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## INO: Interplanetary Network of Optical Lattice Clocks

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The new technique of measuring frequency by optical lattice clocks now approaches to the relative precision of  $(\Delta f/f) = O(10^{-18})$ . We propose to place such precise clocks in space and to use Doppler tracking method for detecting low-frequency gravitational wave below 1 Hz. Our idea is to locate three satellites at one A.U. distance (say at L1, L4 & L5 of the Sun-Earth orbit), and apply the Doppler tracking method by communicating “the time” each other. Applying the current available technologies, we obtain the sensitivity for gravitational wave with three or four-order improvement ( $h_n \sim 10^{-17}$  or  $10^{-18}$  level in  $10^{-5}\text{Hz} - 1\text{Hz}$ ) than that of Cassini satellite in 2001. This sensitivity enables us to observe black-hole mergers of their mass greater than  $10^5 M_\odot$  in the cosmological scale. Based on the hierarchical growth model of black-holes in galaxies, we estimate the event rate of detection will be 20-50 a year. We nickname “INO” (Interplanetary Network

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of Optical Lattice Clocks) for this system, named after Tadataka Ino (1745–1818), a Japanese astronomer, cartographer, and geodesist.

*Keywords:* Gravitational Waves; Detector proposals in Space; Optical Lattice Clock; Super-massive black-holes

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## 1. Introduction

The direct detections of gravitational wave by LIGO/Virgo groups<sup>1</sup> has opened new era for physics and astronomy. With this new method of observing the Universe, we are now able to observe black-holes (BHs) directly with gravitational wave. With help of other observational bands,<sup>2</sup> *i.e.* with the electromagnetic waves from radio to gamma-ray or by neutrino observation, we can discuss the details of high-energy events, the equation of state of nuclear matter, cosmology, and the validity of gravitational theories. LIGO/Virgo groups announced so-far that five events of the coalescences of binary BHs,<sup>1,3–6</sup> and one merger of binary neutron stars.<sup>2</sup> In 2019, LIGO/Virgo detectors will start their observation again with upgraded systems, and KAGRA in Japan will start its observation, too. Such gravitational wave observations will give us the statistics of the events and make the science more precise and trustable.

Among the unsolved problems in the Universe, however, the growth process of large BHs is left untouched. Almost all of the galaxies in the Universe have super-massive black holes (SMBHs) in their center, which mass is over  $10^6 M_{\odot}$ .<sup>7</sup> Observational data show that the masses of such SMBHs are proportional to the masses of their bulge (the center part of the galaxies).<sup>8,9</sup> This fact indicates that the SMBHs in the galaxies coevolved with its mother galaxy, but such an inevitable relation is still a mystery in the history of the Universe.

One of the plausible scenario is the hierarchical growth model of the stars in the galaxy. This model says that a black hole (a seed black hole) is first formed in the center of galaxy when it grows to the size of a dwarf galaxy. The intermediate-mass black holes (IMBHs), which mass is around  $100\sim 1000 M_{\odot}$ , in its star clusters will then accumulate in the center region of the galaxy, merge together, and form to a SMBH.<sup>10</sup> This theory naturally explains central BH-bulge mass relation, described above. A seed black hole might be formed by another process; one possible way is to start from a forming a  $10^5 M_{\odot}$  black hole in the early stage of the Universe. Hierarchical mergers will proceed between galaxies, and such processes will naturally produce SMBHs.<sup>11,12</sup> In result, the mergers of  $10^6 - 10^{10} M_{\odot}$  black holes are expected to be the sources of gravitational wave.<sup>13,14</sup>

Formation of SMBHs are also modeled by accumulations of large amount of gas to a seed BH.<sup>15</sup> Therefore, collecting the detections of gravitational waves from large mass BHs, which means the direct evidence of the growth process of the BHs, is the clue to solve the current mystery of coevolution of a BH and its mother galaxy.

