Wormhole dynamics
in higher-dimensional space-time

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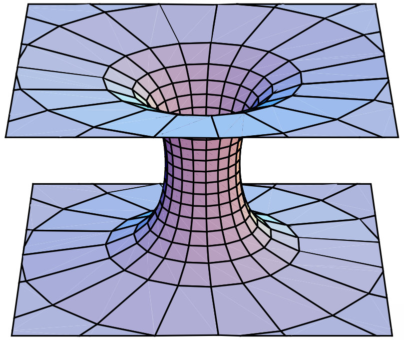


Part I　　Wormholes in 4-dim.

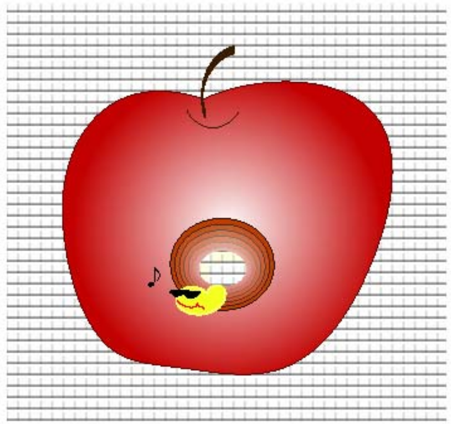
Part II　　1. n-dim. exact solution

　　　　　2. linear stability

　　　　　3. dynamical stability

Part III　　Wormhole in Gauss-Bonnet gravity

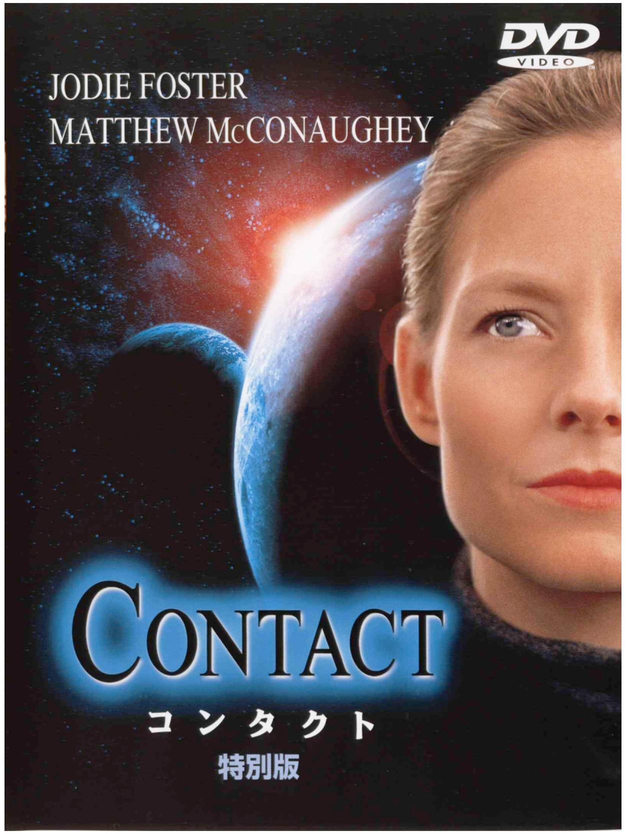
Part IV　　In Search of stable wormhole

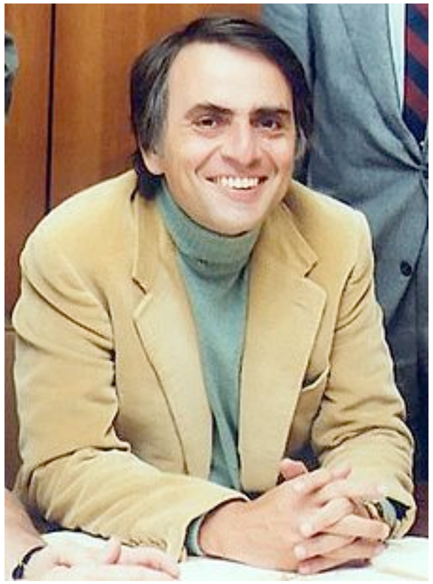


http://www.is.oit.ac.jp/~shinkai/　　2014年11月28日　大阪市立大　重力グループ　コロキウム

Why Wormhole?

They make great science fiction -- short cuts between otherwise distant regions.
Morris & Thorne 1988, Sagan "Contact" etc





US movie 1997

Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity

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(Received 16 March 1987; accepted for publication 17 July 1987)

Rapid interstellar travel by means of spacetime wormholes is described in a way that is useful for teaching elementary general relativity. The description touches base with Carl Sagan's novel *Contact*, which, unlike most science fiction novels, treats such travel in a manner that accords with the best 1986 knowledge of the laws of physics. Many objections are given against the use of black holes or Schwarzschild wormholes for rapid interstellar travel. A new class of solutions of the Einstein field equations is presented, which describe wormholes that, in principle, could be traversed by human beings. It is essential in these solutions that the wormhole possess a throat at which there is no horizon; and this property, together with the Einstein field equations, places an extreme constraint on the material that generates the wormhole's spacetime curvature: In the wormhole's throat that material must possess a radial tension τ_0 with the enormous magnitude $\tau_0 \sim$ (pressure at the center of the most massive of neutron stars) $\times (20\ \text{km})^2/(\text{circumference of throat})^2$. Moreover, this tension must exceed the material's density of mass-energy, $\rho_0 c^2$. No known material has this $\tau_0 > \rho_0 c^2$ property, and such material would violate all the "energy conditions" that underlie some deeply cherished theorems in general relativity. However, it is not possible today to rule out firmly the existence of such material; and quantum field theory gives tantalizing hints that such material might, in fact, be possible.

Box 1. Excerpts from *Contact* by Carl Sagan.[19]

After traveling through some sort of "tunnel" that took them in less than an hour from Earth to an orbit around the star Vega, five of the characters in the novel speculate on the nature of the tunnel:

"You see," Eda explained softly, "if the tunnels are black holes there are real contradictions implied. There is an interior tunnel in the exact Kerr solution of the Einstein Field Equations, but it's unstable. The slightest perturbation would seal it off and convert the tunnel into a physical singularity through which nothing can pass. I have tried to imagine a superior civilization that would control the internal structure of a collapsing star to keep the interior tunnel stable. This is very difficult. The civilization would have to monitor and stabilize the tunnel forever. It would be especially difficult with something as large as the dodecahedron falling through."

"Even if Abonnema can discover how to keep the tunnel open, there are many other problems," Vaygay said. "Too many. Black holes collect problems faster than they collect matter. There are the tidal forces. We should have been torn apart in the black hole's gravitational field. We should have been stretched like people in the paintings of El Greco or the sculptures of . . . Giacometti. Then other problems: As measured from Earth it takes an infinite amount of time for us to pass through a black hole, and we could never, never return to Earth. Maybe this is what happened. Maybe we will never go home. Then, there should be an inferno of radiation near the singularity. This is a quantum mechanical instability. . . "

"And finally," Eda continued, "a Kerr-type tunnel can lead to grotesque causality violations. With a modest change of trajectory inside the tunnel, one could emerge from the other end as early in the history of the universe as you might like—a picosecond after the big bang, for example. That would be a very disorderly universe.

"Look, fellas," she said, "I'm no expert in General Relativity. But didn't we see black holes? Didn't we fall into them? Didn't we emerge out of them? Isn't a gram of observation worth a ton of theory?"

"I know, I know," Vaygay said in mild agony. "It has to be something else. Our understanding of physics can't be so far off. Can it?"

He addressed this last question, a little plaintively, to Eda, who only replied, "A naturally occurring black hole can't be a tunnel; they have impassible singularities at their centers."

pages 347,348

Eda was, considering the circumstances, very relaxed. She soon understood why. While she and Vaygay had been undergoing lengthy interrogations, he had been calculating.

"I think the tunnels are Einstein–Rosen bridges," he said. "General relativity admits a class of solutions, called wormholes, similar to black holes, but with no evolutionary connection—they cannot be generated, as black holes can, by the gravitational collapse of a star. But the usual sort of wormhole, once made, expands and contracts before anything can cross through; it exerts disastrous tidal forces, and it also requires—at least as seen by an observer left behind—an infinite amount of time to get through."

Ellie did not see how this represented much progress, and asked him to clarify. The key problem was holding the wormhole open. Eda had found a class of solutions to his field equations that suggested a new macroscopic field, a kind of tension that could be used to prevent a wormhole from contracting fully. Such a wormhole would pose none of the other problems of black holes; it would have much smaller tidal stresses, two-way access, quick transit times as measured by an exterior observer, and no devastating interior radiation field.

"I don't know whether the tunnel is stable against small perturbations," he said. "If not, they would have to build a very elaborate feedback system to monitor and correct the instabilities."

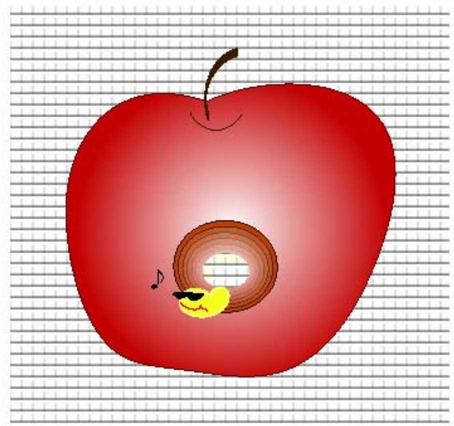
page 406

Morris-Thorne's "Traversable" wormhole

M.S. Morris and K.S. Thorne, Am. J. Phys. 56 (1988) 395
M.S. Morris, K.S. Thorne, and U. Yurtsever, PRL 61 (1988) 3182
H.G. Ellis, J. Math. Phys. 14 (1973) 104
(G. Clément, Am. J. Phys. 57 (1989) 967)

Desired properties of traversable WHs

1. Spherically symmetric and Static ⇒ M. Visser, PRD 39(89) 3182 & NPB 328 (89) 203
2. Einstein gravity
3. Asymptotically flat
4. No horizon for travel through
5. Tidal gravitational forces should be small for traveler
6. Traveler should cross it in a finite and reasonably small proper time
7. Must have a physically reasonable stress-energy tensor
   ⇒ Weak Energy Condition is violated at the WH throat.
   ⇒ (Null EC is also violated in general cases.)
8. Should be perturbatively stable
9. Should be possible to assemble



"Ellis (Morris-Thorne) wormhole"

Interstellar (2014)

Executive Producer: Kip Thorne

https://www.youtube.com/watch?v=qZZ9jRan9eo

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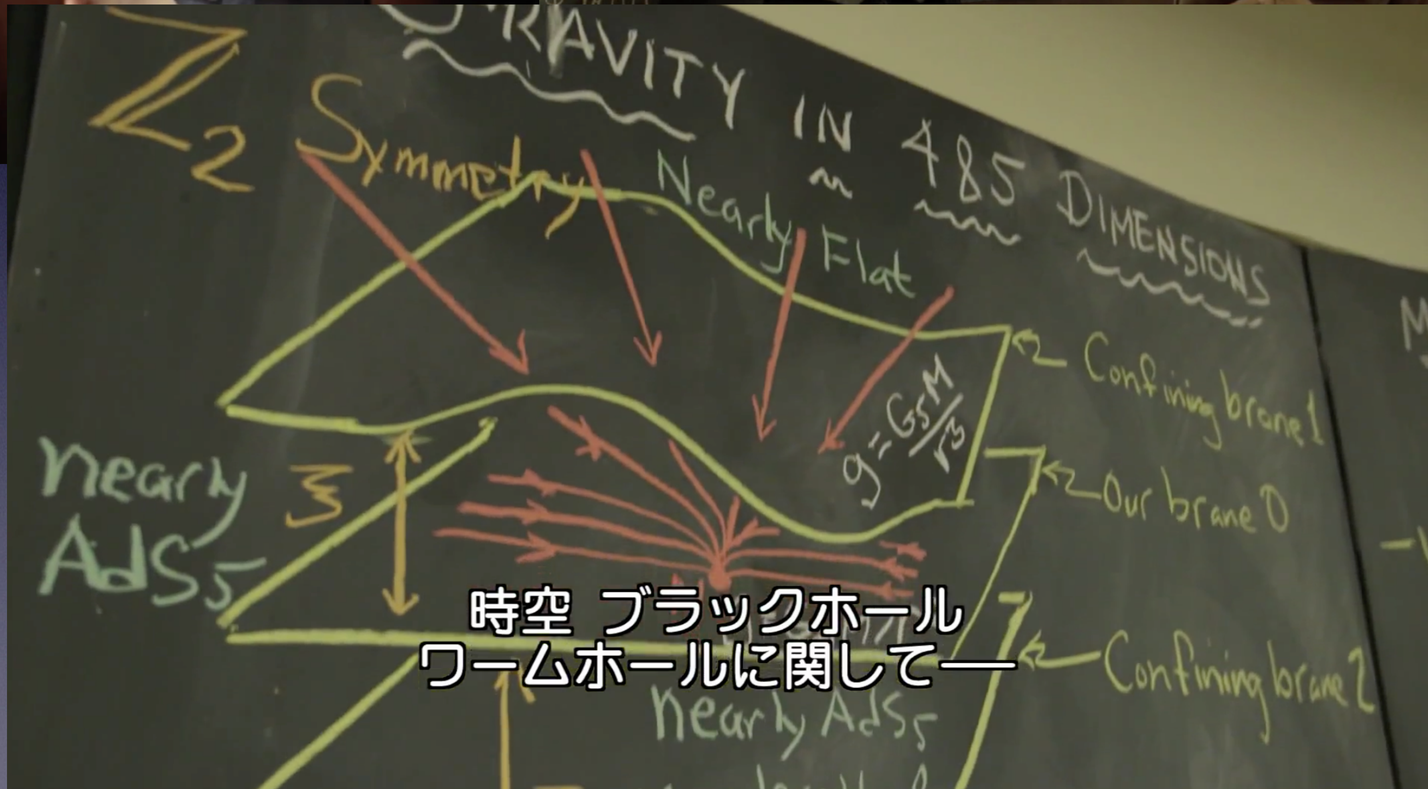


Interstellar (2014)

Executive Producer: Kip Thorne

https://www.youtube.com/watch?v=qZZ9jRan9eo

Why Wormhole?

They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.

They are very similar to black holes --both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

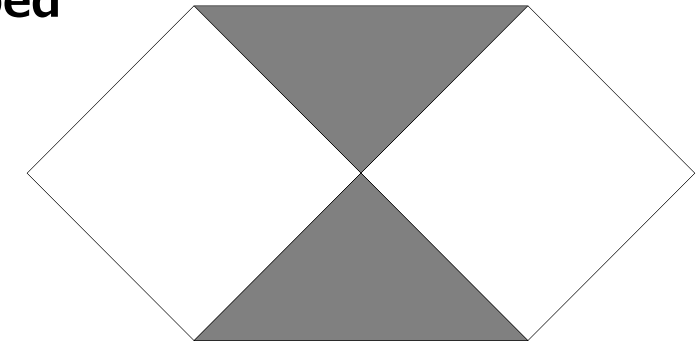
Wormhole = Hypersurface foliated by marginally trapped surfaces

BH and WH are interconvertible? New duality?

BH & WH are interconvertible?

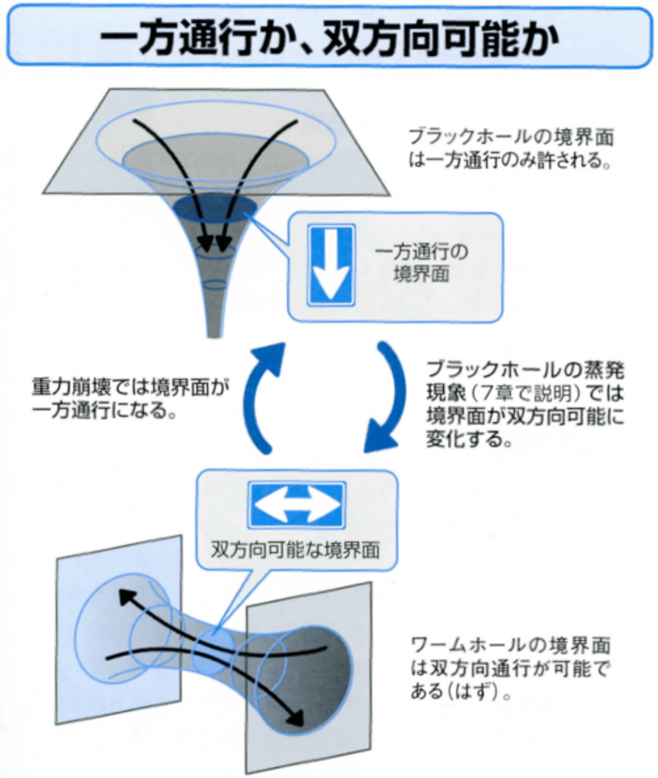
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)



Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.

	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appear-ance	occur naturally	Unlikely to occur naturally. but constructible??



Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\}+\mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated.)

- **new topic in numerical relativity.**
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496

- **much attentions in WH community**
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

Part I Wormhole dynamics in 4-dim GR

PHYSICAL REVIEW D **66**, 044005 (2002)

Fate of the first traversible wormhole: Black-hole collapse or inflationary expansion

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(Received 10 May 2002; published 16 August 2002)

Fate of Morris-Thorne (Ellis) wormhole?

