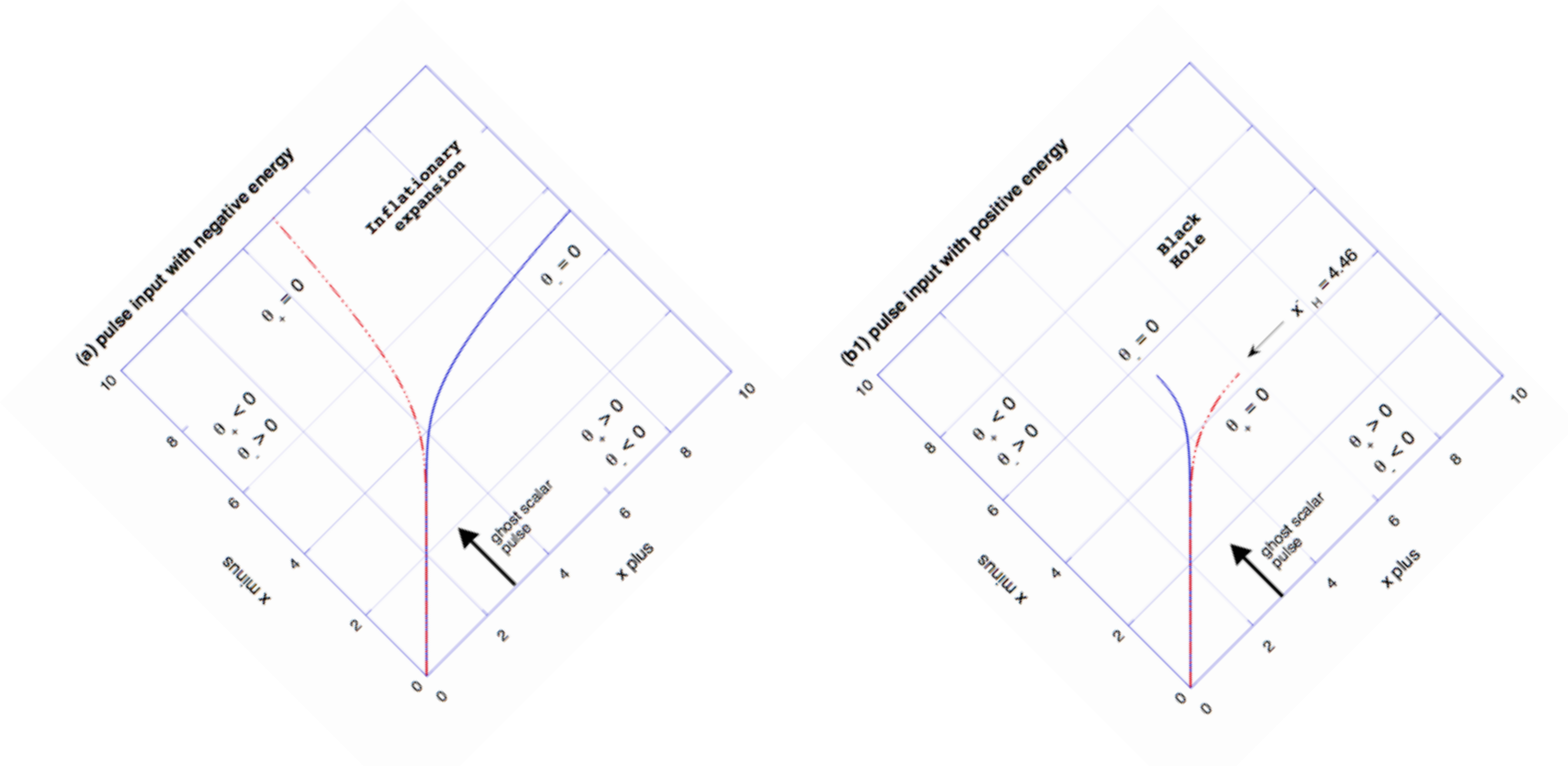
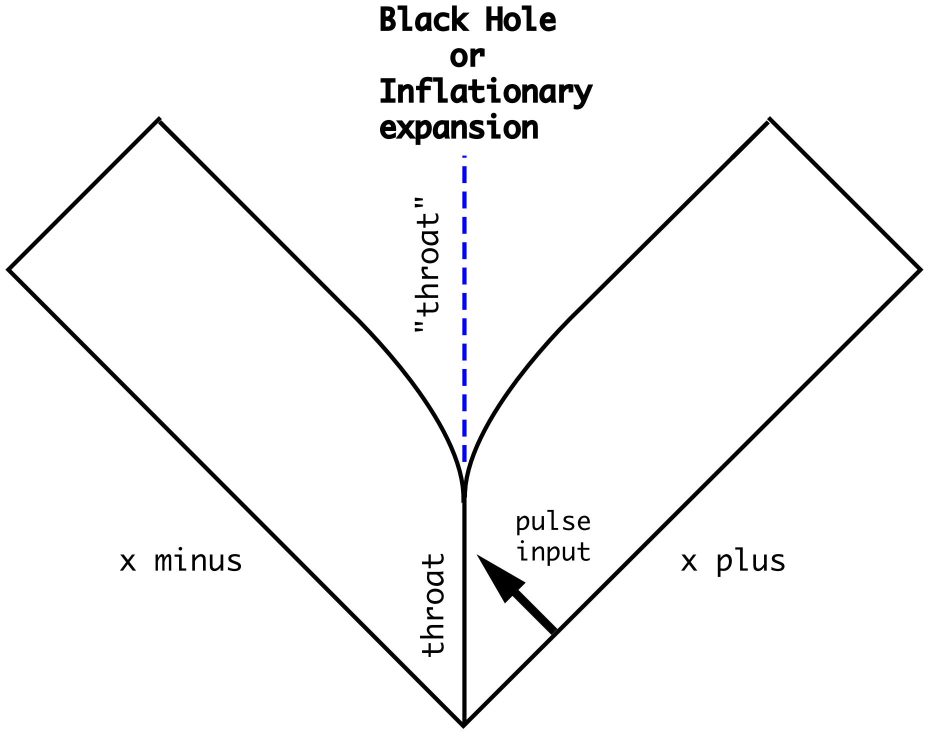


Figure 3: Horizon locations, $\vartheta_{\pm} = 0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