All the gravitational detectors which are under operation today, such as LIGO

and Virgo, are located on the ground, which means that we are hard to detect gravitational wave below 10 Hz since seismic vibration dominates as noise. The mergers of BHs larger than  $10^4 M_{\odot}$ , on the other hand, end up below 1 Hz. Therefore the only possible way to detect such a gravitational wave signal is to place detectors in space.

The project of eLISA by ESA<sup>16</sup> is the plan of constructing laser interferometer in space with the arm length  $2.5 \times 10^6$  km, targeting mainly at milli-Hz range of gravitational wave. Locating three satellites at Lagrange 4 (L4) of the Sun-Earth orbit with drag-free flight motion, and using the light-transponder technique, ESA plans to realize the system in early 2030s.

Japanese group proposed DECIGO (B-DECIGO) project,<sup>17</sup> which plans to construct Fabry-Perot laser interferometer with 1000 km (100 km) arm length, with three satellites on the Sun-Earth orbit (around the Earth orbit) with drag-free flight motion. Their main target is deci-Hz range of gravitational wave.

Space-borne interferometers such as eLISA or B-DECIGO require significant technical breakthroughs. We, in the present article, propose an alternative method for detecting low-frequency gravitational waves, technically feasible with the current technologies. Our idea is to locate three satellites at A.U. scales (say at L1, L4 and L5 of the Sun-Earth orbit), which load the optical lattice clock, the ultimate precise atomic clock. By comparing the time each other, applying the principle of the Doppler tracking, we can detect the passage of gravitational waves of mHz range. The ideas of using atomic clocks for detecting gravitational wave have already been appeared.<sup>18–20</sup> However, our discussion is new on the feasibility of satellites, showing both sensitivity and detectable distance, and expected event rates.

We named this proposal *Interplanetary Network of Optical Lattice Clocks*, with the abbreviation *INO*. The acronym, INO, is named after Tadataka Ino, a Japanese astronomer, cartographer, and geodesist, who made precise map of Japan two centuries ago.

## 2. Gravitational wave detection using optical lattice clocks

The only observations of gravitational wave in space so far are the one by tracking artificial satellite using Doppler effect<sup>21</sup> (actually they showed us the upper-bound constraint of the gravitational wave). The method is to observe the velocity shift (Doppler shift) produced by passing gravitational wave between the Earth and the satellite, by comparing the frequency of the signal sent from the Earth and received at the satellite using their clocks. The sensitivity of this Doppler-tracking method depends on the distance of the signal baseline. Until now, the most strict sensitivity was obtained by the Cassini satellite which was launched for surveying the Saturn.<sup>22</sup>

Armstrong<sup>23</sup> listed up key noises to be improved in the future mission of Cassini-type gravitational wave observation; (i) frequency standard, (ii) ground electronics, (iii) tropospheric scintillation, (iv) plasma scintillation, (v) spacecraft motion, and (iv) antenna mechanical [See Table 1]. We discuss how we can improve them with

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the current technologies one by one.

Table 1. Required improvement in subsystems to improve overall Doppler sensitivity by a factor of 10 relative to Cassini-era performance. (Copy of Table 4 of Armstrong (2006),<sup>23</sup> added the left column.)

Noise source	Comment ( $\sigma_y$ at $\tau = 1000\text{s}$ ) current values	Required improvement
(i) Frequency standard	FTS + distribution $\simeq 8 \times 10^{-16}$	$\simeq 8\text{X}$
(ii) Ground electronics	$\simeq 2 \times 10^{-16}$	$\simeq 2\text{X}$
(iii) Tropospheric scintillation	$\simeq 10^{-15}$ under favorable conditions	$\simeq 10\text{X}$
(iv) Plasma scintillation	Cassini-class radio system probably adequate for calibration to $\simeq 10^{-16}$	$\simeq 1\text{X}$
(v) Spacecraft motion	$\simeq 2 \times 10^{-16}$	$\simeq 2\text{X}$
(iv) Antenna mechanical	$\simeq 2 \times 10^{-15}$ under favorable conditions	$\simeq 20\text{X}$