- "Dynamical wormhole" defined by local trapping horizon
- spherically symmetric, both normal/ghost KG field
- apply dual-null formulation in order to seek horizons
- Numerical simulation

ghost/normal Klein-Gordon fields

$$T_{\mu\nu} = T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) = \underbrace{\left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right]}_{\text{normal}} + \underbrace{\left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right]}_{\text{ghost}}$$

$$\Box\psi = \frac{dV_1(\psi)}{d\psi}, \qquad \Box\phi = \frac{dV_2(\phi)}{d\phi}. \qquad \text{(Hereafter, we set } V_1(\psi) = 0, V_2(\phi) = 0)$$

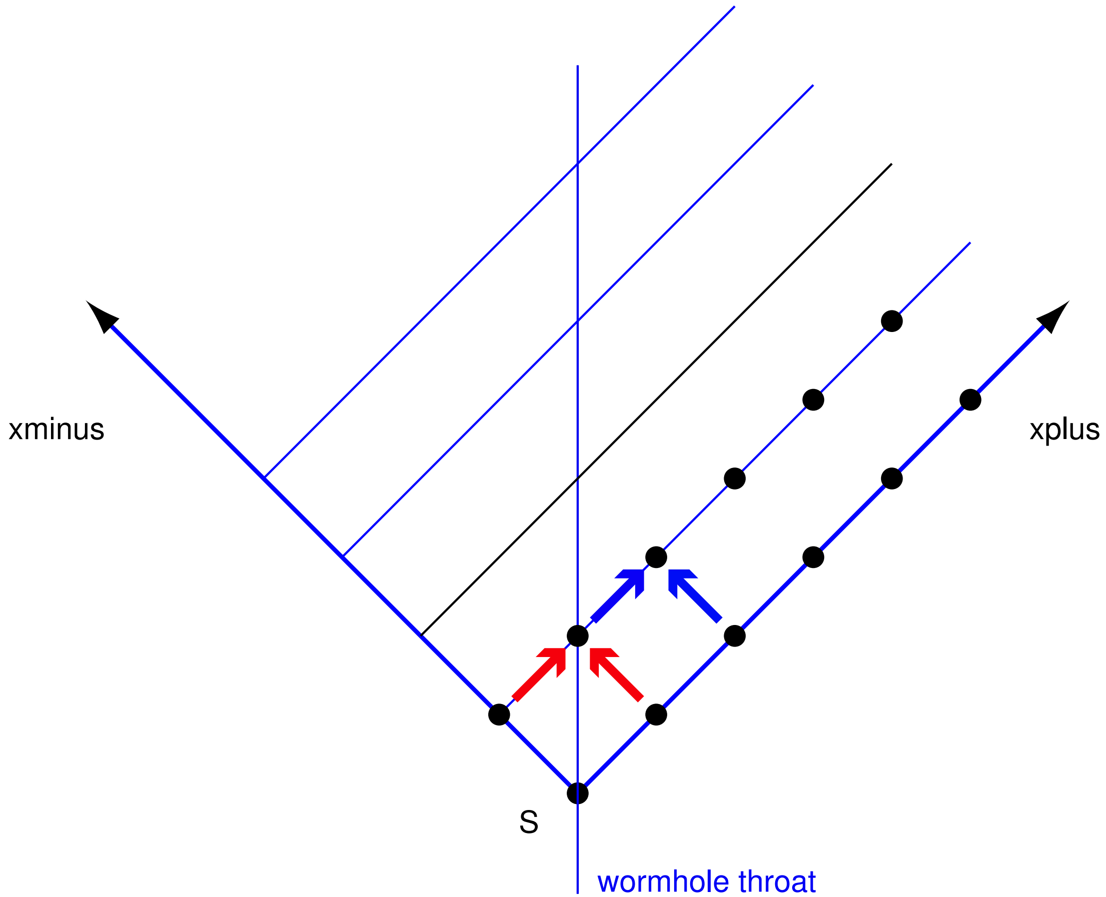
Initial data on $x^+ = 0$, $x^- = 0$ slices and on S

Generally, we have to set :

$$(\Omega, f, \vartheta_\pm, \phi, \psi) \quad \text{on } S\text{: } x^+ = x^- = 0$$

$$(\nu_\pm, \wp_\pm, \pi_\pm) \quad \text{on } \Sigma_\pm\text{: } x^\mp = 0,\ x^\pm \geq 0$$

Grid Structure for Numerical Evolution



dual-null formulation, spherically symmetric spacetime (4D)

- The spherically symmetric line-element:

$$ds^2 = -2e^{-f}dx^+dx^- + r^2 dS^2, \quad \text{where} \quad r = r(x^+, x^-), f = f(x^+, x^-), \cdots$$

- To obtain a system accurate near $\Im^{\pm}$, we introduce the conformal factor $\boxed{\Omega = 1/r}$. We also define first-order variables, the conformally rescaled momenta

expansions	$\vartheta_\pm = 2\partial_\pm r = -2\Omega^{-2}\partial_\pm\Omega \qquad (\theta_\pm = 2r^{-1}\partial_\pm r)$	(1)
inaffinities	$\nu_\pm = \partial_\pm f$	(2)
momenta of ϕ	$\wp_\pm = r\partial_\pm\phi = \Omega^{-1}\partial_\pm\phi$	(3)
momenta of ψ	$\pi_\pm = r\partial_\pm\psi = \Omega^{-1}\partial_\pm\psi$	(4)

The set of equations (remember the identity: $\partial_+\partial_- = \partial_-\partial_+$):

$$\partial_\pm\vartheta_\pm = -\nu_\pm\vartheta_\pm - 2\Omega\pi_\pm^2 + 2\Omega\wp_\pm^2, \tag{5}$$

$$\partial_\pm\vartheta_\mp = -\Omega(\vartheta_+\vartheta_-/2 + e^{-f}), \tag{6}$$

$$\partial_\pm\nu_\mp = -\Omega^2(\vartheta_+\vartheta_-/2 + e^{-f} - 2\pi_+\pi_- + 2\wp_+\wp_-), \tag{7}$$

$$\partial_\pm\wp_\mp = -\Omega\vartheta_\mp\wp_\pm/2, \tag{8}$$

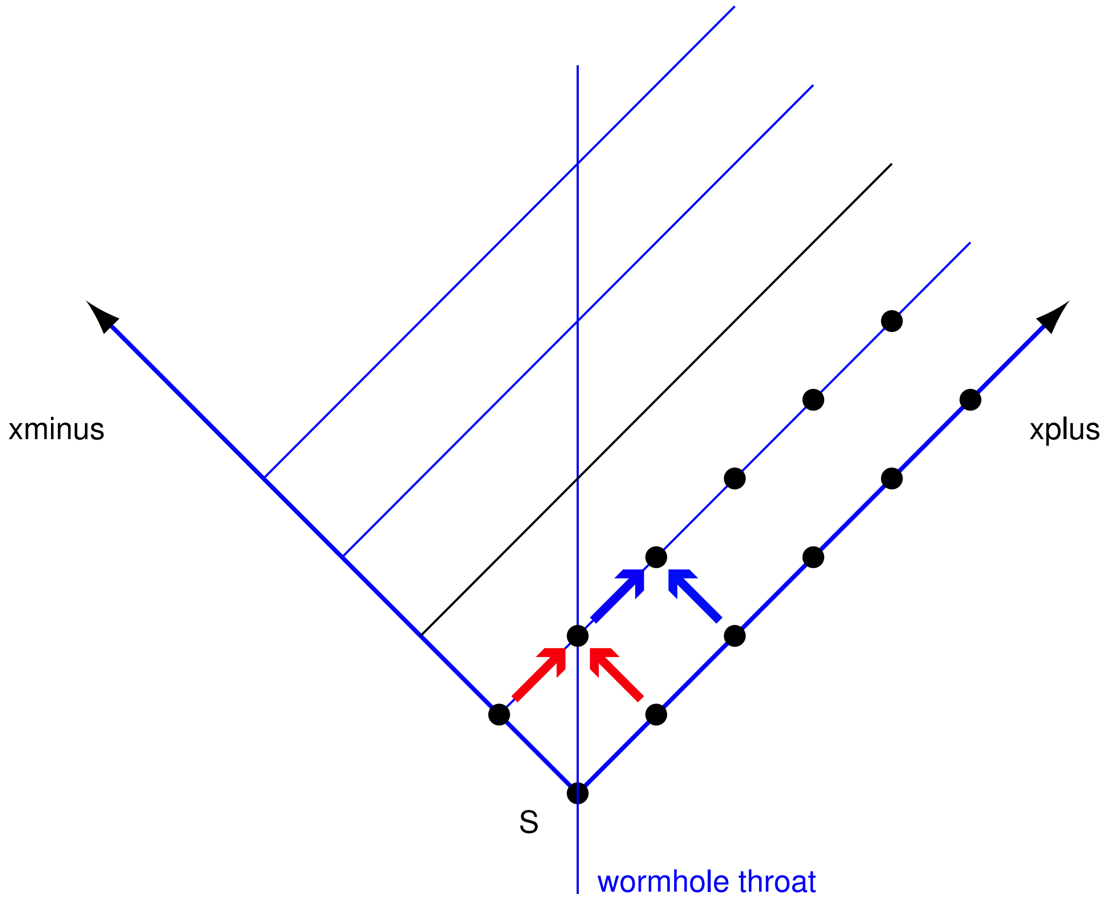
$$\partial_\pm\pi_\mp = -\Omega\vartheta_\mp\pi_\pm/2. \tag{9}$$

Initial data on $x^+ = 0$, $x^- = 0$ slices and on S

Generally, we have to set :

$$(\Omega, f, \vartheta_\pm, \phi, \psi) \quad \text{on } S\text{: } x^+ = x^- = 0$$
$$(\nu_\pm, \wp_\pm, \pi_\pm) \quad \text{on } \Sigma_\pm\text{: } x^\mp = 0,\ x^\pm \geq 0$$

Grid Structure for Numerical Evolution



Ghost pulse input -- Bifurcation of the horizons（4d)

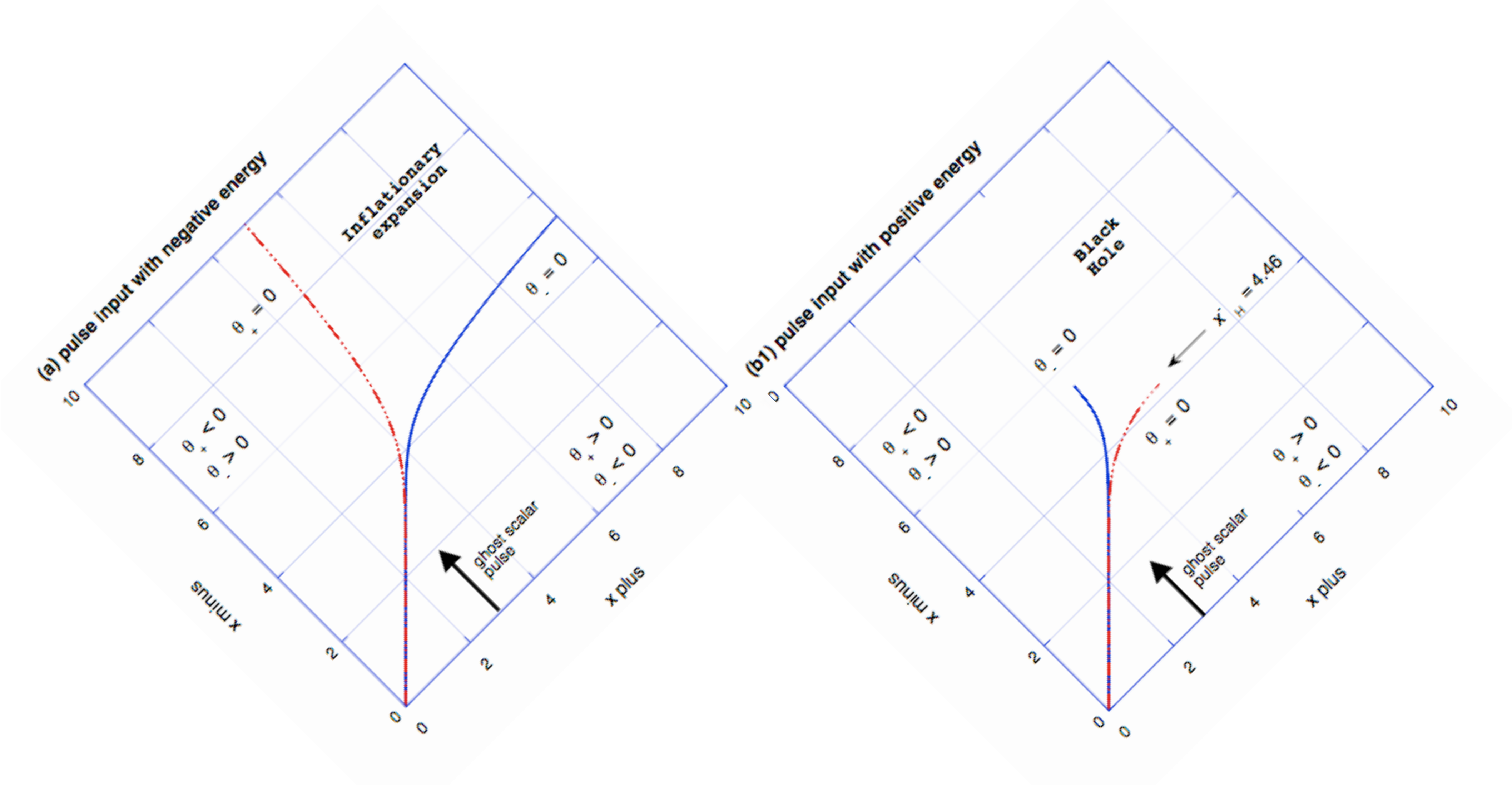
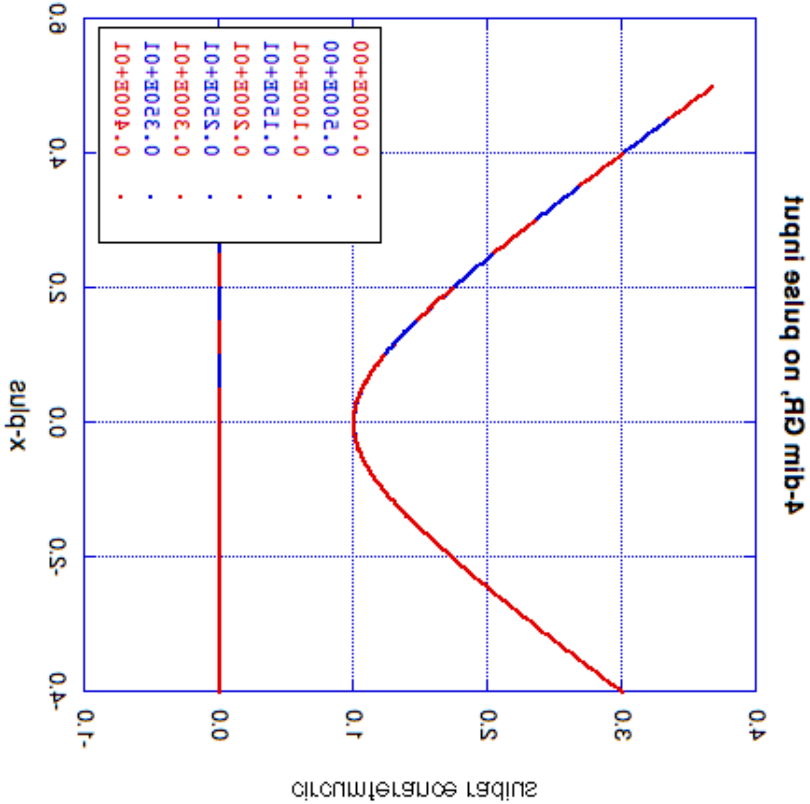


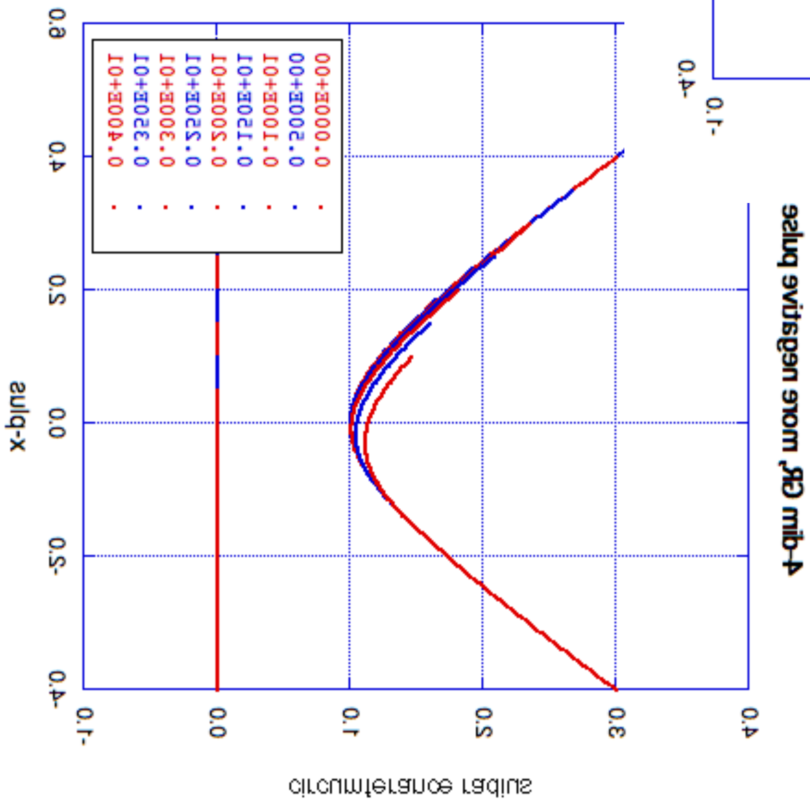
Figure 3: Horizon locations, $\vartheta_\pm = 0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