Bifurcation of the horizons – go to a Black Hole or Inflationary expansion



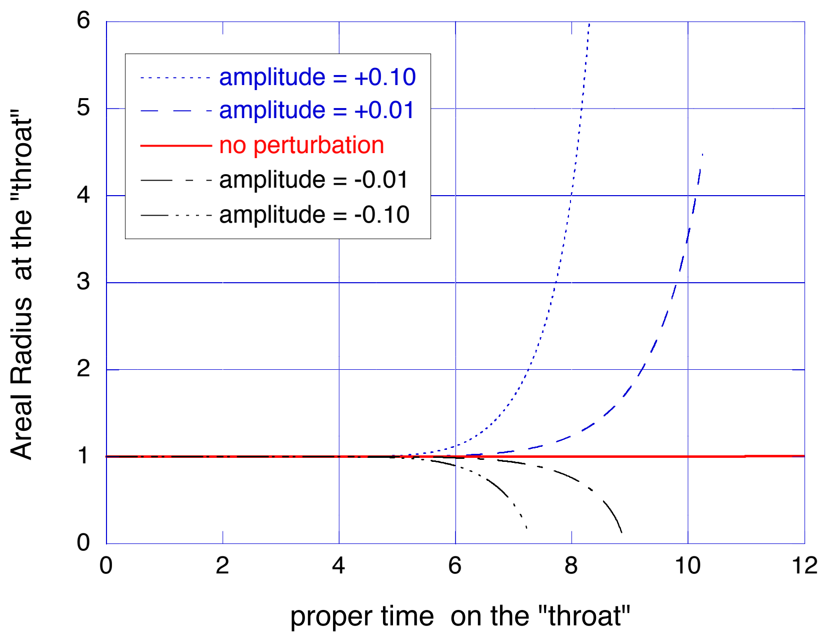


Figure 4: Partial Penrose diagram of the evolved space-time.

Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Local Energy Measure – Determination of the Black Hole Mass

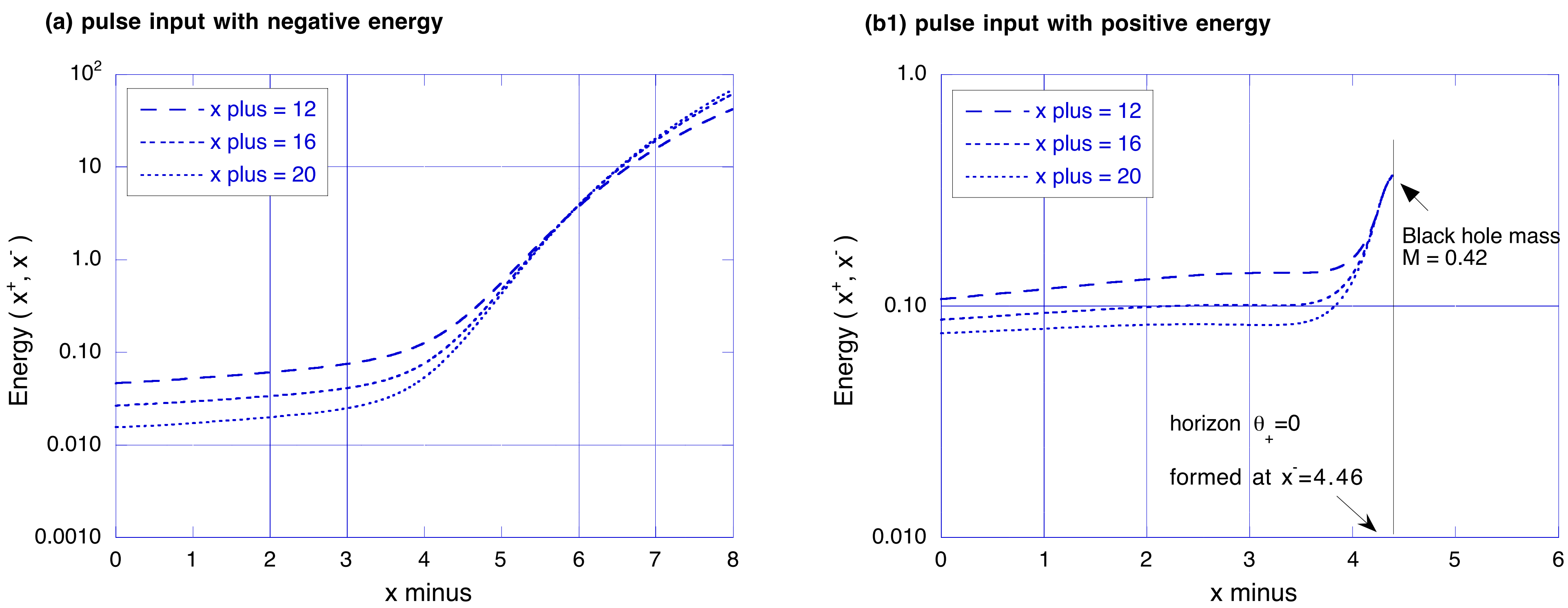


Figure 7: Energy $E(x^+, x^-)$ as a function of x^-, for $x^+ = 12, 16, 20$. Here c_a is (a) 0.05, (b1) -0.1 and (b2) -0.01. The energy for different x^+ coincides at the final horizon location x_H^-, indicating that the horizon quickly attains constant mass $M = E(\infty, x_H^-)$. This is the final mass of the black hole or cosmological horizon.

Is there a Minimum Black Hole Mass to be formed?