### 2.1. Usage of optical lattice clocks

The key technology of the Doppler-tracking method is the stability of clocks [the list (i)]. We propose to use “optical lattice clocks” that allow significant improvement in atomic clocks’ stability. Atomic clocks steer the frequency of local oscillators, such as cavity-stabilized lasers, by referencing atomic transitions. Stability of such atomic clocks are limited by the quantum projection noise<sup>24</sup> that is given by the number of atoms  $N$ . By interrogating  $N \sim 10^6$  atoms trapped at the anti-nodes of a standing-wave laser, which is referred to as an optical lattice, and by eliminating the Stark shift perturbation by tuning the laser to the magic frequency,<sup>25,26</sup> the optical lattice clocks achieve<sup>27</sup> high stability and accuracy approaching  $10^{-18}$ . By applying an operational magic frequency, the accuracy of  $10^{-19}$  is in scope.<sup>28</sup> If this level of the accuracy is obtained, then the noise from clock can be completely ignored.

Kolkowitz *et al*<sup>20</sup> proposed to use optical lattice clocks to detect gravitational waves. Their idea is to measure the Doppler shift between the two optical lattice clocks in space which are communicating with lasers. By controlling two mirrors located apart in the drag-free state, they propose to measure the frequency difference between two optical lattice clocks using precise laser which are linked with these mirrors. The core idea is the same with that of the Doppler-tracking method, but with the technology of drag-free control, they say the sensitivity is greatly improved at  $0.01 \sim 1$  Hz. However, there is a disadvantage in drag-free technology. The remained acceleration of free mass is controlled with magnetic field, but when cosmic-ray hits the device, the photoelectric effect charges the free-mass and this fluctuation behaves noise. Especially at the lower frequency, the residual error is inversely proportional to the square of frequency,<sup>29</sup> and as a result the sensitivity of their proposal is at the same level with eLISA.

We therefore propose not to use drag-free control, but to improve Doppler-tracking method with advanced optical lattice clock and the light-linking technology

for constructing a gravitational wave detector.

## 2.2. Location of INO satellites

If we measure all the difference of the clock between the satellites in space, then we do not need to care the noises due to ground electronics [the list (ii)] and tropospheric scintillation [the list (iii)].

As we already mentioned, it is preferable to locate the satellites for Doppler tracking at far distance such as beyond the orbit of Jupiter or Saturn. This request, however, is severe for keeping power and fuel. We therefore propose to locate three satellites at the  $L_1$ ,  $L_4$ , and  $L_5$  of the Sun-Earth orbit, which enable us to take the baselines between each satellites at the order of A.U. (see Fig.1).

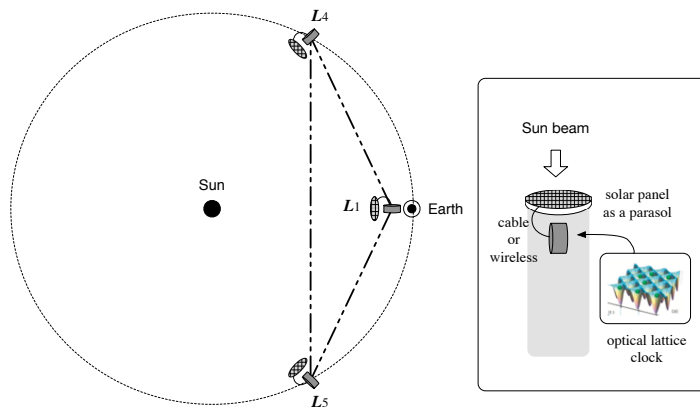


Fig. 1. A planned location of the satellites: Lagrangian points  $L_1$ ,  $L_4$  and  $L_5$  of the Sun-Earth orbit. The  $L_1$  is at 1/100 A.U. from the Earth, while  $L_4$  and  $L_5$  forms equilateral triangle with the Sun and the Earth respectively; the distance between  $L_1$ - $L_4$  ( $L_5$ ) is 1 A.U., while that of  $L_4$ - $L_5$  is  $\sqrt{3}$  A.U. Two-frequency radio or light will be used for communication between satellites. The inset explains that the solar panel of the satellite is separated as a parasol from the main body, in order to prevent acceleration noise due to solar wind.