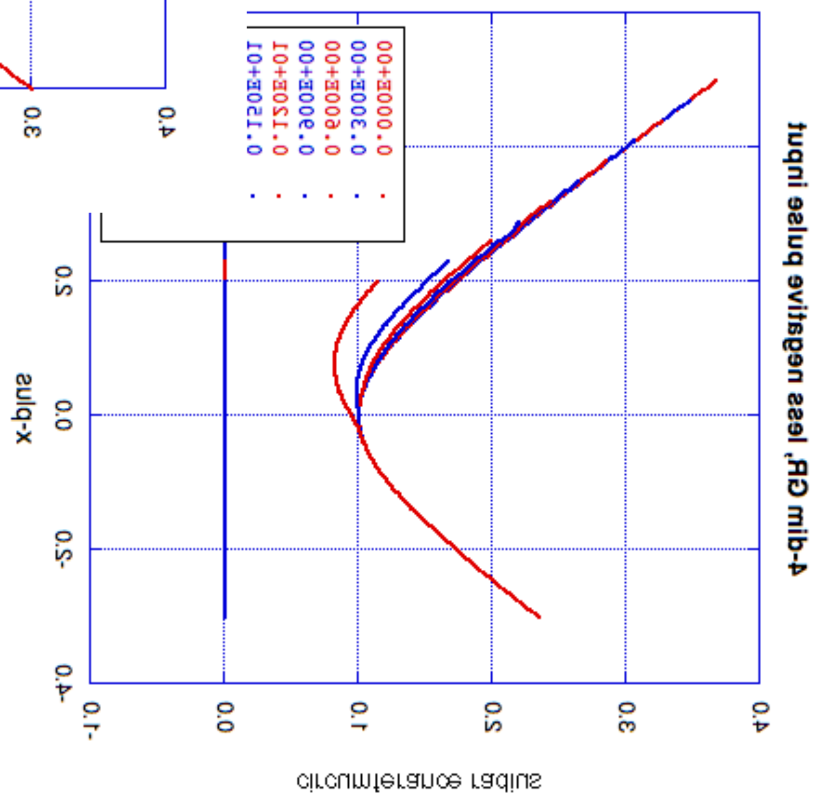
wormhole configurations (4dim. GR)



more negative field
-> throat expansion

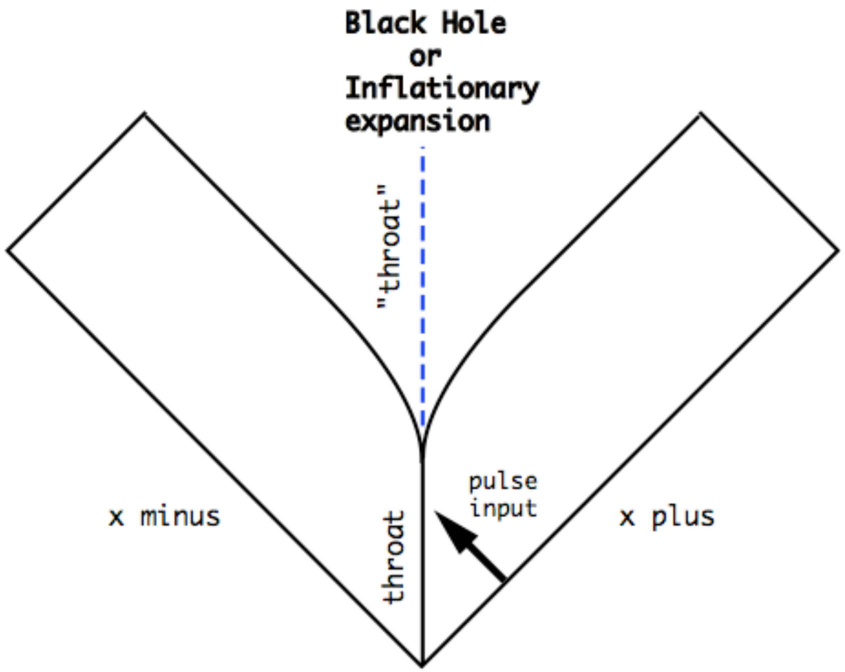
less negative field
-> throat shrink





Bifurcation of the horizons

-- go to a Black Hole or Inflationary expansion



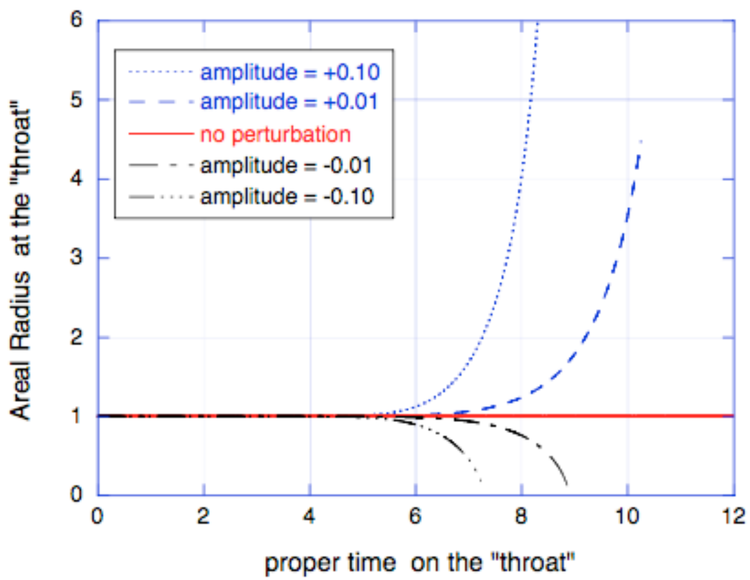


Figure 4: Partial Penrose diagram of the evolved space-time.
Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Normal pulse (a traveller) input -- Forming a Black Hole

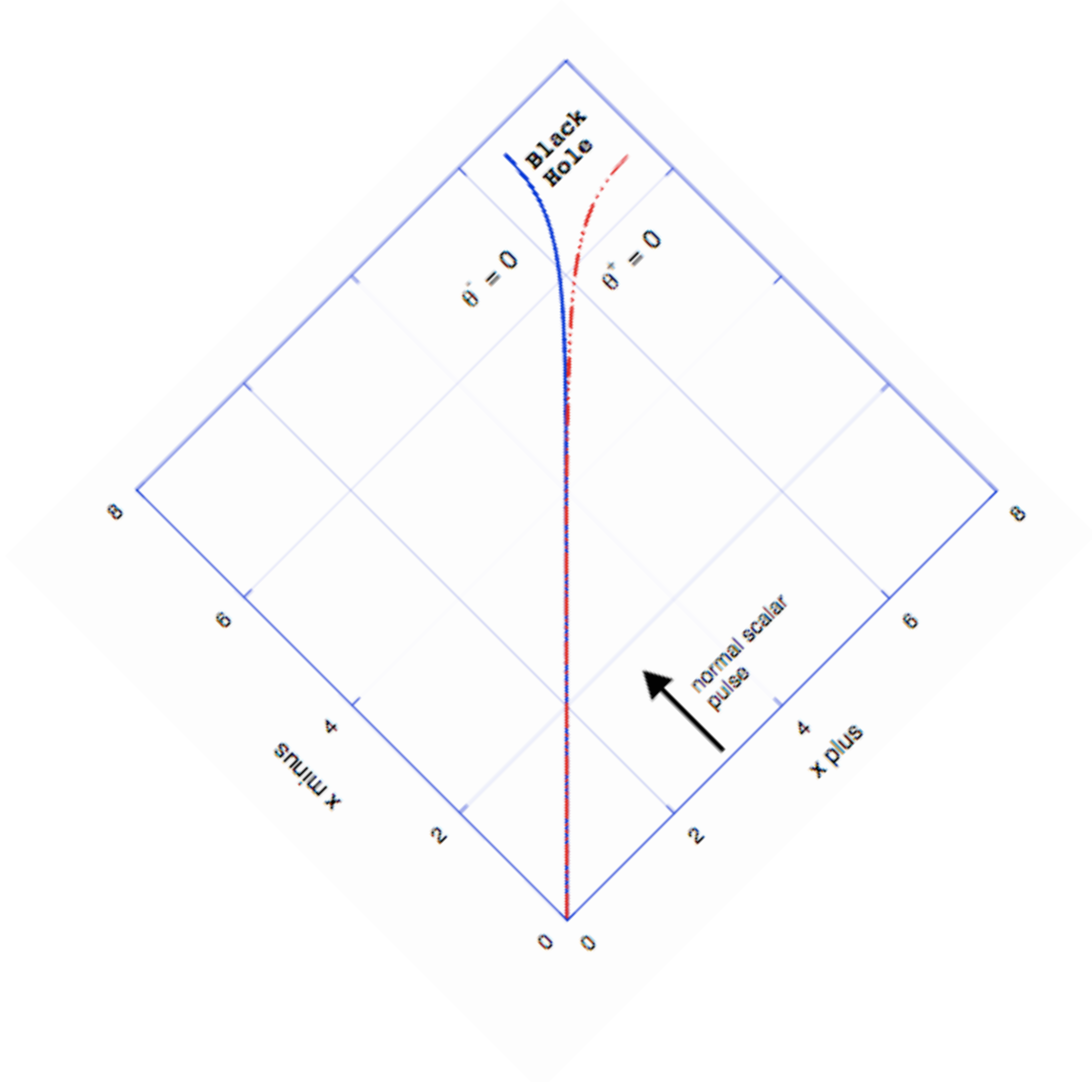
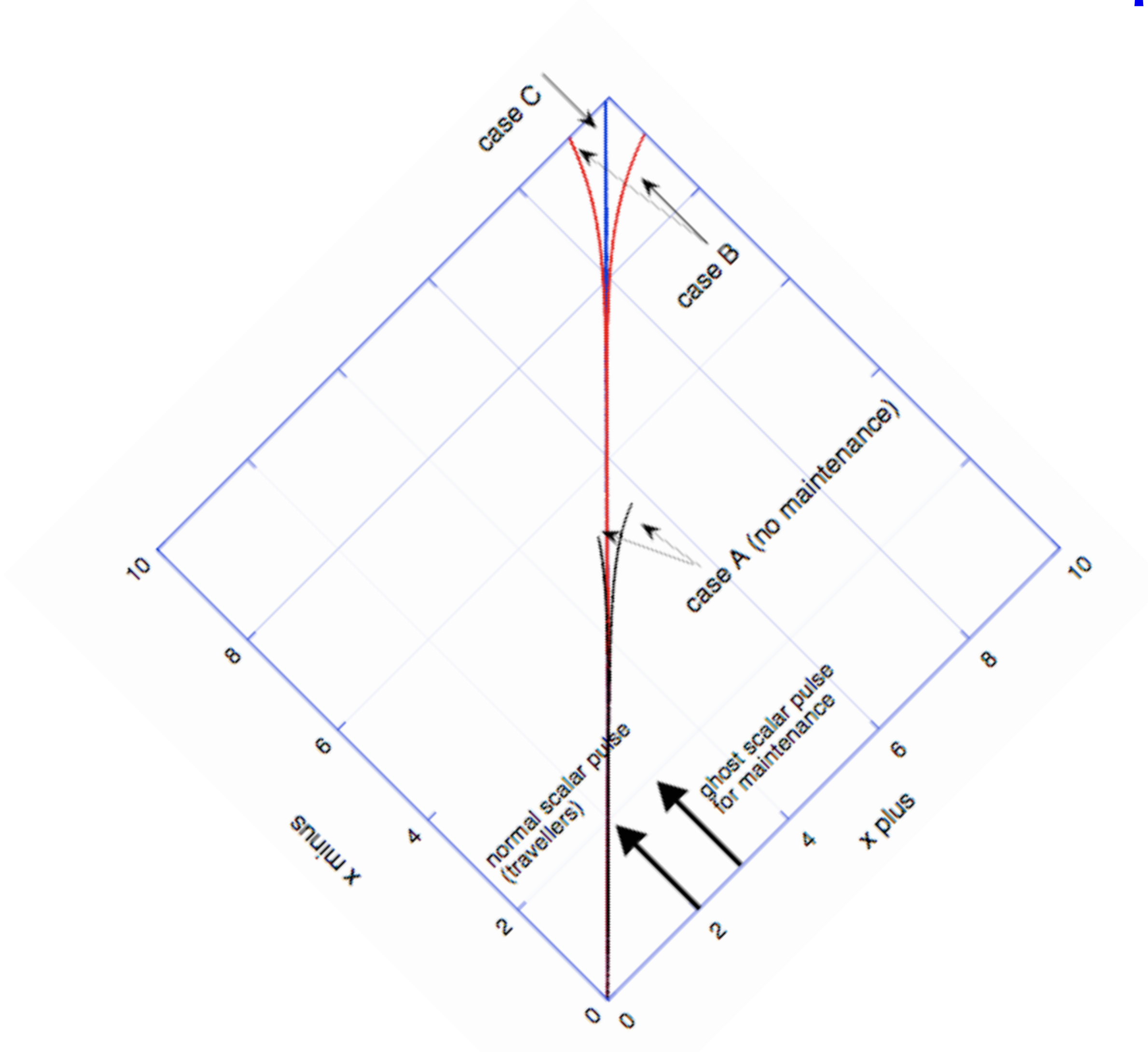


Figure 9: Evolution of a wormhole perturbed by a normal scalar field. Horizon locations: dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively.

Travel through a Wormhole
-- with Maintenance Operations!



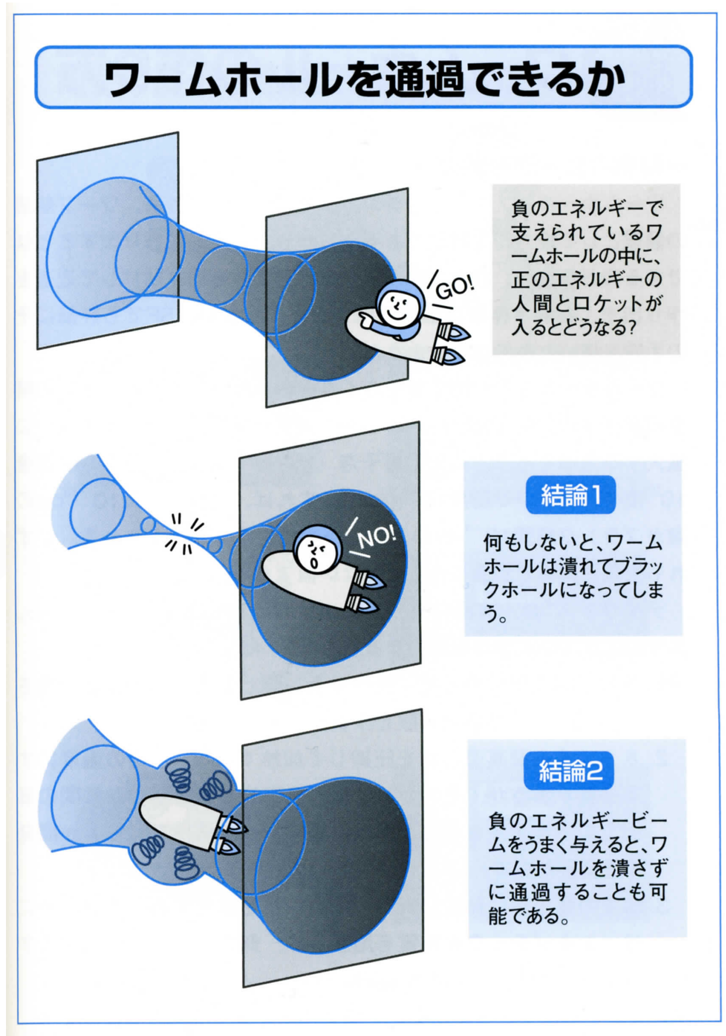


Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. The travellers pulse are commonly expressed with a normal scalar field pulse, $(\tilde{c}_a, \tilde{c}_b, \tilde{c}_c) = (+0.1, 6.0, 2.0)$. Horizon locations $\vartheta_+ = 0$ are plotted for three cases:
(A) no maintenance case (results in a black hole),
(B) with maintenance pulse of $(c_a, c_b, c_c) = (0.02390, 6.0, 3.0)$ (results in an inflationary expansion),
(C) with maintenance pulse of $(c_a, c_b, c_c) = (0.02385, 6.0, 3.0)$ (keep stationary structure upto the end of this range).

Summary of Part I

HS & Hayward, PRD66 (2002) 044005

Dynamics of Ellis (Morris-Thorne) traversible WH

WH is Unstable

(A) with positive energy pulse ---> BH

---> confirms duality conjecture between BH and WH.

(B) with negative energy pulse ---> Inflationary expansion

---> provides a mechanism for enlarging a quantum WH
to macroscopic size

(C) can be maintained by sophisticated operations

---> a round-trip is available for our hero/heroine

The basic behaviors has been confirmed by

A Doroshkevich, J Hansen, I Novikov, A Shatskiy, IJMPD 18 (2009) 1665
J A Gonzalez, F S Guzman & O Sarbach, CQG 26 (2009) 015010, 015011
J A Gonzalez, F S Guzman & O Sarbach, PRD80 (2009) 024023
O Sarbach & T Zannias, PRD 81 (2010) 047502

observation?

Abe, APJ 725 (2010) 787.

The Astrophysical Journal, 725:787–793, 2010 December 10 doi:10.1088/0004-637X/725/1/787

GRAVITATIONAL MICROLENSING BY THE ELLIS WORMHOLE

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Received 2010 February 21; accepted 2010 October 7; published 2010 November 19

ABSTRACT

A method to calculate light curves of the gravitational microlensing of the Ellis wormhole is derived in the weak-field limit. In this limit, lensing by the wormhole produces one image outside the Einstein ring and another image inside. The weak-field hypothesis is a good approximation in Galactic lensing if the throat radius is less than 10^{11} km. The light curves calculated have gutters of approximately 4% immediately outside the Einstein ring crossing times. The magnification of the Ellis wormhole lensing is generally less than that of Schwarzschild lensing. The optical depths and event rates are calculated for the Galactic bulge and Large Magellanic Cloud fields according to bound and unbound hypotheses. If the wormholes have throat radii between 100 and 10^7 km, are bound to the galaxy, and have a number density that is approximately that of ordinary stars, detection can be achieved by reanalyzing past data. If the wormholes are unbound, detection using past data is impossible.

Key words: gravitational lensing: micro

Online-only material: color figures

(1) Exact Solution : Basic eqns.