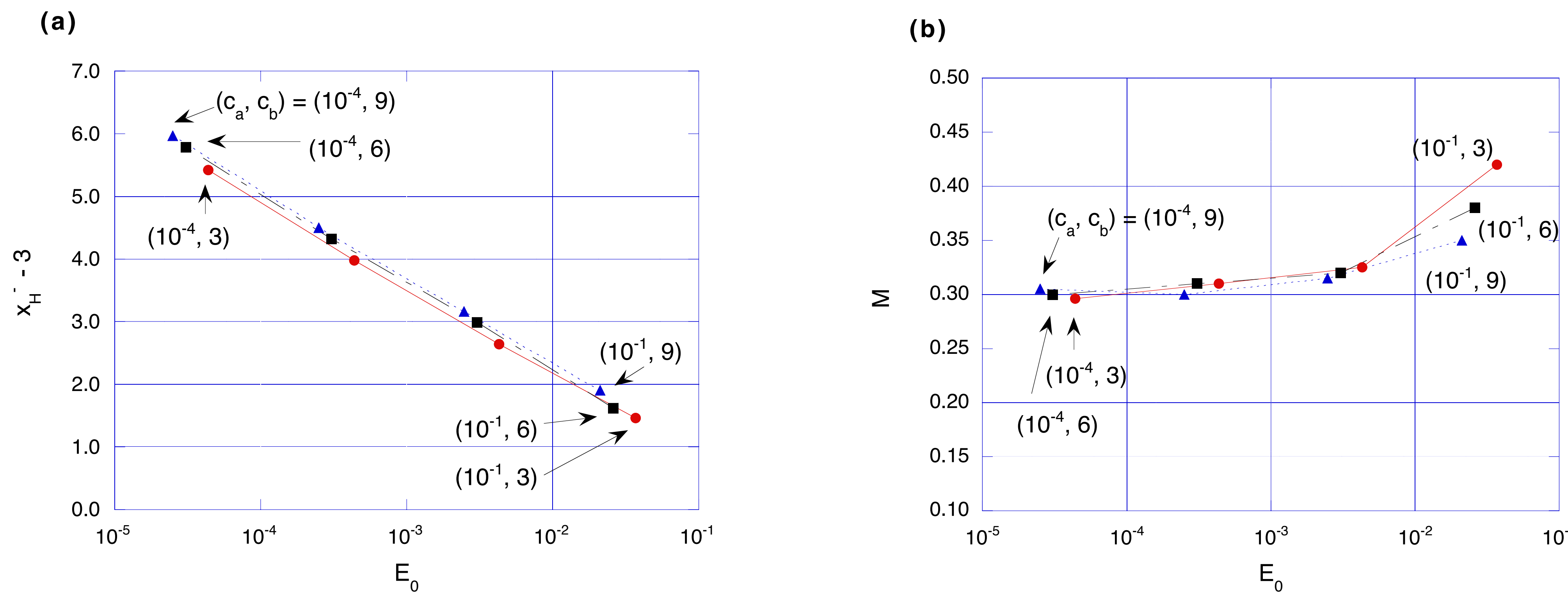


Figure 8: Relation between the initial perturbation and the final mass of the black hole. (a) The trapping horizon ($\vartheta_+ = 0$) coordinate, $x_H^- - 3$ (since we fixed $c_c = 3$), versus initial energy of the perturbation, E_0. We plotted the results of the runs of $c_a = 10^{-1}, \cdots, 10^{-4}$ with $c_b = 3, 6$, and 9. They lie close to one line. (b) The final black hole mass M for the same examples. We see that M appears to reach a non-zero minimum for small perturbations.

Normal pulse (a traveller) input -- Forming a Black Hole

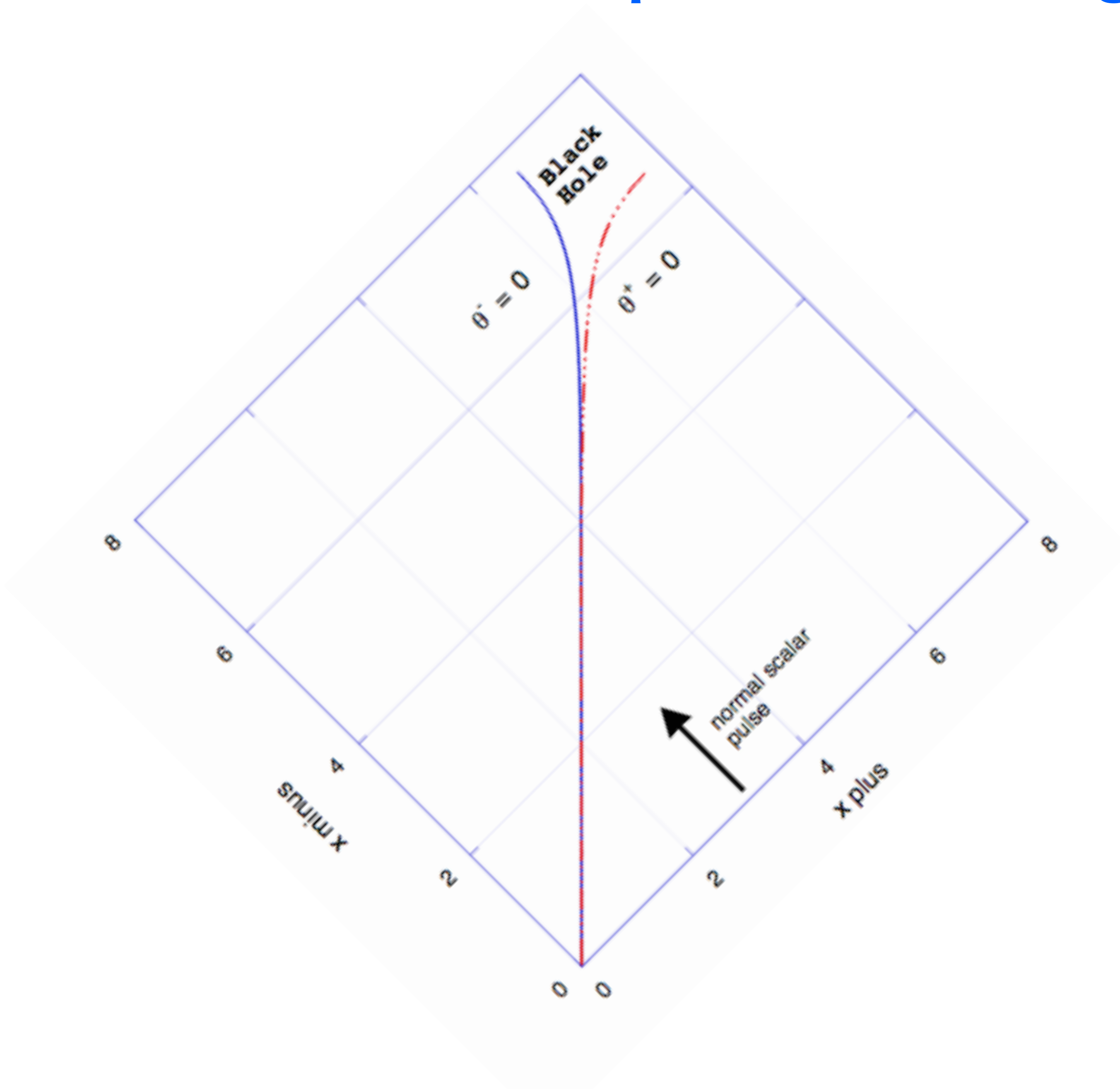


Figure 9: Evolution of a wormhole perturbed by a normal scalar field. Horizon locations: dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively.

Travel through a Wormhole

-- with Maintenance Operations!