## 2.3. Communication between the satellites

The noise list (iv) by Armstrong is plasma scintillation in the Solar system. We may have two ways to communicate each satellite; radio or light. If we link them with light, the phase fluctuation by the plasma effects is negligible. However, light communication requires precise directivity than radio. In the current technology, for communication over  $10^7$  km, radio is preferable. If we link them with radio, then double tracking method which uses two-frequency bands will compensate the phase shift due to interplanetary plasma.

#### 2.4. Separation of the main body and the solar-cell panel

The noise list (v) by Armstrong is spacecraft stability against radiation pressure of the Sun beam, which dominates the noise in the lower frequency range. Suppose we require the sensitivity of  $h_n = 10^{-17}$  for the baseline of  $1.5 \times 10^8$  km ( $\sim 1$  A.U.), which is comparable with the amplitude 0.7 mm; that corresponds to the measurement of the velocity  $2 \times 10^{-8}$  m s $^{-1}$ , or the acceleration  $1.4 \times 10^{-12}$  m s $^{-2}$  at  $10^{-5}$  Hz.

The radiation pressure force is  $F = P/c$ , where  $P$  is the power of the Sun beam and  $c$  is the speed of light. Around the Earth orbit,  $P$  is 1.3 kW m $^{-2}$  per unit area. If we suppose the solar cell panel as 10 m $^2$  and the mass of the satellite as 1000 kg, then the acceleration of the satellite due to the beam pressure is about  $5 \times 10^{-8}$  m s $^{-2}$ . The pressure of the Sun fluctuates at the order of  $10^{-3}$ , so that the acceleration fluctuates at the order of  $10^{-11}$ . In order to detect the gravitational wave of which acceleration is at the order of  $10^{-12}$ , we should reduce the fluctuation one order smaller. This is attainable by mechanically separating the satellite main body from its solar-cell panel and use the solar panel as a parasol for shielding from the Sun beam (see the inset of Fig.1). The area of the solar-cell is around 270 W/m $^2$ , effective than the normal satellite since this network always receive the Sun beam. If the cable connection is a source of vibration, then the wireless transmission of electricity can be used. In the current technology, wireless transmission of 12 kW with 10 cm is obtained at experiments on the ground. This structure removes the dynamical interactions which may reduce the acceleration noise two-orders of magnitude.

### 3. Sensitivity of INO

We estimate the reachable sensitivity for gravitational wave detection with current known technologies. In order to make the most feasible discussion, we do not consider to use drag-free control, nor precise laser control, but simply apply the advanced optical lattice clock to the Doppler-tracking method.

The sensitivity of the Doppler-tracking method is well understood by the report of Cassini satellite,<sup>23,29</sup> which keeps the best record as  $h_n \sim 3 \times 10^{-15}$  at  $10^{-4}$  Hz, where  $h_n$  is the noise amplitude, which is given by the square root of the combination of the power spectrum of the noise times frequency  $f$ . The noise amplitude is the standard quantity since it can be compared directly with the characteristic strain  $h_c$  which expresses the strength of the gravitational wave signal. Cassini's sensitivity showed the curve  $f^{-1}$  below  $10^{-4}$  Hz.

The origins of noise in Cassini are identified mostly from the accuracy of the atomic clock and from the fluctuation of troposphere of the Earth.<sup>23</sup> As we discussed in the previous section, if we use the advanced optical lattice clock instead of the atomic clock, and let the satellites communicate each other directly, and with a Sun-beam shield, the sensitivity will be dramatically improved. From the Table.4 in,<sup>23</sup> we estimate that the three or four-order improved version of Cassini satellite

(i.e. the minimum sensitivity is around  $h_n = 10^{-17}$  or  $10^{-18}$ ) will be available.