Torii & HS, PRD88 (2013) 064027

- general relativity, *n-dimension*

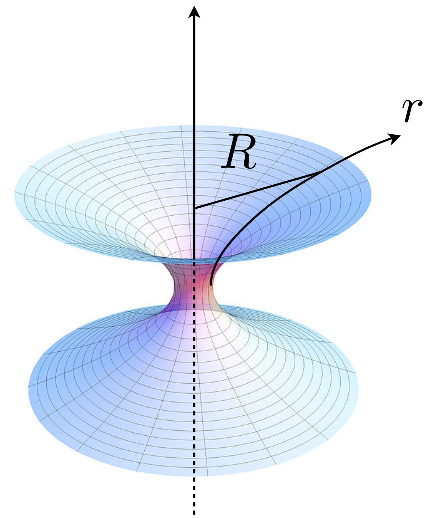
massless scalar field (ghost)

$$S = \int d^n x \sqrt{-g} \left[\frac{1}{2\kappa_n^2} R - \frac{1}{2}\epsilon (\partial\phi)^2 - \cancel{V(\phi)} \right], \qquad \epsilon = -1$$

- static, spherical sym., asymptotically flat

$$ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 h_{ij}dx^i dx^j$$

$(k = 1)$



- Basic equations

$$(t,t): \quad -\frac{n-2}{2} f^2 \left[\frac{2R''}{R} + \frac{f'R'}{fR} + \frac{(n-3)R'^2}{R^2} \right] + \frac{(n-2)(n-3)kf}{2R^2} = \kappa_n^2 f \left[\frac{1}{2}\epsilon f \phi'^2 + \cancel{V(\phi)} \right],$$

$$(r,r): \quad \frac{n-2}{2}\frac{R'}{R} \left[\frac{f'}{f} + \frac{(n-3)R'}{R} \right] - \frac{(n-2)(n-3)k}{2fR^2} = \frac{\kappa_n^2}{f} \left[\frac{1}{2}\epsilon f \phi'^2 - \cancel{V(\phi)} \right],$$

$$(i,j): \quad \frac{f''}{2} + (n-3)f \left(\frac{R''}{R} + \frac{f'R'}{fR} + \frac{n-4}{2}\frac{R'^2}{R^2} \right) - \frac{(n-3)(n-4)k}{2R^2} = \kappa_n^2 \left[\frac{1}{2}\epsilon f \phi'^2 + \cancel{V(\phi)} \right],$$

$$(KG): \quad \frac{1}{R^{n-2}} \left(R^{n-2} f \phi' \right)' = -\epsilon \cancel{\frac{dV}{d\phi}}. \quad \Rightarrow \quad \phi' = \frac{C}{fR^{n-2}}$$

constant

Solution

$$ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 h_{ij}dx^i dx^j$$

- regularity at the throat ($r = 0$)

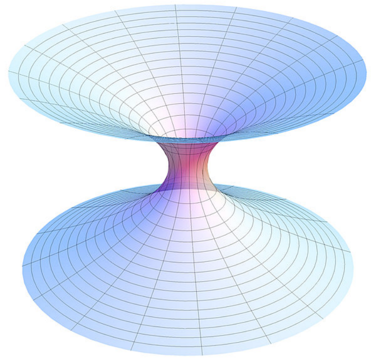
$R = a$ ← throat radius

$R' = 0, \quad f = f_0, \quad f' = 0, \quad \phi = 0$

★ from the scaling rule

$a = 1 \qquad f_0 = 1$

Basics eqns. → $\kappa_n^2 C^2 = (n-2)(n-3)a^{2(n-3)}$



- Exact solution

$$f \equiv 1$$

$$r(R) = -mB_z\left[-m, \frac{1}{2}\right] - \frac{\sqrt{\pi}\Gamma[1-m]}{\Gamma[m(n-4)]}$$

$$\phi = \frac{\sqrt{(n-2)(n-3)}}{\kappa_n} a^{n-3} \int \frac{1}{R(r)^{n-2}} dr$$

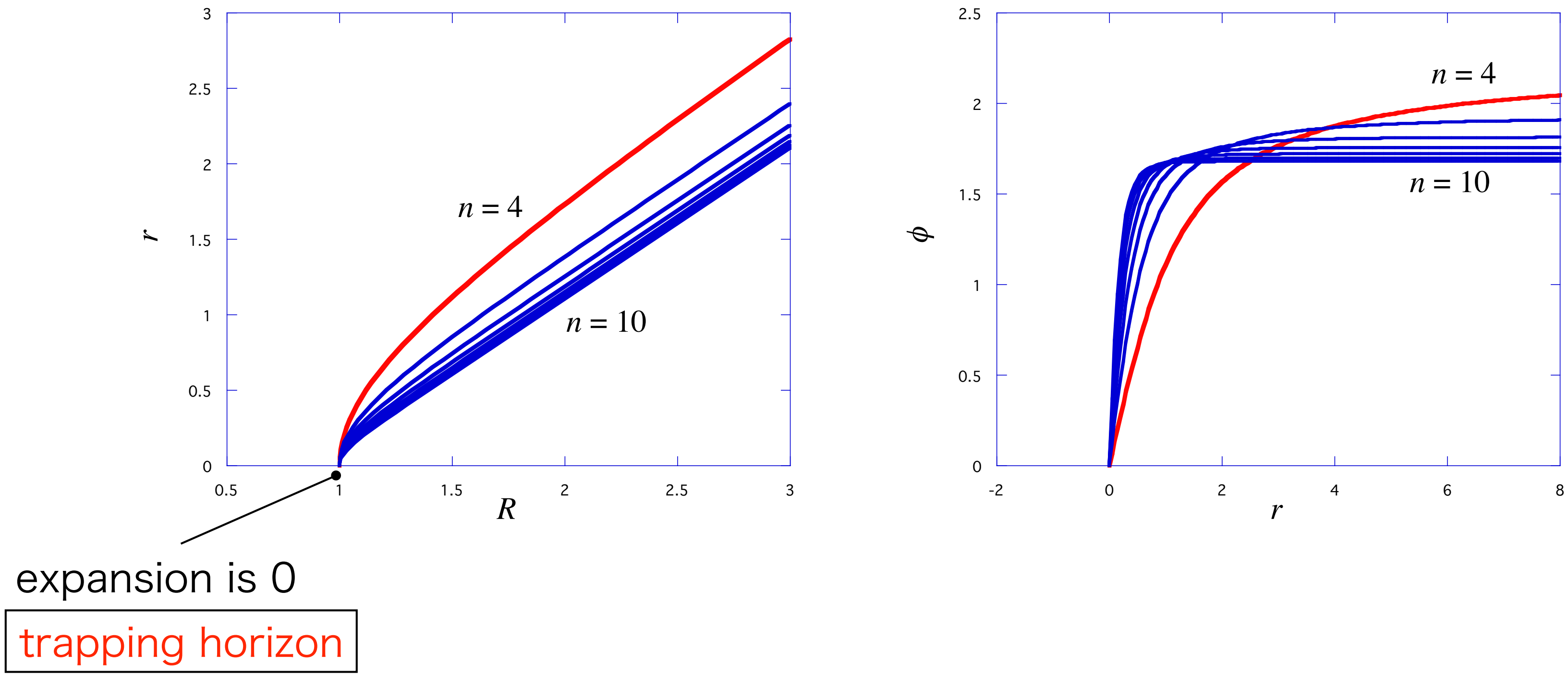
$$m = \frac{1}{2(n-3)}, \quad z = R^m \quad B_z(p,q) := \int_0^z t^{p-1}(1-t)^{q-1}dt$$

Incomplete Beta func.

★ in another metric form: V. Dzhunushaliev+, 2013

Configurations

- configurations $ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 h_{ij} dx^i dx^j$



★ large curvature near the throat.

★ scalar field goes steep if n is large.

★ In the $n \to \infty$ limit

$$R = r + 1 \qquad \phi = 0 \ \ (r = 0) \quad \frac{\pi}{2} \ \ (r > 0)$$

(2) Linear Stability: Master eqn.

Torii & HS, PRD88 (2013) 064027

- metric

$$ds_n^2 = -f(t,r)e^{-2\delta(t,r)}dt^2 + f(t,r)^{-1}dr^2 + R(t,r)^2 h_{ij}dx^i dx^j$$

- linear perturbation

static solution

$$f = f_0(r) + f_1(r)e^{i\omega t}, \quad R = R_0(r) + R_1(r)e^{i\omega t},$$
$$\delta = \delta_0(r) + \delta_1(r)e^{i\omega t}, \quad \phi = \phi_0(r) + \phi_1(r)e^{i\omega t}.$$

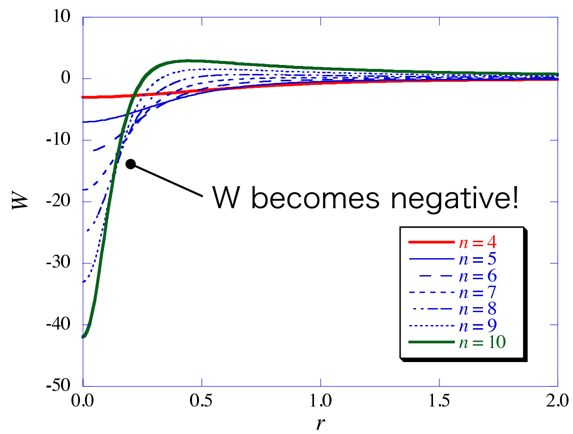
- master equation

$$-\Psi_1'' + W(r)\Psi_1 = \omega^2\Psi_1,$$

$$W(r) = -\frac{1}{4R_0^2}\left[\frac{3(n-2)^2}{R_0^{2(n-3)}} - (n-4)(n-6)\right].$$

$$\Psi_1 = \mathcal{D}_+\psi_1 \quad \mathcal{D}_+ = \frac{d}{dr} - \frac{\bar{\psi}_1'}{\bar{\psi}_1} \quad \psi_1 = R_0^{\frac{n-2}{2}}\left(\phi_1 - \frac{\phi_0'}{R_0'}R_1\right),$$

★ Ψ_1: Gauge invariant in spherical sym.



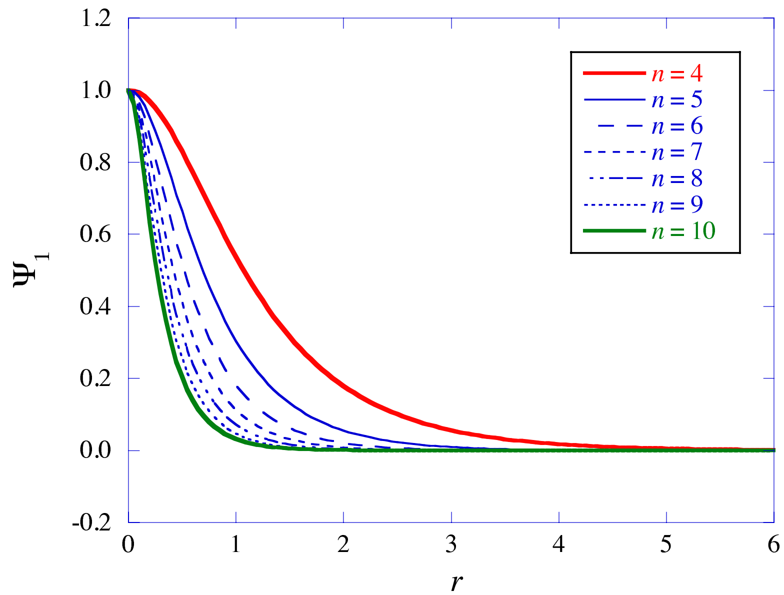
★ potential W

Unstable!

- exist negative mode

n	ω^2
4	-1.39705243371511
5	-2.98495893027790
6	-4.68662054299460
7	-6.46258414126318
8	-8.28975936306259
9	-10.1535530451867
10	-12.0442650147438
11	-13.9552091676647
20	-31.5751101285105
50	-91.3457759137153
100	-191.283017729717

eigenvalues of negative mode



eigenfunction of negative mode

★ In all dimensions, we found negative modes.

➡ Ellis's wormhole is unstable

★ Higher dimension, instability appears in short time scale

(3) Numerical Evolution

HS & Torii, in preparation

n-dim., Spherical Symmetry, Dual-null coordinate

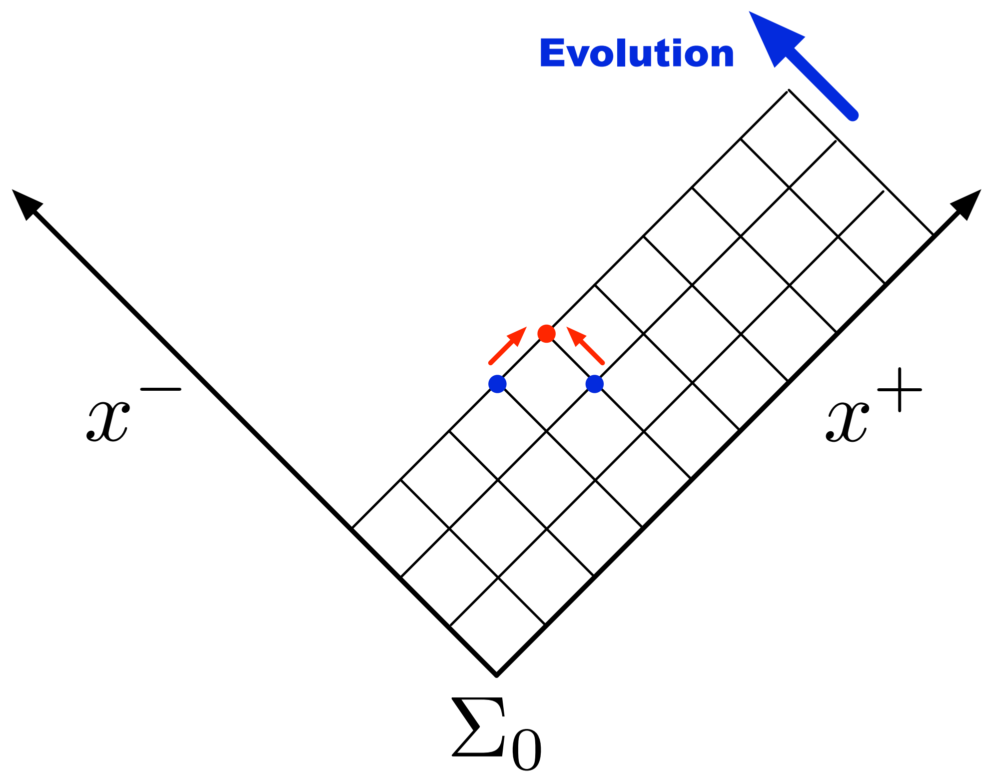
$$ds^2 = -2e^{-f(x^+,x^-)}dx^+\,dx^- + r^2(x^+,x^-)d\Omega_3^{(n-2)}$$

Space-time Variables

$$\Omega = \frac{1}{r}$$
$$\vartheta_\pm \equiv (n-2)\partial_\pm r$$
$$\nu_\pm \equiv \partial_\pm f$$

We also define η as

$$Z \equiv k + \frac{2e^f}{(n-2)^2}\vartheta_+\vartheta_- \equiv k + W$$



Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated.)

- **new topic in numerical relativity.**
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496

- **much attentions in WH community**
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

matter variables

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) \\ &= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \end{aligned}$$

this derives Klein-Gordon equations

$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}.$$

Scalar field variables

$$\begin{aligned} \pi_\pm &\equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi \\ p_\pm &\equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi \end{aligned}$$

Klein-Gordon eqs.

$$\begin{aligned} \Box\phi &= -\frac{e^f}{r}\left(2r\phi_{uv} + (n-2)r_u\phi_v + (n-2)r_v\phi_u\right) \\ &= -2e^f\phi_{uv} - e^f\Omega^2\left(\vartheta_- p_+ + \vartheta_+ p_-\right) \end{aligned}$$

Energy-momentum tensor

$$\begin{aligned} T_{++} &= \Omega^2(\pi_+^2 - p_+^2) \\ T_{--} &= \Omega^2(\pi_-^2 - p_-^2) \\ T_{+-} &= -e^{-f}\left(V_1(\psi) + V_2(\phi)\right) \\ T_{zz} &= e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}\left(V_1(\psi) - V_2(\phi)\right) \end{aligned}$$

evolution equations (1)

x^+-direction

Let $\tilde{\alpha} = (n-3)(n-4)\alpha_2$, $\tilde{\Lambda} = \dfrac{2\Lambda}{(n-1)(n-2)}$, and $A = \alpha_1 + 2\tilde{\alpha}\Omega^2(k+W)$.

$$
\begin{aligned}
\partial_+\Omega &= -\frac{1}{n-2}\vartheta_+\Omega^2 && (1)\\
\partial_+\vartheta_+ &= -\vartheta_+\nu_+ - \frac{1}{A}\kappa^2\Omega(\pi_+^2 - p_+^2) && (2)\\
\partial_+\vartheta_- &= \frac{1}{A}\frac{e^{-f}}{\Omega}\left\{-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}(k+W) + \Lambda + \kappa^2(V_1+V_2)\right\} - \frac{\tilde{\alpha}}{A}\Omega^3 e^{-f}\frac{(n-2)(n-5)}{2}\left\{(k+W)^2 + W\right\} && (3)\\
\partial_+ f &= \nu_+ && (4)\\
\partial_+\nu_+ &= \text{no equation}\\
\partial_+\nu_- &= \frac{\alpha_1}{A}Ze^{-f}\Omega^2\frac{(n-3)}{2}\left\{-\frac{\alpha_1}{A}2(n-3) + n - 4\right\}\\
&\quad + \frac{1}{A}\Omega^2 e^{-f}\kappa^2(\pi_+\pi_- - p_+p_-) + \frac{1}{A}e^{-f}\left\{\frac{\alpha_1}{A}\frac{2(n-3)}{(n-2)} - 1\right\}\{\Lambda + \kappa^2(V_1+V_2)\}\\
&\quad - \frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\frac{\alpha_1}{A}\Omega^2(n-3)\left\{k^2 + 2WZ + 2Z^2\right\} - \frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\frac{\tilde{\alpha}}{A}\Omega^4 2(n-5)\left\{k^2 + 2WZ\right\}Z\\
&\quad + \frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\Omega^2\frac{1}{2}\left\{(n-2)k^2 + 2WZ - 4Z^2\right\} + \frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\frac{1}{A}\frac{4}{n-2}Z\left\{\Lambda + \kappa^2(V_1+V_2)\right\}\\
&\quad - \frac{\tilde{\alpha}}{A}e^{f}\Omega^2\frac{4}{(n-2)^2}\left\{\nu_+\vartheta_+(\partial_-\vartheta_-) + \nu_-\vartheta_-(\partial_+\vartheta_+) + (\partial_+\vartheta_+)(\partial_-\vartheta_-) + \nu_+\nu_-\vartheta_+\vartheta_- - (\partial_-\vartheta_+)^2\right\} && (5)\\
\partial_+\psi &= \Omega\pi_+ && (6)\\
\partial_+\phi &= \Omega p_+ && (7)\\
\partial_+\pi_+ &= \text{no equation}\\
\partial_+\pi_- &= \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} && (8)\\
\partial_+ p_+ &= \text{no equation}\\
\partial_+ p_- &= \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_+ p_- - \frac{1}{2}\Omega\vartheta_- p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} && (9)
\end{aligned}
$$

evolution equations (2)

x^--direction

$$\begin{aligned}
\partial_- \Omega &= -\frac{1}{n-2}\vartheta_- \Omega^2 && (10)\\
\partial_- \vartheta_+ &= \partial_+ \vartheta_- && (11)\\
\partial_- \vartheta_- &= -\vartheta_- \nu_- - \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2) && (12)\\
\partial_- f &= \nu_- && (13)\\
\partial_- \nu_+ &= \partial_+ \nu_- && (14)\\
\partial_- \nu_- &= \text{no equation} &&\\
\partial_- \psi &= \Omega \pi_- && (15)\\
\partial_- \phi &= \Omega p_- && (16)\\
\partial_- \pi_+ &= -\frac{1}{2}\Omega\vartheta_+\pi_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} && (17)\\
\partial_- \pi_- &= \text{no equation} &&\\
\partial_- p_+ &= -\frac{1}{2}\Omega\vartheta_+ p_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_- p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} && (18)\\
\partial_- p_- &= \text{no equation} &&
\end{aligned}$$