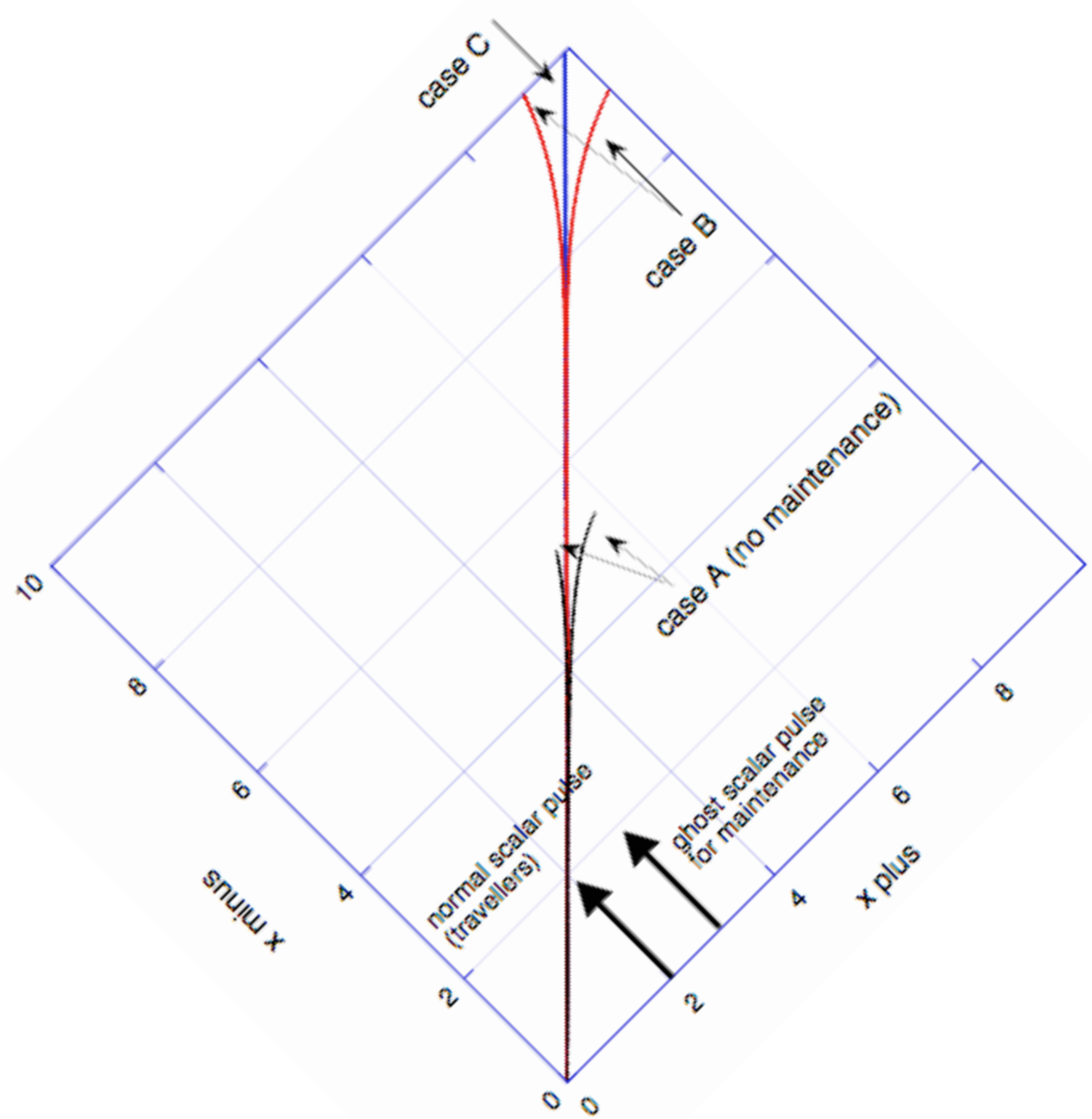


Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. The travellers pulse are commonly expressed with a normal scalar field pulse, $(\tilde{c}_a, \tilde{c}_b, \tilde{c}_c) = (+0.1, 6.0, 2.0)$. Horizon locations $\vartheta_+ = 0$ are plotted for three cases:

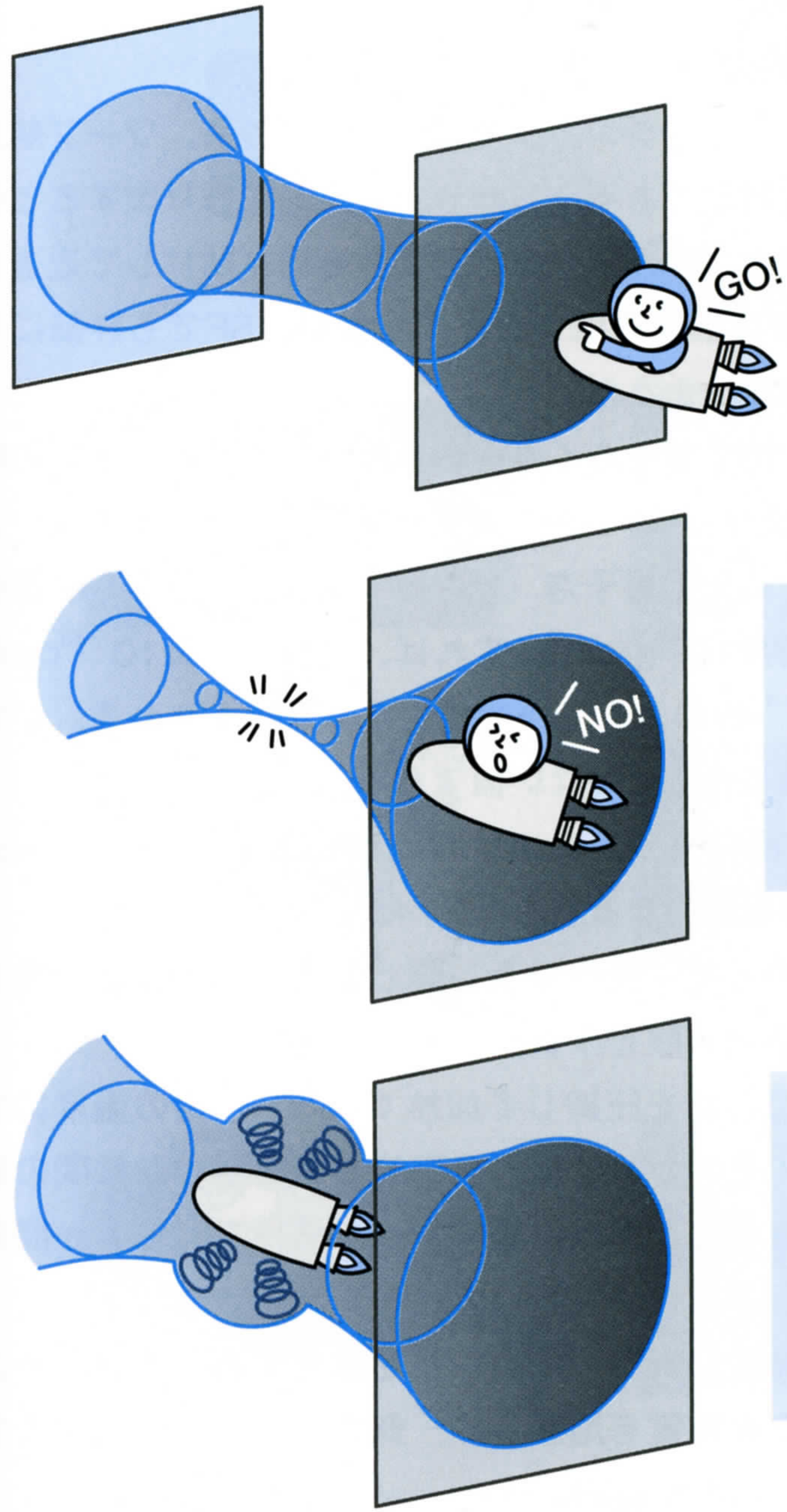
(A) no maintenance case (results in a black hole),

(B) with maintenance pulse of $(c_a, c_b, c_c) = (0.02390, 6.0, 3.0)$ (results in an inflationary expansion),

(C) with maintenance pulse of $(c_a, c_b, c_c) = (0.02385, 6.0, 3.0)$ (keep stationary structure upto the end of this range).