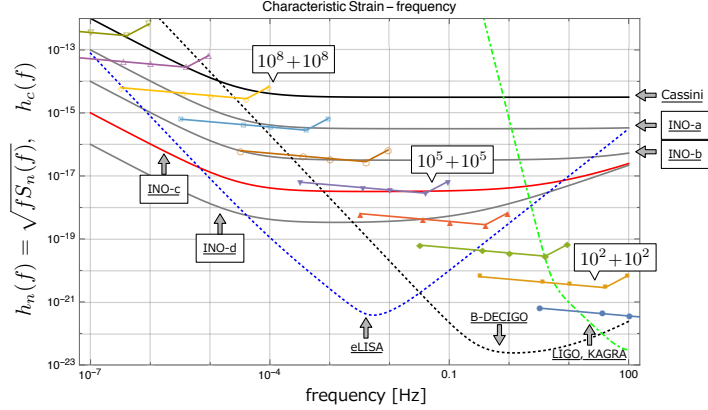


Fig. 2. Sensitivity of Doppler-tracking satellites and expected strains of gravitational wave. The most upper solid curves indicates the sensitivity of Cassini satellite (2001), while the other solid curves are those of 1~4-order improved version (we named INO-a, INO-b, INO-c and INO-d, respectively). The dotted line is the sensitivity curve of eLISA. Almost horizontal lines with symbols indicate the characteristic strain of gravitational wave from a merger of equal-mass binary BHs at 1 Gpc. Each line is for the inspiral phase; starts from its separation 50 times of the event horizon radius, and ends at their merger (frequency moves up higher for smaller separation).

Fig. 2 shows the sensitivity curve of Cassini satellite and their one to four-order improved version (we named INO-a, INO-b, INO-c and INO-d, respectively), together with that of eLISA, B-DECIGO and advanced LIGO/KAGRA. Since the frequency dependence at the lower frequency is different, INO achieves the same sensitivity with eLISA at  $10^{-5}$  Hz, and better than eLISA in the range less than that.

In Fig. 2, we also plot the characteristic strain of the gravitational wave ( $h_c$ ) from a merger of the binary BHs with its distance 1 Gpc from the Earth. We plotted for mergers of equal-mass BHs for several different masses. Each line starts from its frequency when the binary's separation is 50 times of their event horizon radius, and ends at the frequency when they merge.

We see that the mergers of SMBHs of  $10^7 \sim 10^8 M_\odot$  produce gravitational wave around  $10^{-4}$  Hz, which is detectable with INO at the signal-to-noise ratio (SNR) 10.

## 4. Expected gravitational wave events

### 4.1. Detectable distance for BH mergers

Once the detector's sensitivity is given, then we can calculate the detectable distance (observational distance, or horizon of the detector) for typical BH merger events.

In Fig. 3, we plot them for Cassini and its improved version. We see INO-c already covers the Universe for the mergers of SMBHs of their chirp mass  $10^7 \sim 10^8 M_\odot$ . (For the binary of masses  $m_1$  and  $m_2$ , the chirp mass,  $M_c$ , is given by  $M_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ .  $M_c$  determines the leading-order amplitude and frequency evolution of the gravitational-wave signal from inspiral binary.)

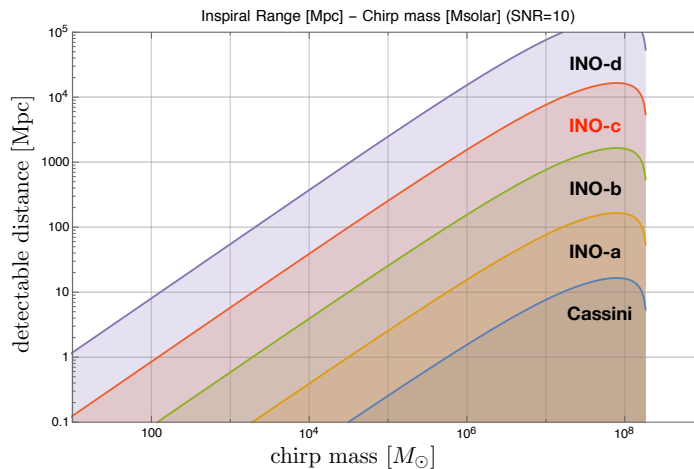


Fig. 3. Detectable distance (observational distance, or horizon of the detector) of Cassini, INO-a,  $\dots$ , INO-d as a function of the binary's chirp mass. The distance is the luminosity distance. All lines are for SNR=10.