This constitutes the first-order dual-null form, suitable for numerical coding.

initial data

- Static condition

$$(\partial_+ + \partial_-)\Omega = 0 \implies \vartheta_+ + \vartheta_- = 0$$
$$(\partial_+ + \partial_-)\psi = 0 \implies \pi_+ + \pi_- = 0$$
$$(\partial_+ + \partial_-)\phi = 0 \implies p_+ + p_- = 0$$
$$\left.\begin{array}{l}(\partial_+ + \partial_-)\vartheta_+ = 0\\ (\partial_+ + \partial_-)\vartheta_- = 0\end{array}\right\} \implies \vartheta_+\nu_+ + \frac{1}{A}\Omega\kappa^2(\pi_+^2 - p_+^2) = \vartheta_-\nu_- + \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2)$$

- Solve x^+ and x^- equations with the starting condition at the throat

$$\vartheta_+ = \vartheta_-(=0)$$
$$\nu_+ = \nu_-(=0)$$
$$-\kappa^2\Omega(\pi_+^2 - p_+^2)e^f = -\frac{1}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}k + \Lambda + \kappa^2(V_1 + V_2)\right] + \tilde{\alpha}\Omega^3\frac{(n-2)(n-5)}{2}k^2$$

If we assume only ghost field ϕ, then

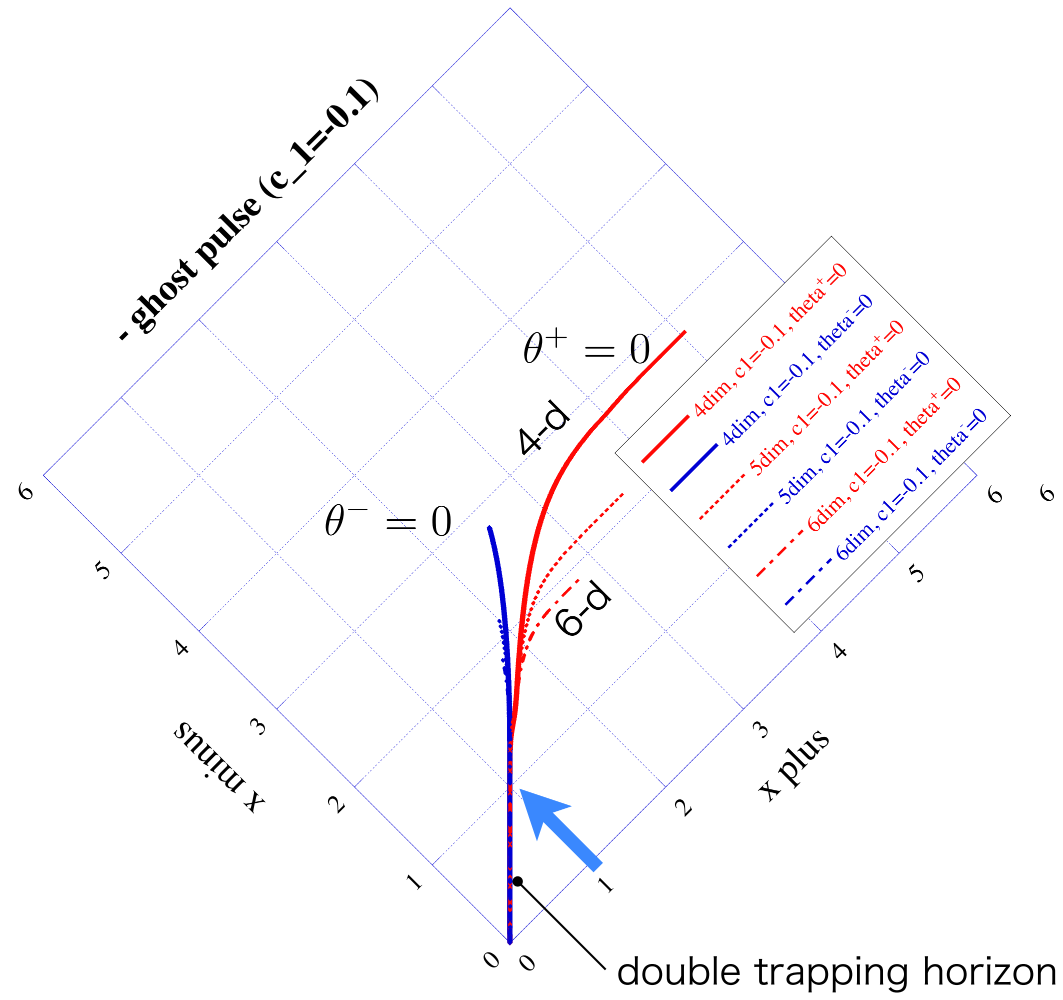
$$p_+ = -p_- = \sqrt{\frac{1}{\kappa^2 e^f}\left[\alpha_1\frac{(n-2)(n-3)}{2}k - \frac{1}{\Omega^2}(\Lambda + \kappa^2 V_2) + \tilde{\alpha}\Omega^2\frac{(n-2)(n-5)}{2}k^2\right]}$$

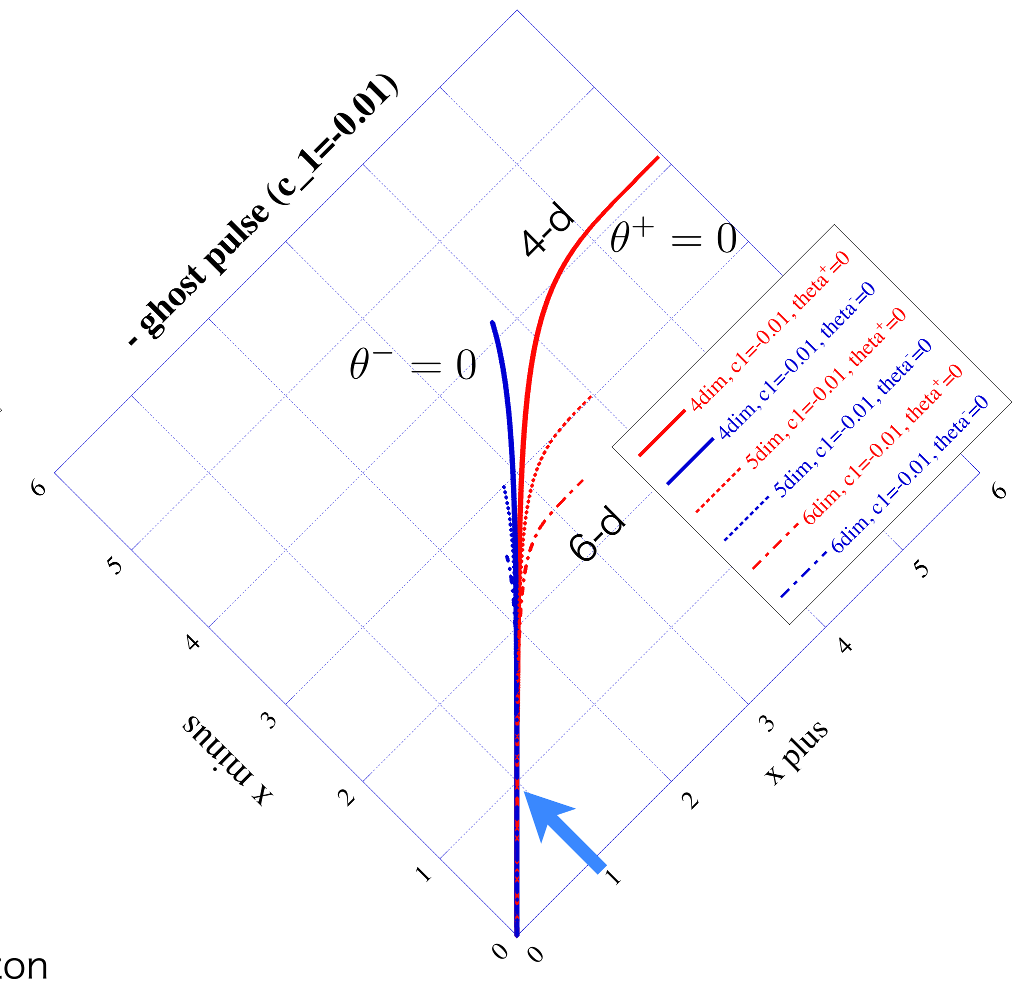
- add perturbation

$$p_+(x^+ = x, x^- = 0) = p_+(\text{solution}) + a\exp[-100(x-0.5)^2]$$

4d 5d 6d GR

ghost pulse (negative amp.) input

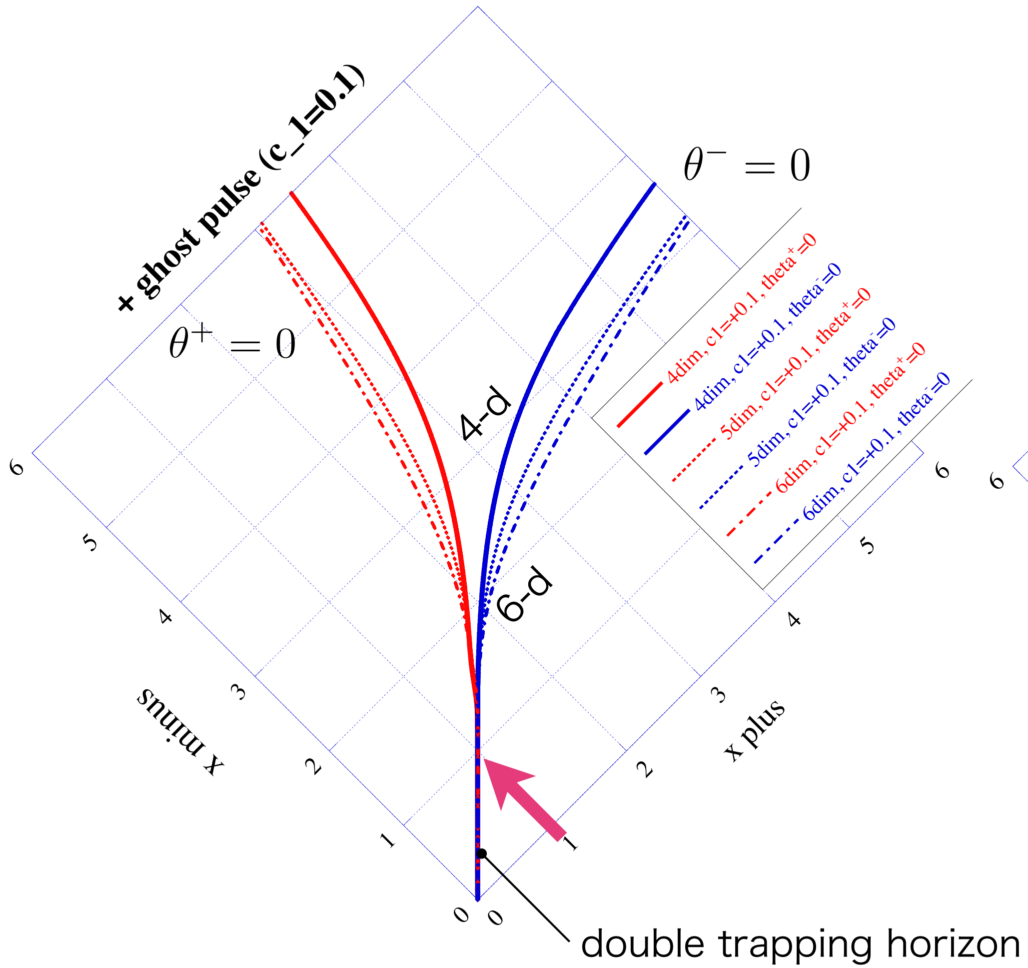


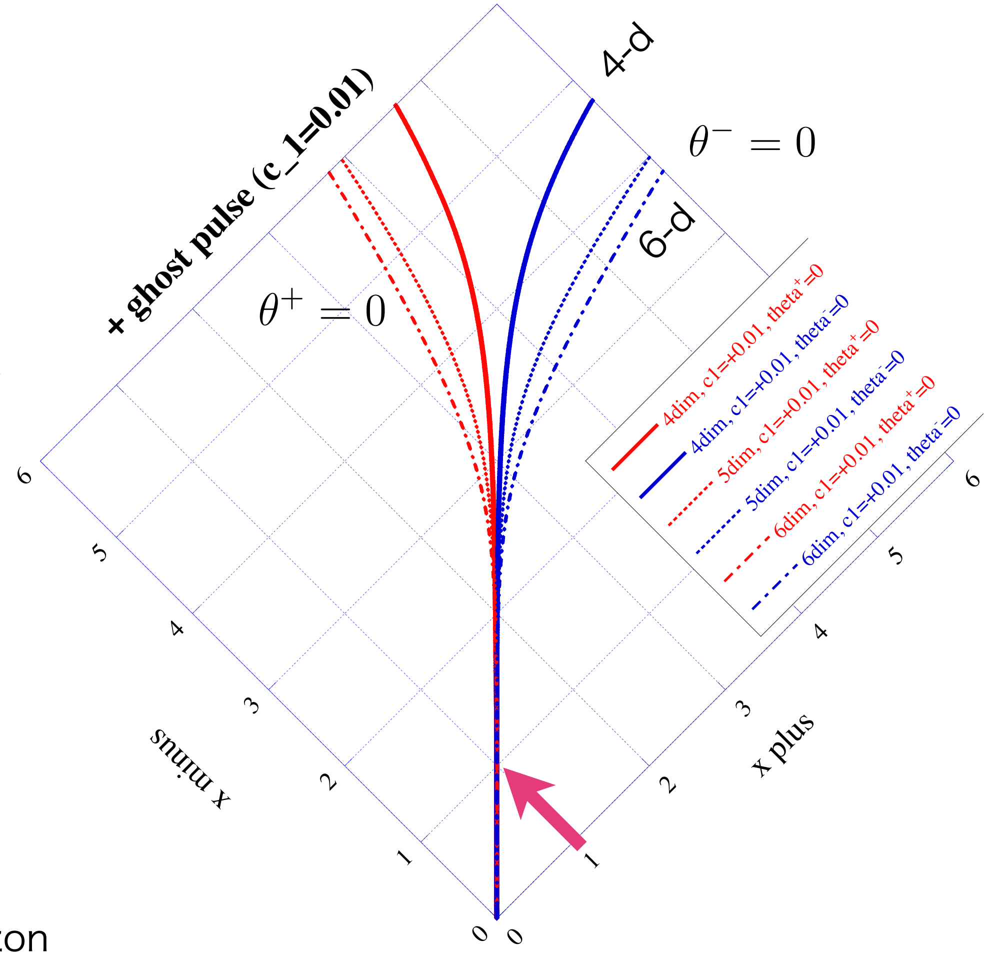


positive energy input --> BH formation

4d 5d 6d GR

ghost pulse (positive amp.) input





negative energy input --> throat inflates

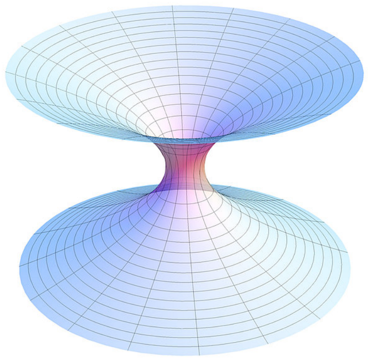
existence of the minimum mass of BH

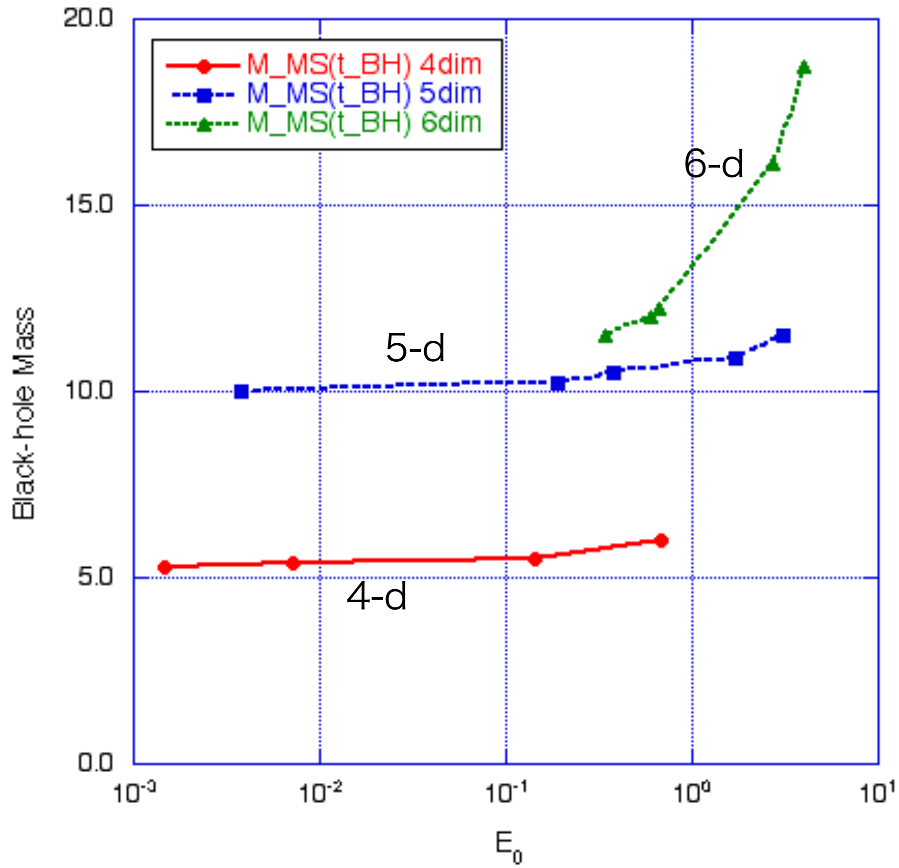
- BH mass (Misner-Sharp mass)

$$E_n = \frac{(n-2)A_{n-2}}{2\kappa_n^2}\Omega\left[-\frac{1}{\Omega^2}\tilde{\Lambda} + \left(k + \frac{2}{(n-2)^2}e^f\vartheta_+\vartheta_-\right)\right]$$

(Maeda & Nozawa, 2008)

→ existence of minimum mass





Dynamics in Gauss-Bonnet gravity?