Time Machine &
Science of Space-time
(HS, 2011)

ワームホールを通過できるか



負のエネルギーで支えられているワームホールの中に、正のエネルギーの人間とロケットが入るとどうなる?

結論1

何もしないと、ワームホールは潰れてブラックホールになってしまう。

結論2

負のエネルギービームをうまく与えると、ワームホールを潰さずに通過することも可能である。

Summary of Part I

Dynamics of Ellis (Morris-Thorne) traversible WH

WH is Unstable

(A) with positive energy pulse ---> BH

---> confirmes duality conjecture between BH and WH.

(B) with negative energy pulse ---> Inflationary expansion

---> provides a mechanism for enlarging a quantum WH
to macroscopic size

(C) can be maintained by sophisticated operations

---> a round-trip is available for our hero/heroine

The basic behaviors has been confirmed by

A Doroshkevich, J Hansen, I Novikov, A Shatskiy, IJMPD 18 (2009) 1665
J A Gonzalez, F S Guzman & O Sarbach, CQG 26 (2009) 015010, 015011
J A Gonzalez, F S Guzman & O Sarbach, PRD80 (2009) 024023
O Sarbach & T Zannias, PRD 81 (2010) 047502

Part II 5-dimensional numerical simulations

[1] **How the stability changes in 5-d GR?**

[2] **How the stability changes in Gauss-Bonnet gravity?**

Gauss-Bonnet gravity

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\left(\mathcal{R}^2 - 4\mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta} + \mathcal{R}_{\alpha\beta\gamma\delta}\mathcal{R}^{\alpha\beta\gamma\delta}\right)\} + \mathcal{L}_{\rm matter}\Big]$$

- **has GR correction terms from String Theory.**
- **has two solution branches (GR/non-GR).**
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated.)

- **new topic in numerical relativity.**
  (S Golod & T Piran, PRD 85 (2012) 104015;
  F Izaurieta & E Rodriguez, 1207.1496; N Deppe+ 1208.5250)

Wormholes in Einstein-Gauss-Bonnet gravity

- B Bhawal & S Kar, PRD 46 (1992) 2464
  WH sols and a-α relations.
- G Dotti, J Oliva & R Troncoso, PRD 76 (2007) 064038
  exhaustive classification of sols
- M G Richarte & C Simeone, PRD 76 (2007) 087502
  thin-shell WHs supported by ordinary matter.
- H Maeda & M Nozawa, PRD 78 (2008) 024005
  WH sols and energy conditions.
- M H Dehghani & Z Dayyani, PRD 79 (2009) 064010
  WH sols and a-α relations in Lovelock.
- S H Mazharimousavi$+$, CQG 28 (2011) 025004
  thin-shell WHs in Einstein-Yang-Mills-Gauss-Bonnet.
- P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101, PRD 85 (2012) 044007
  WH sols in Dilatonic-Gauss-Bonnet.

Field Equations

- **Action**

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\}+\mathcal{L}_{\rm matter}\Big] \quad (1)$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- **Field equation**

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu} \quad (2)$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **matter**

normal field $\psi(u,v)$ **and/or ghost field** $\phi(u,v)$

$$\begin{aligned} T_{\mu\nu} &= T^{\psi}_{\mu\nu} + T^{\phi}_{\mu\nu} \quad (3) \\ &= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] - \left[\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \end{aligned}$$

this derives Klein-Gordon equations

$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}. \quad (4)$$

Assumptions

- **5-dim.**
- **Spherical Symmetry**
- **Dual-null coordinate**

$$ds^2 = -2e^{-f(x^+,x^-)}dx^+\,dx^- + r^2(x^+,x^-)d\Omega_3^2 \tag{5}$$

- **Variables**

$$\Omega = \frac{1}{r} \tag{6}$$

$$\vartheta_\pm \equiv 3\partial_\pm r \tag{7}$$

$$\nu_\pm \equiv \partial_\pm f \tag{8}$$

$$\pi_\pm \equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi \tag{9}$$

$$p_\pm \equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi \tag{10}$$

We also define η as

$$\eta = \Omega^2\left(e^{-f} + \frac{2}{9}\vartheta_+\vartheta_-\right) \tag{11}$$

• **Klein-Gordon eqs.**

$$(4d) \qquad \Box\phi = -\frac{e^f}{r}\left(2r\phi_{+-} + 2r_+\phi_- + 2r_-\phi_+\right) \tag{12}$$

$$(5d) \qquad \Box\phi = -\frac{e^f}{r}\left(2r\phi_{+-} + 3r_+\phi_- + 3r_-\phi_+\right) \tag{13}$$

4-dim

$$\partial_+\pi_- = -\frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \tag{14}$$

$$\partial_-\pi_+ = -\frac{1}{2}\Omega\vartheta_+\pi_- - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \tag{15}$$

$$\partial_+p_- = -\frac{1}{2}\Omega\vartheta_-p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \tag{16}$$

$$\partial_-p_+ = -\frac{1}{2}\Omega\vartheta_+p_- - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \tag{17}$$

5-dim

$$\partial_+\pi_- = -\frac{1}{6}\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \tag{18}$$

$$\partial_-\pi_+ = -\frac{1}{2}\Omega\vartheta_+\pi_- - \frac{1}{6}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \tag{19}$$

$$\partial_+p_- = -\frac{1}{6}\Omega\vartheta_+p_- - \frac{1}{2}\Omega\vartheta_-p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \tag{20}$$

$$\partial_-p_+ = -\frac{1}{2}\Omega\vartheta_+p_- - \frac{1}{6}\Omega\vartheta_-p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \tag{21}$$

Equations in 5-D with Gauss-Bonnet corrections

$(\Omega, \vartheta_{\pm} = 3\partial_{\pm} r, f, \nu_{\pm} = \partial_{\pm} f, \psi, \pi_{\pm} = r\partial_{\pm}\psi, \phi, p_{\pm} = r\partial_{\pm}\phi)$

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

$$\eta = \Omega^2\left(e^{-f} + \frac{2}{9}\vartheta_+\vartheta_-\right), \quad \tilde{A} = (\alpha_1 + 4\alpha_2\eta e^f), \quad B = \kappa^2 T_{+-} + e^{-f}\Lambda$$

x^+-direction

$$\partial_+\Omega = -\frac{1}{3}\vartheta_+\Omega^2 \tag{1}$$

$$\partial_+\vartheta_+ = -\nu_+\vartheta_+ - \frac{1}{\tilde{A}\Omega}\kappa^2 T_{++} \tag{2}$$

$$\partial_+\vartheta_- = \frac{1}{\tilde{A}\Omega}(-3\alpha_1\eta + B) \tag{3}$$

$$\partial_+ f = \nu_+ \tag{4}$$

$$\partial_+\nu_- = \frac{\alpha_1}{\tilde{A}}\left\{\eta - \frac{4\left(3\alpha_1\eta - B\right)}{3\tilde{A}}\right\} + \frac{(\kappa^2 T_{zz}\Omega^2 - \Lambda)}{\tilde{A}e^f} + \frac{8\alpha_2}{9\tilde{A}^3}\left\{e^f\left(3\alpha_1\eta - B\right)^2 - \kappa^4 T_{++}T_{--}\right\} \tag{5}$$