#### 4.2. Event rate of BH mergers by hierarchical growth model

If we further assume the distribution model of BH mass (i.e. the evolution model of BHs), and the distribution model of galaxies, together with cosmological model, then we can estimate the event rate per year.

We calculate the event rate based on the hierarchical growth model.<sup>14</sup> This model assumes that the formation of SMBHs are from mergers of BHs in a hierarchical sequence. The number of BH mergers are estimated from the giant molecular cloud model, of its total numbers are depend on the size of galaxies. The distribution and the size of galaxies are modeled from the number density of galaxies from the halo formation model. Although there are several unknown factors in the model (such as contributions of BH spins, mass ratio of binaries, merger ratio as a function of mass, etc), but the simplest model predicts that detection profile of the ground-based gravitational wave interferometers has a peak at  $60 M_\odot$ , which is actually the same with the first detection, GW150914.

The result of the event rate for INO is shown in Figure 4. We show both for INO-c and INO-d. The number is of per year per bin. If we integrated for the



standard SNR=10 case, then we get 19.1 mergers for  $< 10^3 M_\odot$  and 0.35 mergers for  $> 10^3 M_\odot$  per year for INO-c, while we get 19.2 and 29.8 per year respectively for INO-d.

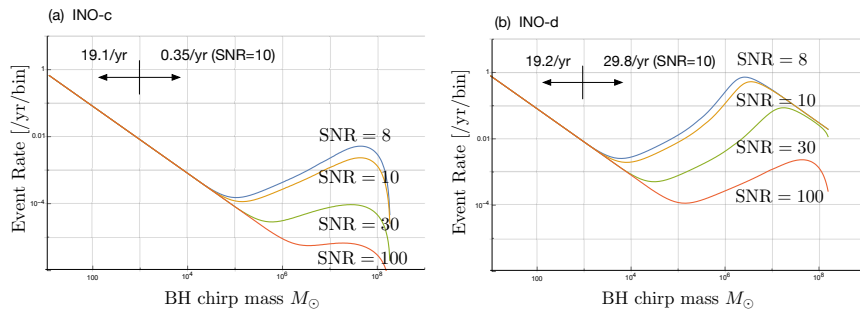


Fig. 4. Event rates of mergers of BHs by INO-c (a) and INO-d (b). Event rates are per year per bin, which we plot the number with 20 bins in log-scale in one order of the chirp mass. Each plot has four lines, for signal-to-noise ratio (SNR) of 8, 10, 30 and 100, respectively. The integrated event rates are also shown in the figure for upper and lower than  $10^3 M_\odot$ .

## 5. Closing comments

We proposed a new method for detecting gravitational wave in space, named INO (Interplanetary Network of Optical Lattice Clocks). We discussed that, with the current technologies, Cassini's Doppler tracking method (2001-2002) can be improved 3 to 4-order magnitudes. Although even at INO-d level the best sensitivity is around  $h_n \sim 10^{-18}$ , which is worse than the that of ongoing eLISA project, but we showed that INO-c and INO-d (which are of three and four-order improved sensitivity than Cassini, respectively) has better sensitivity range than eLISA at lower frequency range. We also showed that INO covers cosmological scale for observing BH mergers larger than  $10^5 M_\odot$ . We calculated event rates based on a hierarchical growth model of SMBHs, which say we could observe stellar-mass BH mergers 20 events per year at INO-c and if we can reach one order sensitivity up then we could observe more 30 BH mergers above  $10^3 M_\odot$  range. We think this number is worth trying to consider seriously.

The detection of gravitational wave in space will give us the first clue to the process of the formation of SMBHs which is totally unknown now. Our proposal is complementary to the method of interferometers, and the ultimate application of the optical lattice clocks.

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