- Action

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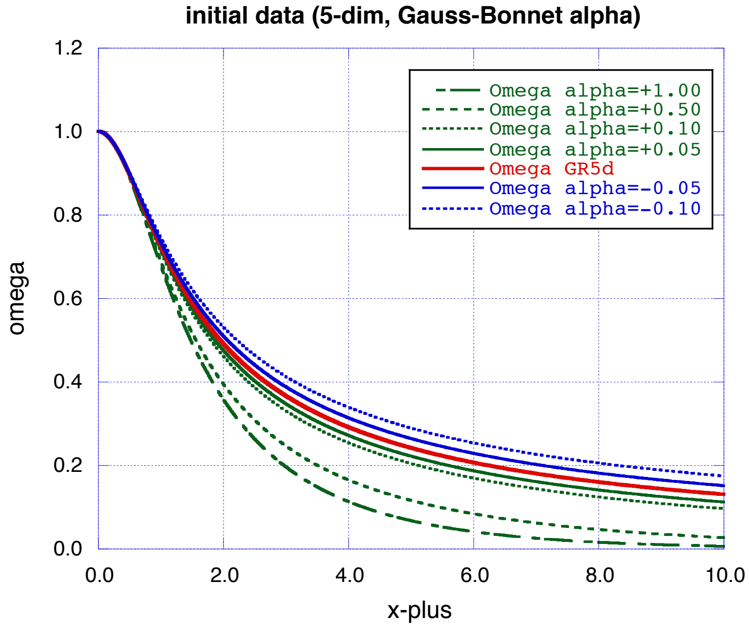
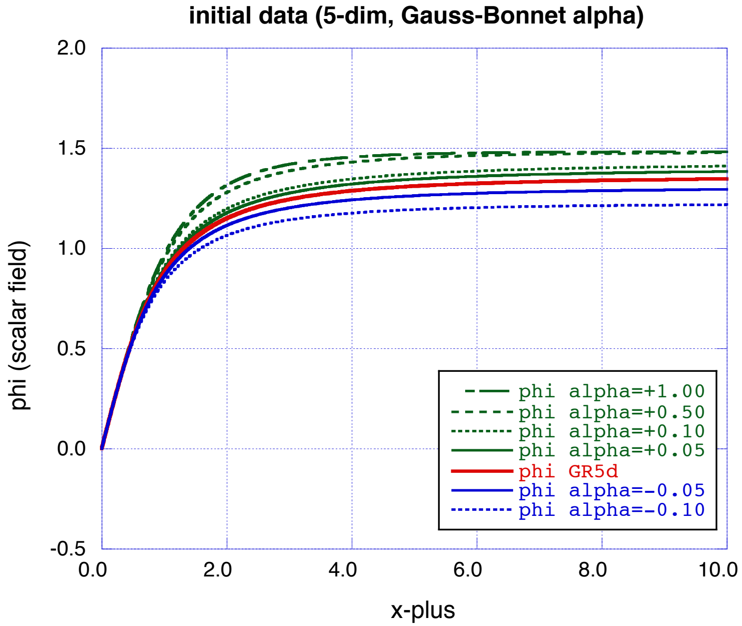
Initial Data





conformal factor

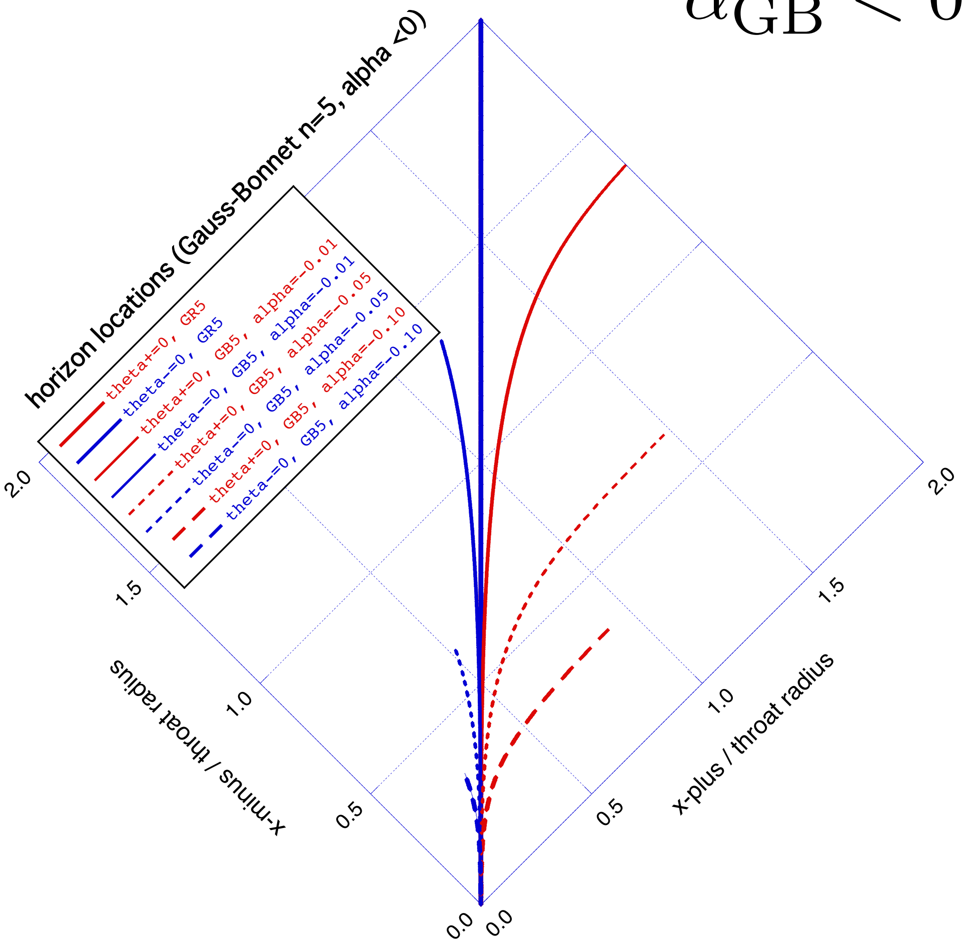
scalar field

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\}+\mathcal{L}_{\rm matter}\Big]$$

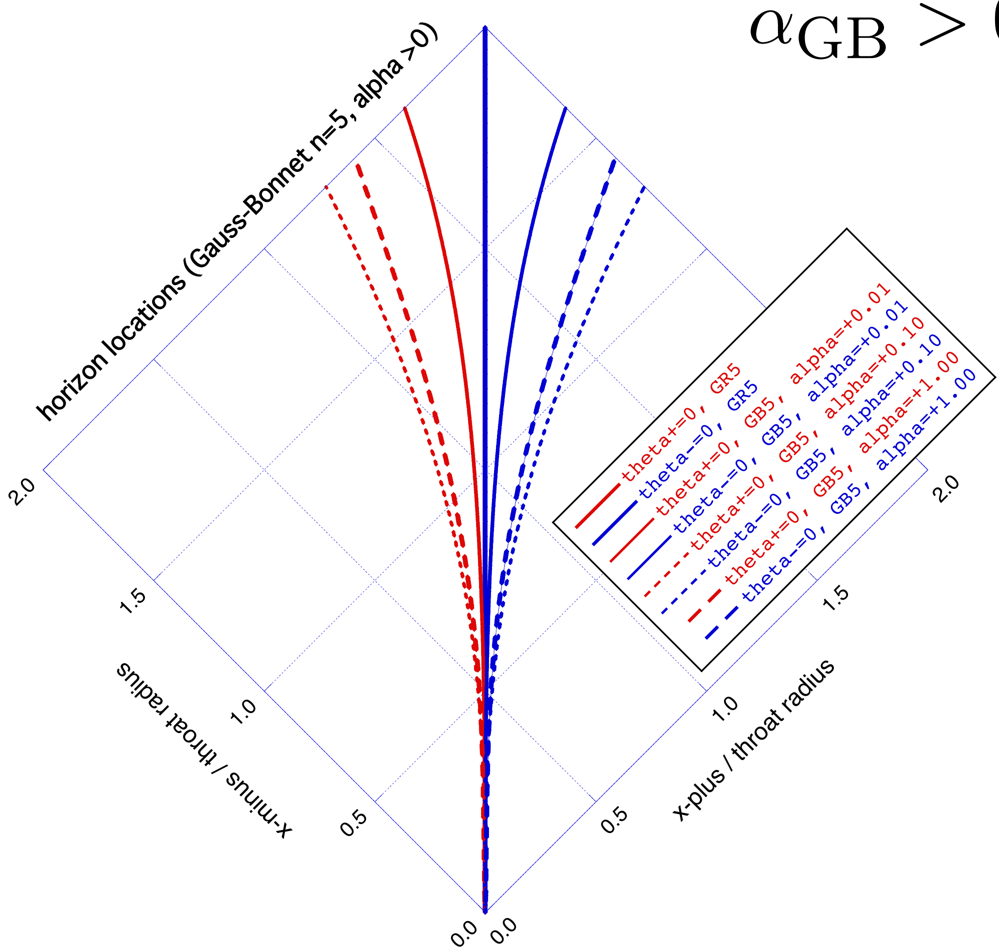
where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

5d GR vs Gauss-Bonnet instability appears

$\alpha_{\rm GB} < 0$



$\alpha_{\rm GB} > 0$



BH formation

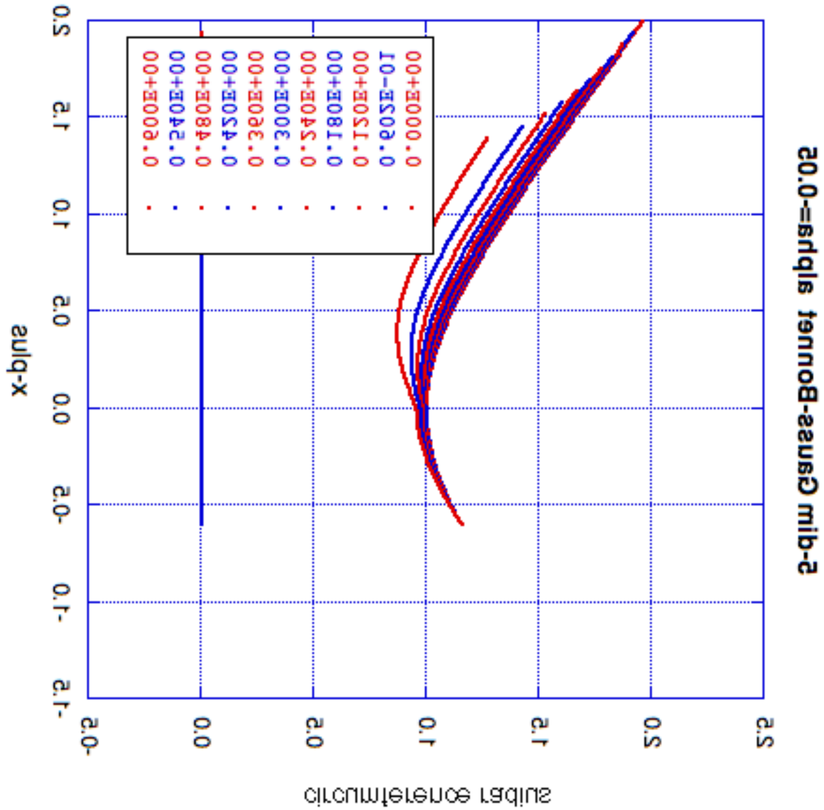
throat inflates

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\Big]$$

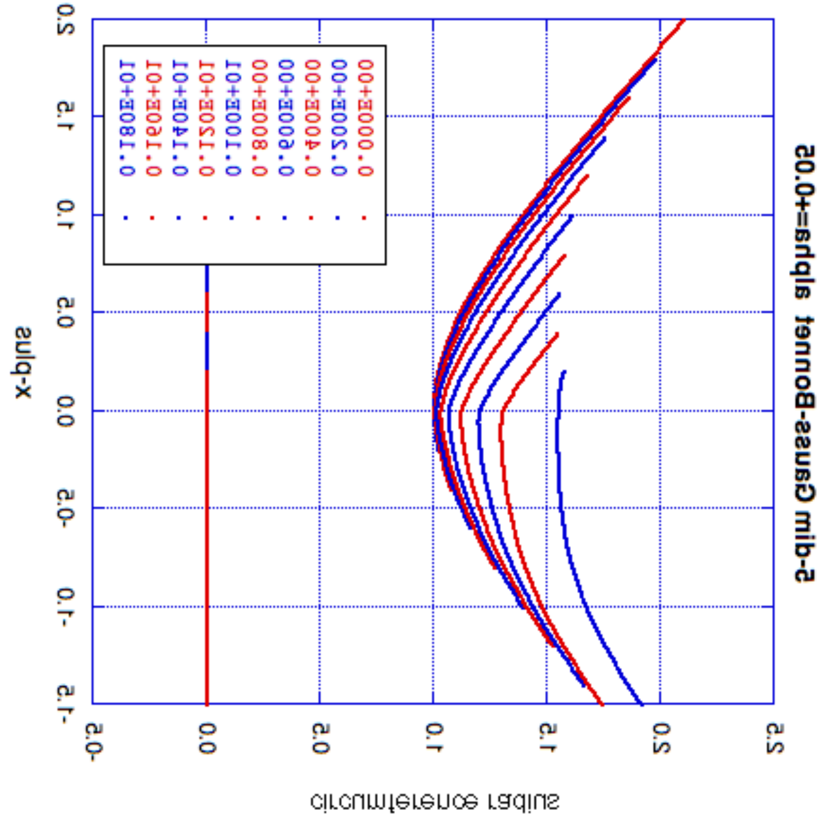
where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

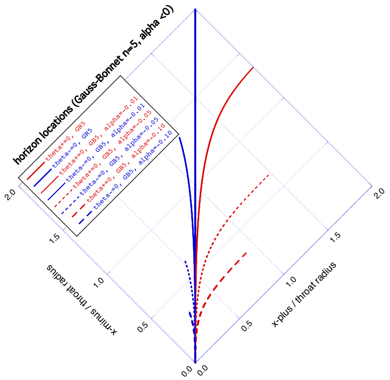
wormhole configurations (5dim. GaussBonnet)