$$\partial_+\psi = \Omega\pi_+ \tag{6}$$

$$\partial_+\phi = \Omega p_+ \tag{7}$$

$$\partial_+\pi_- = -\frac{1}{6}\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \tag{8}$$

$$\partial_+ p_- = -\frac{1}{6}\Omega\vartheta_+ p_- - \frac{1}{2}\Omega\vartheta_- p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \tag{9}$$

x^--direction

$$\partial_- \Omega = -\frac{1}{3}\vartheta_- \Omega^2 \tag{10}$$

$$\partial_- \vartheta_+ = \frac{1}{\tilde{A}\Omega}\left(-3\alpha_1 \eta + B\right) \tag{11}$$

$$\partial_- \vartheta_- = -\nu_- \vartheta_- - \frac{1}{\tilde{A}\Omega}\kappa^2 T_{--} \tag{12}$$

$$\partial_- f = \nu_- \tag{13}$$

$$\partial_- \nu_+ = (5) \tag{14}$$

$$\partial_- \psi = \Omega \pi_- \tag{15}$$

$$\partial_- \phi = \Omega p_- \tag{16}$$

$$\partial_- \pi_+ = -\frac{1}{2}\Omega \vartheta_+ \pi_- - \frac{1}{6}\Omega \vartheta_- \pi_+ - \frac{1}{2e^f \Omega}\frac{dV_1}{d\psi} \tag{17}$$

$$\partial_- p_+ = -\frac{1}{2}\Omega \vartheta_+ p_- - \frac{1}{6}\Omega \vartheta_- p_+ - \frac{1}{2e^f \Omega}\frac{dV_2}{d\phi} \tag{18}$$

Energy-momentum tensor

$$T_{++} = \Omega^2 (\pi_+^2 - p_+^2) \tag{19}$$

$$T_{--} = \Omega^2 (\pi_-^2 - p_-^2) \tag{20}$$

$$T_{+-} = -e^{-f}\left(V_1(\psi) + V_2(\phi)\right) \tag{21}$$

$$T_{zz} = e^f (\pi_+ \pi_- - p_+ p_-) - \frac{1}{\Omega^2}\left(V_1(\psi) - V_2(\phi)\right) \tag{22}$$

Ellis solution (4D)

- **massless ghost scalar field ϕ.**
- **static, spherical symmetry.**

$$ds^2 = -2e^{-f(x^+,x^-)}dx^+dx^- + r^2(x^+,x^-)d\Omega^2$$

metric ($z = \dfrac{x^+ - x^-}{\sqrt{2}}$, a: throat radius.)

$$r = \sqrt{a^2 + z^2} \qquad \vartheta_\pm = \pm\sqrt{2}z/r \tag{1}$$

$$e^{-f} = 1, \qquad f = 0, \qquad \nu_\pm = 0 \tag{2}$$

scalar field

$$\phi = \tan^{-1}(z/a), \qquad \wp_\pm = \pm\frac{a}{\sqrt{2}r} \tag{3}$$

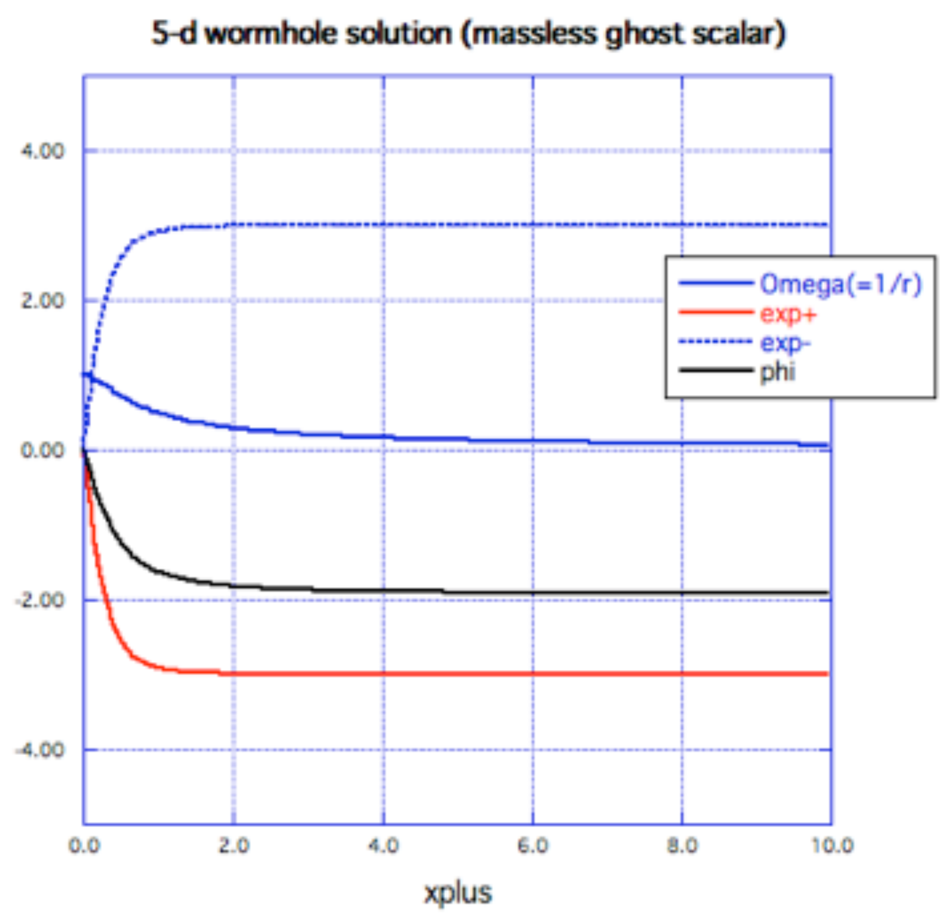
A Wormhole Solution (n-Dim, massless ghost scalar)

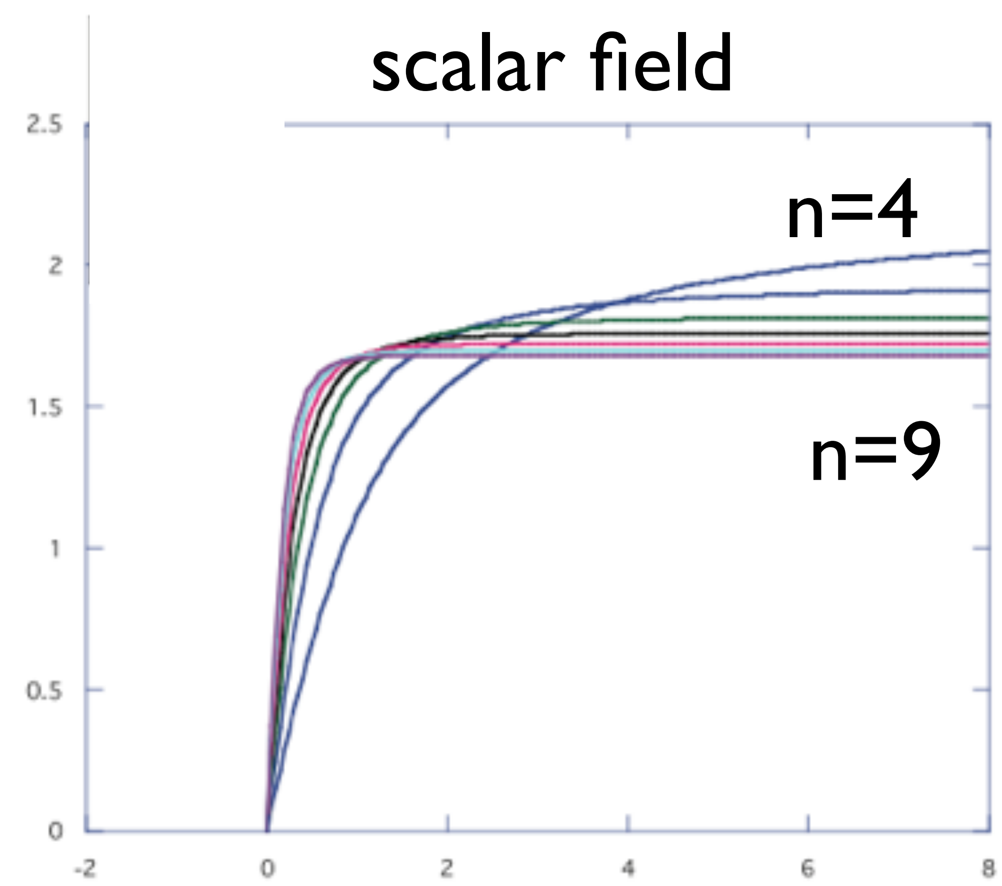
- massless ghost scalar field ϕ.
- static, spherical symmetry.

$$ds^2 = -dt^2 + dz^2 + r^2(z)d\Omega^2$$

$$\frac{d^2r}{dz^2} = \frac{(n-3)a^{2(n-3)}}{r^{2n-5}} \quad (1)$$

$$\frac{d\phi}{dz} = \sqrt{(n-2)(n-3)}\frac{a^{n-3}}{r^{n-2}} \quad (2)$$





A Wormhole Solution (5-Dim, massive ghost scalar)

- **massive ghost scalar field ϕ.**
- **static, spherical symmetry.**

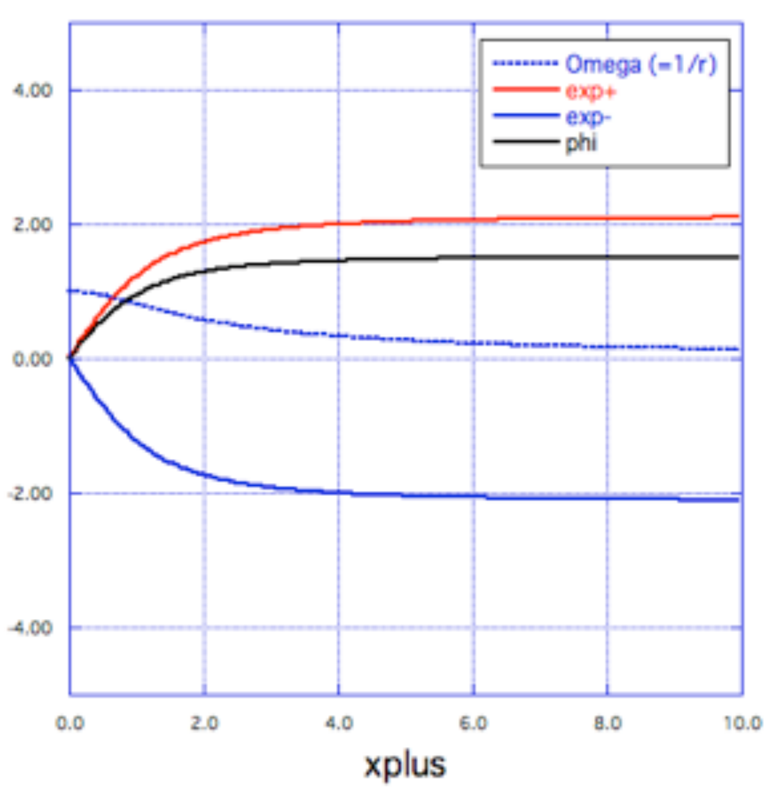
$$ds^2 = -2e^{-f(x^+,x^-)}dx^+dx^- + r^2(x^+,x^-)d\Omega^2$$

metric ($z = \dfrac{x^+ - x^-}{\sqrt{2}}$, a: **throat radius**)

$$r = \sqrt{a^2 + z^2} \qquad \vartheta_\pm = \pm\frac{3}{\sqrt{2}}\frac{z}{r} \tag{1}$$