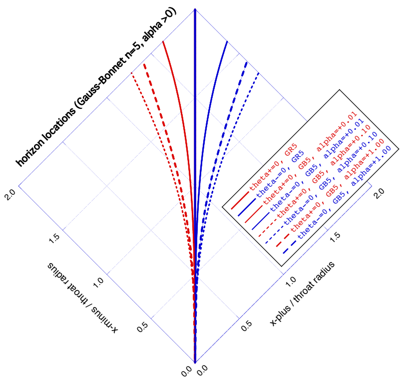
$\alpha_{\rm GB} < 0$



$\alpha_{\rm GB} > 0$

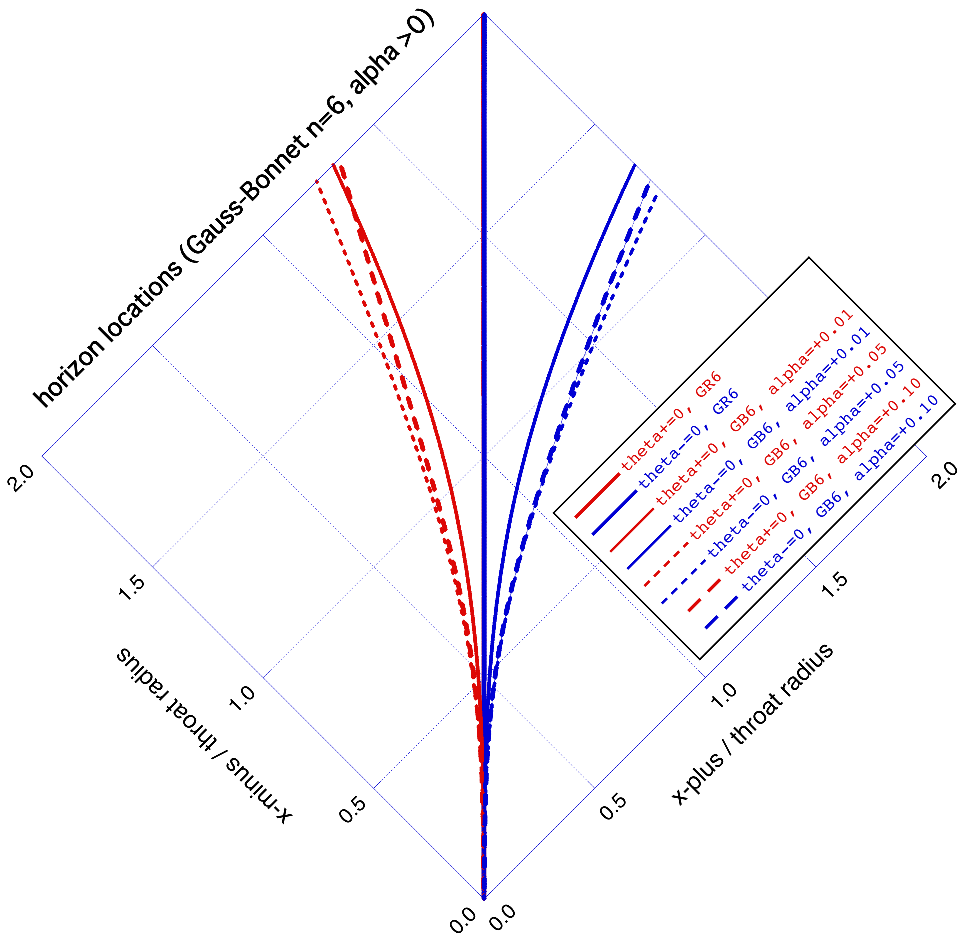






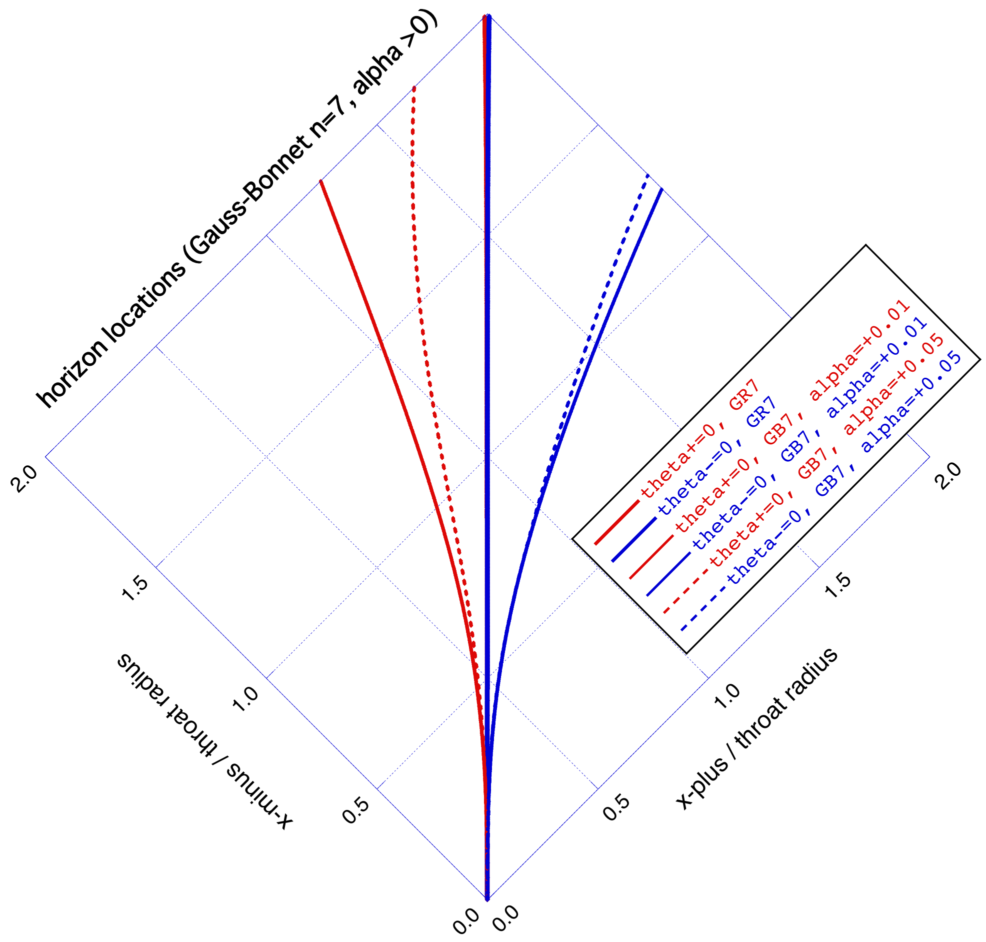
6d 7d Gauss-Bonnet　　instability appears easily

$\alpha_{\rm GB} > 0$



6 dim.

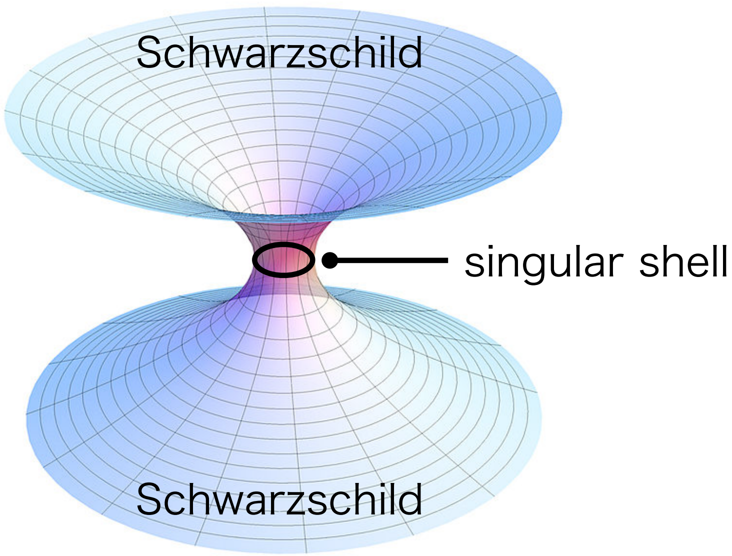
$\alpha_{\rm GB} > 0$



7 dim.

Thin-shell wormhole

‣ Poisson & Visser, PRD52 (1995) 7318



Two Schwarzschild spacetimes are connected by a singular thin shell using the Israel's junction condition.

★ They defined the parameter β_0, which corresponds to the sound velocity in the shell.

$$\beta_0^2 := \frac{\partial p}{\partial \sigma}$$

pressure (p), surface density (σ)

★ They found the parameter regions where the solution is stable.

$$\beta_0^2 \geq \frac{3}{2} + \sqrt{3} \qquad (a_0^- < a_0 < a_0^+)$$

$$\beta_0^2 \leq -\frac{1}{2} \qquad (a_0 > a_0^-)$$

Sound speed is faster than the light speed or become imaginary.

★ "There is no guarantee that β_0 actually is the speed of sound because the matter is exotic (negative energy)!"

(peculiar) EOS

- Bronnikov, et al (Grav. Cosmol. 19 (2013) 269, arXiv:1312.692

★In 4-dim. GR. perfect fluid and source free electro-magnetic field.

★The pressure of the fluid is zero for the static solution. However, if we perturb it, the pressure appears ! → stable wormhole

This is the key!!

★However, the matter field must satisfy a certain EOS.

Does the matter behaves like this?

dilatonic Einstein-Gauss-Bonnet theory

- Kanti, Kleihaus and Kunz, (PRL107 (2011) 271101)

★No exotic matter and linearly stable !

★However, they fix the throat radius.

The stability analysis is insufficient.

Wormhole in GR with Λ

- general relativity, n-dimensions

massless scalar field

$$S = \int d^n x \sqrt{-g}\left[\frac{1}{2\kappa_n^2}(R-2\Lambda) - \frac{1}{2}\epsilon(\nabla\phi)^2 - V(\phi)\right]$$

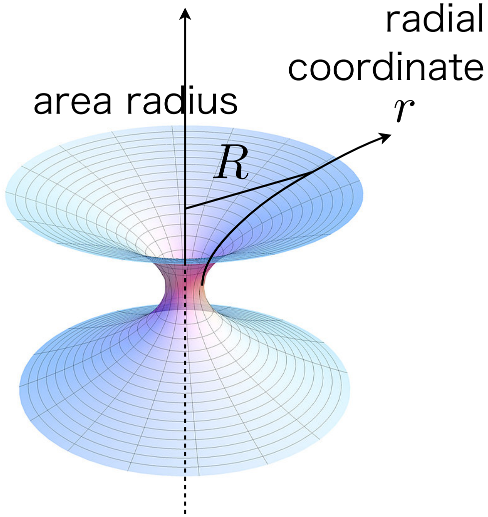
$\epsilon = -1$ (ghost)

- static spacetime

R is the area radius.

$$ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 h_{ij}dx^i dx^j$$

the line element of the (n-2)-dimensional sub-manifold. It is assumed to be a constant curvature space with curvature k.



- Λ = 0
  - 4-dim : Ellis wormhole (1973)
  - n-dim : Torii & HS (2013)

equations

- Einstein equations and the Klein-Gordon equation

$$(t,t) \quad -\frac{n-2}{2}f^2\left[\frac{2R''}{R}+\frac{f'R'}{fR}+\frac{(n-3)R'^2}{R^2}\right]+\frac{(n-2)(n-3)kf}{2R^2}-\Lambda f=\frac{\kappa_n^2}{2}\epsilon f^2\phi'^2,$$

$$(r,r) \quad \frac{n-2}{2}\frac{R'}{R}\left[\frac{f'}{f}+\frac{(n-3)R'}{R}\right]-\frac{(n-2)(n-3)k}{2fR^2}+\frac{\Lambda}{f}=\frac{\kappa_n^2}{2}\epsilon\phi'^2,$$

$$(i,j) \quad \frac{f''}{2}+(n-3)f\left(\frac{R''}{R}+\frac{f'R'}{fR}+\frac{n-4}{2}\frac{R'^2}{R^2}\right)-\frac{(n-3)(n-4)k}{2R^2}=\frac{\kappa_n^2}{2}\epsilon f\phi'^2.$$

$$(\mathrm{KG}) \quad \frac{1}{R^{n-2}}\left(R^{n-2}f\phi'\right)'=0.$$

The Klein-Gordon equation can be integrated, and the scalar field is obtained by integrating the metric functions. ➡

$$\phi'=\frac{C}{fR^2}, \quad ①$$

integration constant (C)

The Einstein equations are reduced to two equations. ➡

$$\frac{(n-2)R'}{R}\left[\frac{f'}{f}+\frac{(n-3)R'}{R}\right]-\frac{(n-2)(n-3)k}{fR^2}+\frac{2\Lambda}{f}=-\frac{\kappa_n^2C^2}{f^2R^{2(n-2)}} \quad ②$$

$$\frac{(n-2)R''}{R}=\frac{\kappa_n^2C^2}{f^2R^{2(n-2)}} \quad ③$$

boundary conditions

- regularity condition (+ symmetry) at the throat $r = 0$

throat radius → $R = a$

$R' = 0$

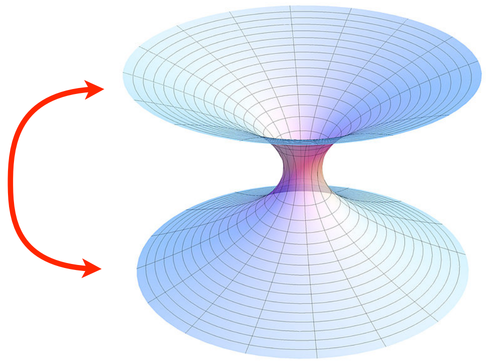
$f = f_0$

$f' = 0$

shift symmetry → $\phi = 0$

We also assume the mirror symmetry at the throat. We can extend the solution to non-symmetric one.

mirror sym.



- Asymptotically AdS

existence of solutions

- At the throat, Einstein equation ② becomes

② ➡ $\kappa_n^2 C^2 = f_0\left[(n-2)(n-3)ka^{2(n-3)} - 2\Lambda a^{2(n-2)}\right]$ $\therefore\ \Lambda < \frac{(n-2)(n-3)}{2a^2}k.$ ④

- For the positive c.c., k is positive and the cosmological horizon should appear.

④ ➡ $k=1$ and $f=0$ at $r=r_C$

① ③ ➡ $\phi' \to \infty,\ \ R'' \to \infty$ at $r=r_C$ The spacetime becomes singular!

There is no regular wormhole solution for positive cosmological constant.

- For the negative c.c.,

there is no constraint for $k=1,\ 0$.

$k=-1$ ④ ➡ $a > \sqrt{\frac{(n-2)(n-3)}{2|\Lambda|}}.$

Throat radius has the lower limit.

	$\Lambda=0$	$\Lambda>0$	$\Lambda<0$
$k=1$	exist	×	exist
$k=0$	×	×	exist
$k=-1$	×	×	exist

linear stability analysis

In the rest of this section, we examine the linear stability of the higher-dimensional Ellis wormhole.

- metric ansatz

$$ds_n^2 = -f(t,r)e^{-2\delta(t,r)}dt^2 + f(t,r)^{-1}dr^2 + R(t,r)^2 h_{ij}dx^i dx^j$$

We consider only the spherically symmetric perturbations.

- These functions are expanded.

The variables with 0 are the static solutions.

$$f = f_0(r) + f_1(r)e^{i\omega t}, \quad R = R_0(r) + R_1(r)e^{i\omega t},$$
$$\delta = \delta_0(r) + \delta_1(r)e^{i\omega t}, \quad \phi = \phi_0(r) + \phi_1(r)e^{i\omega t}.$$

ω is a frequency.

The variables with 1 are the perturbations.

- By taking linear combination, we can find the single master equation.

$$\psi_1 = R_0^{\frac{n-2}{2}}\left(\phi_1 - \frac{\phi_0'}{R_0'}R_1\right),$$

gauge invariant under spherical symmetry

linear stability analysis (2)

- By taking linear combination, we can find the single master equation.

$$\psi_1 = R_0^{\frac{n-2}{2}}\left(\phi_1 - \frac{\phi_0'}{R_0'}R_1\right),$$

gauge invariant under spherical symmetry

$$-\frac{d^2\psi_1}{dr_*^2} + V(r)\psi_1 = \omega^2\psi_2$$

$$V(r) = \frac{2C^2R_0^{-2n+4}}{(n-2)f_0R_0'^2}\left[(n-3)k - \frac{2\Lambda R_0^2}{n-2}\right] - \Lambda f_0 + \frac{(n-2)f_0}{4R_0^2}\left[2(n-3)k - (n-2)f_0R_0'^2\right].$$

diverges at the throat !

➡ The potential is positive definite. ∴ stable ?

- 0-mode solution $\bar{\psi}_1$ ⬅ The mode which changes the throat radius.

The 0-mode diverges at the throat.

This divergence is canceled by the divergence of the potential function.

linear stability analysis (3)

- regularize the perturbation equation by the 0-mode

$$\mathcal{D}_\pm = \pm\frac{d}{dr} - \frac{1}{\bar{\psi}_1}\frac{\bar{\psi}_1}{dr_*}$$

the perturbation equation

$$\mathcal{D}_-\mathcal{D}_+\psi_1 = \omega^2\phi_1.$$

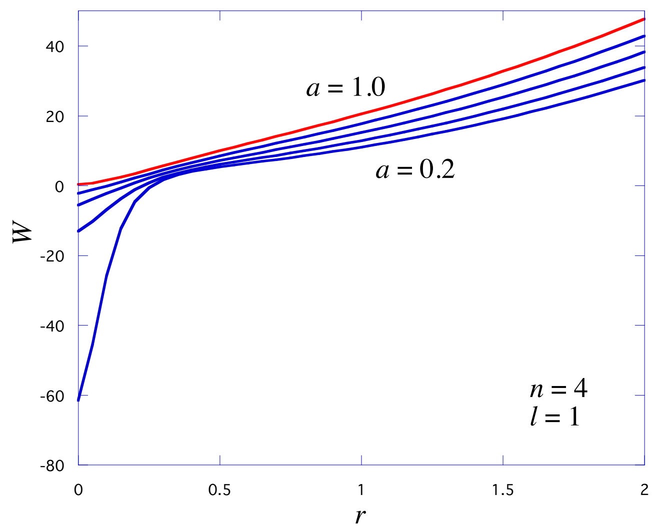
- Operating D+ on the equation and defining $\Psi_1 = \mathcal{D}_+\psi_1$, ...