$$e^{-f} = \frac{r^2 + z^2}{2r^2} \tag{2}$$

scalar field



$$\phi = -\sqrt{3}\,\tanh^{-1}\frac{-z}{\sqrt{r^2 + z^2}} \tag{3}$$

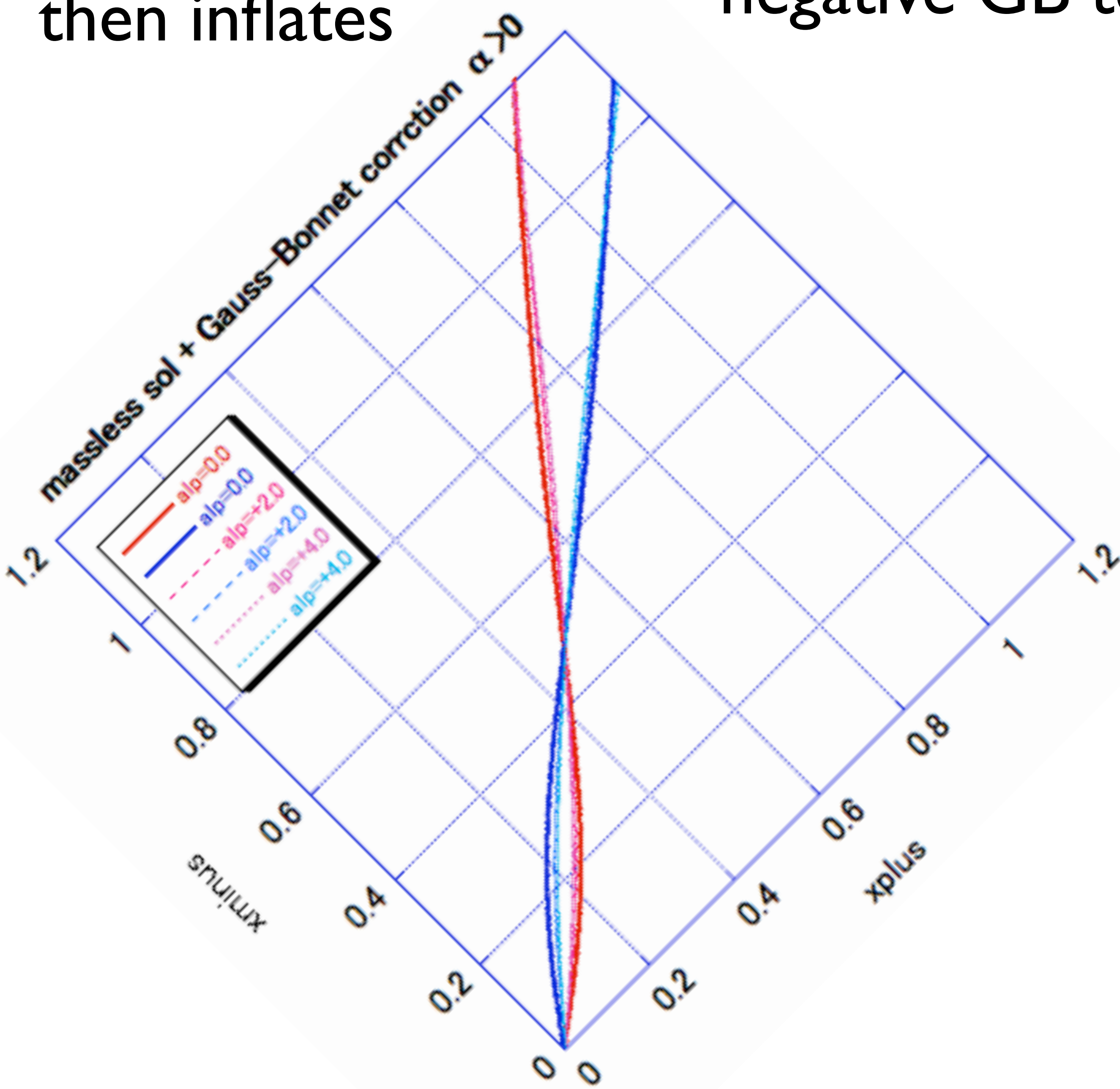
$$\frac{dV_2}{d\phi} = -\frac{\sqrt{3}}{a^2}\sinh\frac{2\phi}{\sqrt{3}}\left(1 - 2\tanh^2\frac{\phi}{\sqrt{3}}\right)^3 \tag{4}$$

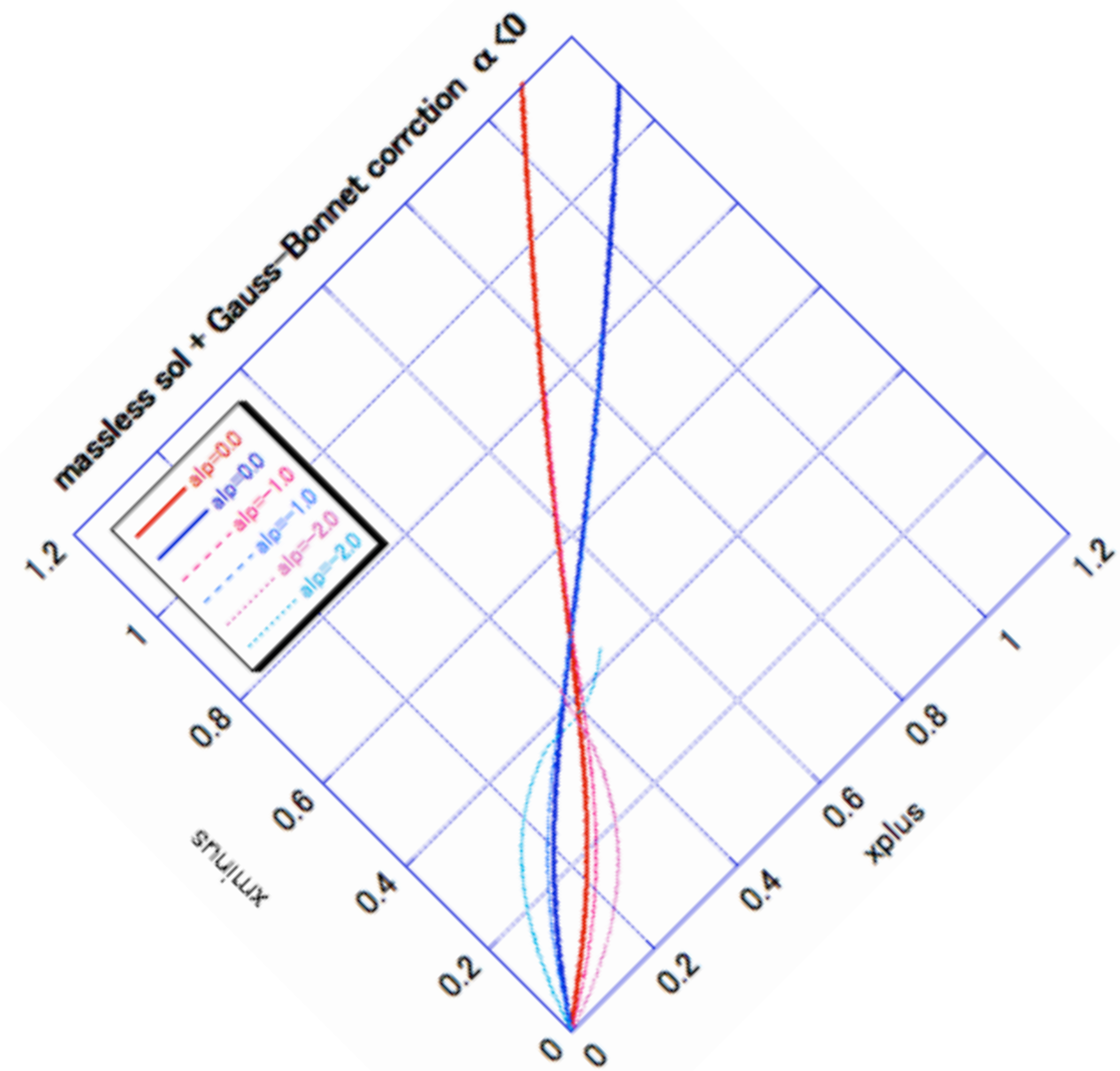
WH evolution in 5D Gauss-Bonnet gravity

Preliminary

temporal BH,
then inflates

positive GB term prevents BH collapse
negative GB term accelerates BH collapse





$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

Summary of Introduction

WH is Dangerous

Summary of Part I

Dynamics of Ellis (Morris-Thorne) traversible WH

WH is Unstable

(A) with positive energy pulse ---> BH

(B) with negative energy pulse ---> Inflationary expansion

(C) can be maintained by sophisticated operations

Summary of Part II (preliminary)

Dynamics in 5-d GR

basically the same with 4-dim

Dynamics in 5-d Gauss-Bonnet gravity

positive GB term prevents BH collapse

negative GB term accelerates BH collapse