- We find the regularized equation.

$$-\frac{d^2\Psi_1}{dr_*^2} + W(r)\Psi_1 = \omega^2\Psi_1$$

$$W(r) = 2f_0^2\left(\frac{1}{\bar{\psi}_1}\frac{d\bar{\psi}_1}{dr_*}\right)^2 - V(r)$$

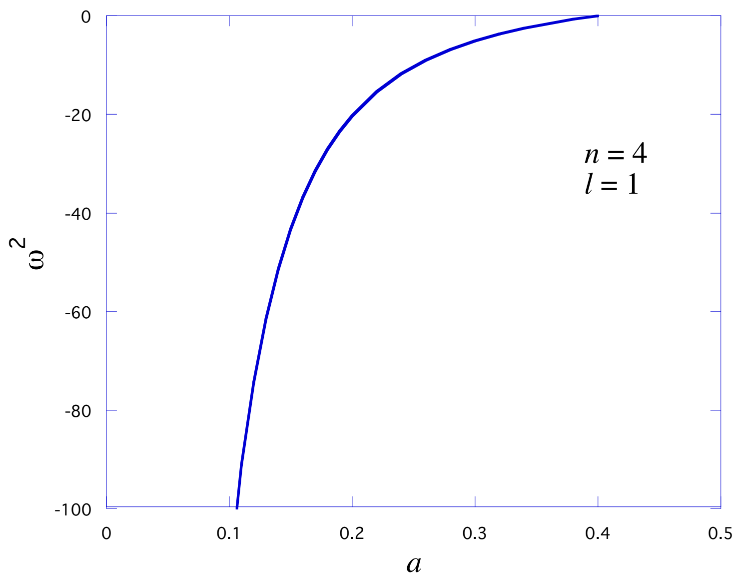
$n = 4,\ \ell_{ads} = 1.0$



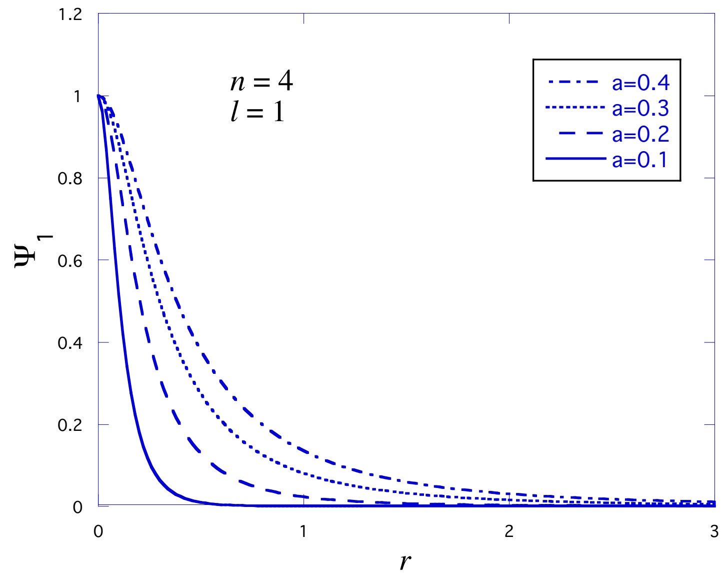
For $n = 4$ and $l_{\rm ads} = 1$, the potential W is positive definite for $a > 1$. Hence these wormholes are stable !!

linear stability analysis (4)

- Solving this equation numerically, we can find a negative mode for $a < 0.4$.



eigenvalue of negative mode



eigenfunction of the negative mode

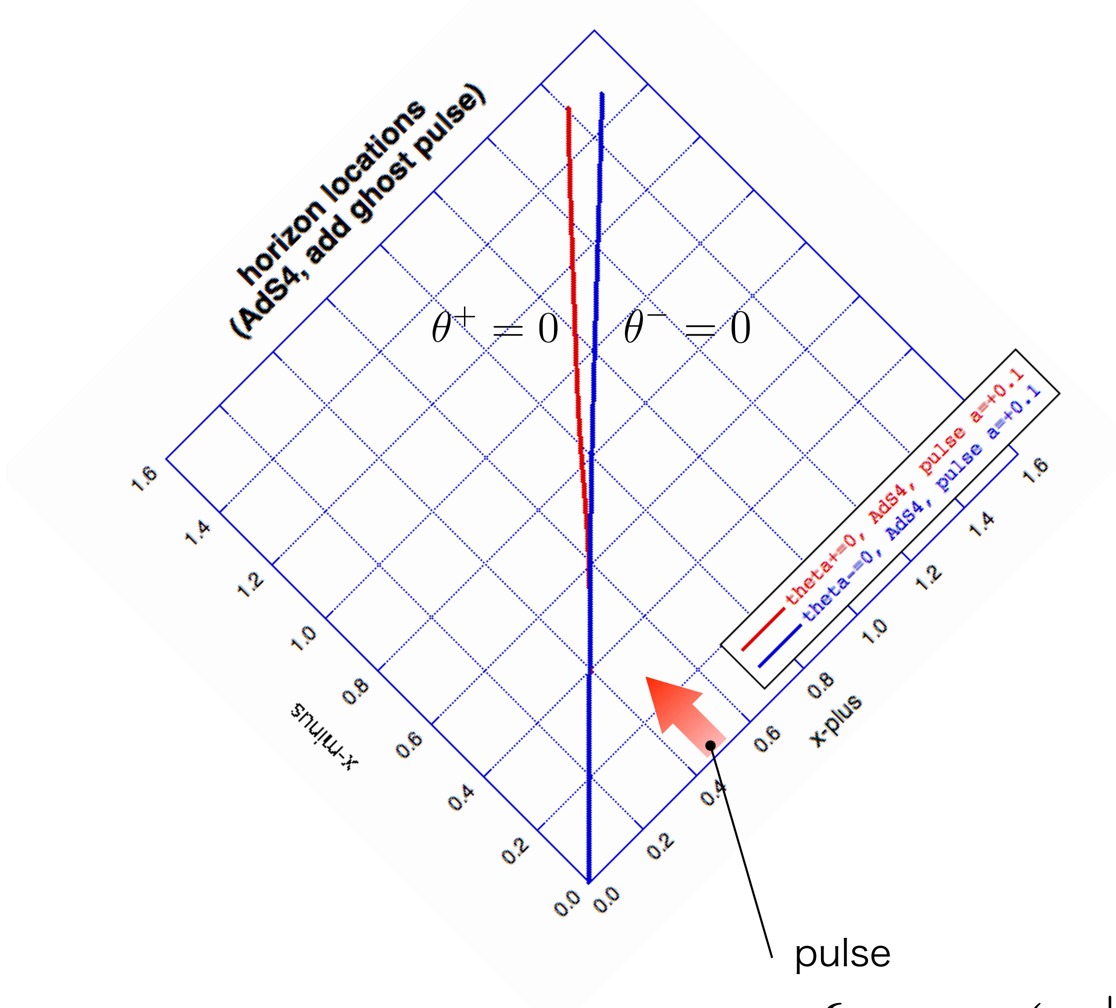
★ For n=4 and l_{ads}=1,

$a > 0.4$	stable
$a < 0.4$	unstable

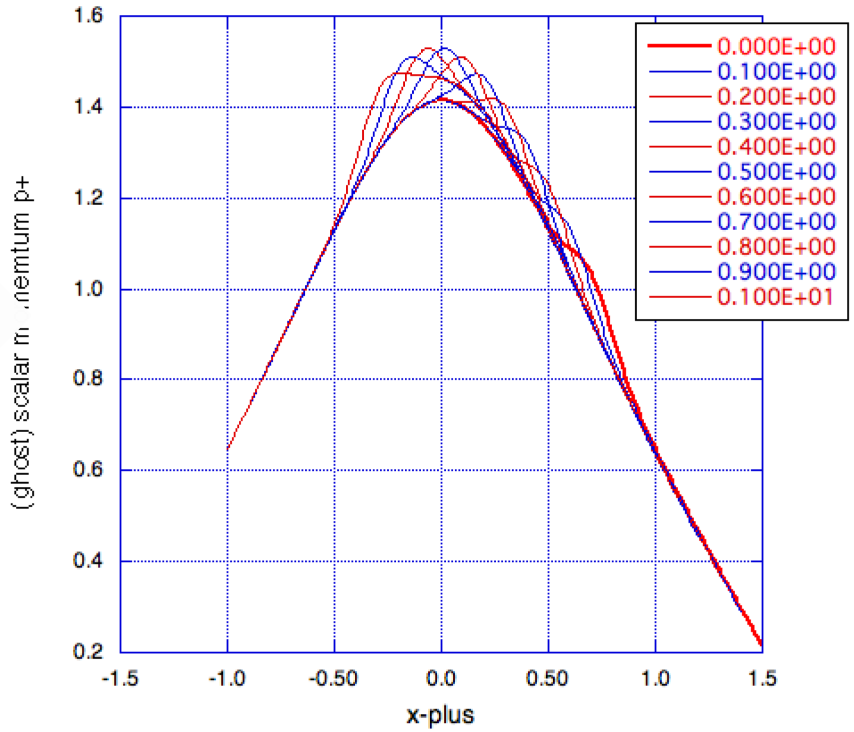
AdS wormhole evolution

Ellis WH with negative Λ, n=4

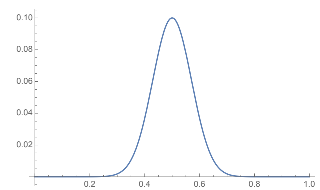
with pulse (added ghost field momentum)



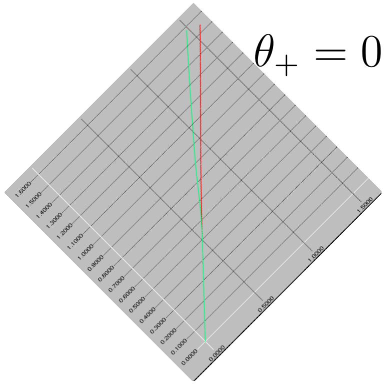
pulse



$$p_+ = p_{+\mathrm{sol}} + a\exp\{-100(x^+ - 0.5)^2\}$$



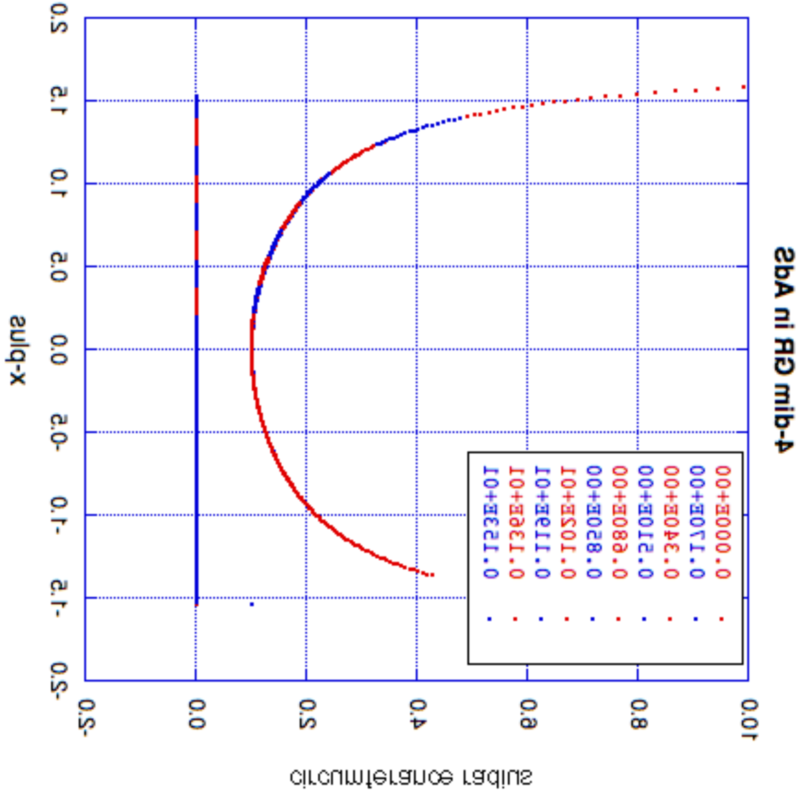
wormhole configurations (4dim. GR, AdS)

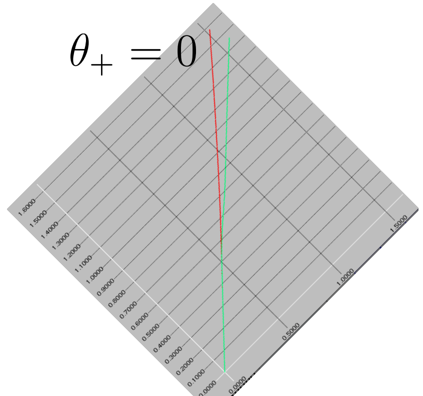


more negative field

-> throat broaden

but stay there

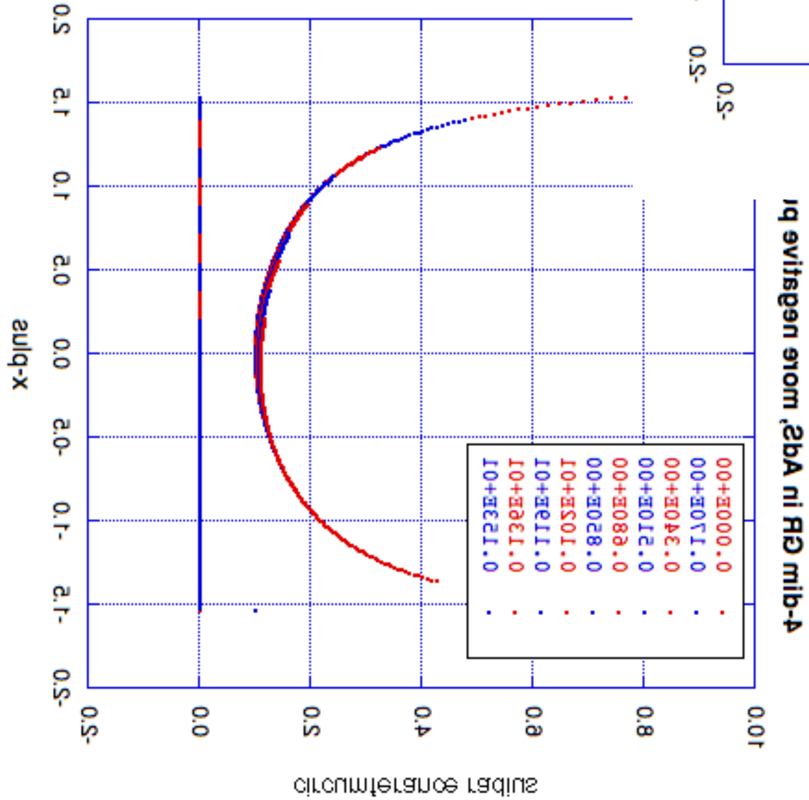


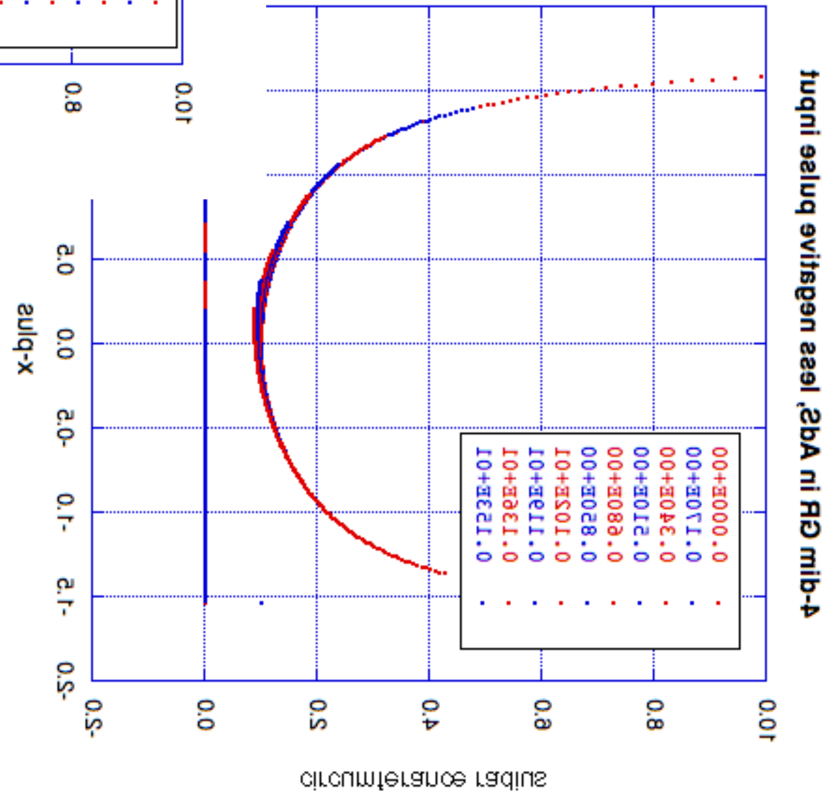


less negative field

-> throat shrink

but stay there





Summary

Ellis (Morris-Thorne) traversable WH解　線形摂動 & 時間発展

WH は 不安定である　　高次元ほど不安定

(A) 正のエネルギーパルス ---> BH

(B) 負のエネルギーパルス ---> Inflationary expansion

(C) 頑張ればメンテナンス可能

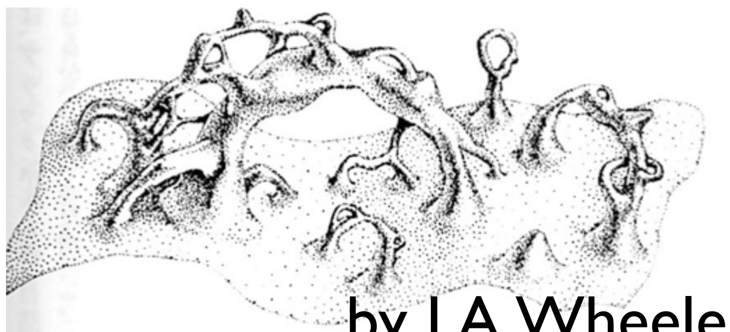
5,6,7次元 Gauss-Bonnet 項入り発展方程式での時間発展

負αの GB coupling --> BH collapse

正αの GB coupling --> Inflationary expansion

宇宙項は，負のときのみ解がありえる

throat半径がAdS半径と同じ程度のorderであれば，安定（っぽい）



by J.A.Wheeler

2002年センター試験「総合理科」第5問

B

さとし　じゃあ，めぐみはどんなものが欲しいの？

めぐみ　そうね。もう少し現実味があるという意味で，「どこでもドア」なんてあるといいわね。

さとし　あれって，そんなに現実的かなあ。まちがって出口が海の底にでもなっていたら，部屋中が水だらけだよ。

めぐみ　でも便利よ。たとえば，エベレストの山頂に行ってみたいと思ったとき，ここにドアをおいて，パッと開けたらそこがエベレストよ。

さとし　「そこがエベレストよ」って，ドアを開けたらどうなると思うの？

めぐみ　どうなるの？

さとし　あのね，人やものが自由に出入りできるのだったら，空気だって自由に出入りできるんでしょ。もしそうなら，山頂の方がこの部屋より気圧が［ウ］すごい風が起きて，みんな吹き飛ばされてしまうよ。

めぐみ　そうね，エベレストはやめましょう。でも，ちょっとした旅行には便利よ。たとえば，東京から那覇へ行くくらいなら便利よね。

さとし　そんなことないよ。ここに(b)<u>今日の天気図</u>があるんだけれど，これによると東京と那覇の間の気圧差は 25 hPa もあるよ。

めぐみ　そんなの微々たるものじゃない？

さとし　計算してごらんよ。ドアの大きさを 1 m × 2 m とすれば，ドアにかかる力は［エ］kg 重にもなるんだよ。

めぐみ　わかったわ。でも，さとしの方こそ夢がないのね。

どこでもドア：行きたいところにすぐ行ける便利なドア。

（藤子・F・不二雄『大長編ドラえもん⑨のび太の日本誕生』）

問 4　文章中の空欄［ウ］に入れるのに最も適当なものを，次の①～④のうちから一つ選べ。［4］

① 高いのだから，部屋からエベレストに向かって
② 高いのだから，エベレストから部屋に向かって
③ 低いのだから，部屋からエベレストに向かって
④ 低いのだから，エベレストから部屋に向かって

問 5　文章中の空欄［エ］に入れる数値として最も適当なものを，次の①～④のうちから一つ選べ。ただし，ドアにかかる力は気圧差によって引き起こされるものとし，1 hPa は 10 kg 重/m^2 とする。［5］

① 25　② 50　③ 250　④ 500

http://school.js88.com/sd_article/dai/dai_center_data/jsc_dai_center_exam:subject_id:66:year:2002:exam_no:5.htm