Wormhole Evolutions in Higher-dim. Gravity
--- Effects of Gauss-Bonnet gravity terms

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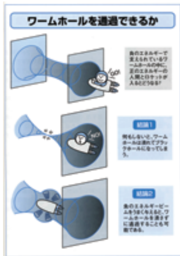
Outline & Summary

(a) "Fate of Morris-Thorne (Ellis) wormhole" was numerically investigated in 2002. [HS & Hayward, PRD66, 044005].
The fate is either black-hole collapse or inflationary expansion, depending on the excessed energy.

(b) The higher-dimensional Ellis wormhole solutions are obtained.
Perturbation study suggests instability. Numerical evolutions confirm its instability.

(c) The wormholes in 5-dim. Gauss-Bonnet gravity are numerically obtained.
Evolutions suggest that positive GB term accelerates black hole collapse.

Time Machine & Science of Space-time (HS, 2011)



Motivations

Why wormholes?

- They make great science fiction – short cuts between otherwise distant regions. Morris & Thorne 1988, Sagan "Contact" etc
- They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.
- They are very similar to black holes –both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole ≡ Hypersurface foliated by marginally trapped surfaces

- BH and WH are interconvertible? New duality?

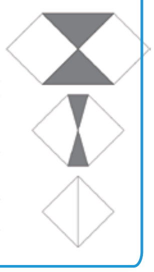
[1] How the stability changes in 5-d GR?
[2] How the stability changes in Gauss-Bonnet gravity?

BH and WH are interconvertible ? (New Duality?)

S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

- They are very similar – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)
- Only the causal nature of the THs differs, whether THs evolve in plus / minus density.

	Black Hole	Wormhole
Locally defined by	Achronal(spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally. but constructible ???



N-dim. Ellis Wormhole sol.

in prep.

A Wormhole Solution (n-Dim, massless ghost scalar)

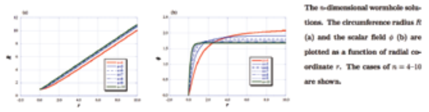
- spherical symmetry.

$$ds^2 = -f(t,r)e^{-2\delta(t,r)}dt^2 + f(t,r)^{-1}dr^2 + R(t,r)^2 d\Omega^2 \quad (1)$$

- with massless ghost scalar field ϕ.
- static, $f \equiv 1$, and throat radius $R(0) = a$; Just solve

$$\frac{d^2R}{dr^2} = \frac{(n-3)a^{2(n-3)}}{R^{2n-5}} \quad (2)$$

$$\frac{d\phi}{dr} = \sqrt{(n-2)(n-3)}\frac{a^{n-3}}{R^{n-2}} \quad (3)$$



The n-dimensional wormhole solutions. The circumference radius R (a) and the scalar field ϕ (b) are plotted as a function of radial coordinate r. The cases of $n = 4$–10 are shown.

Perturbation Analysis

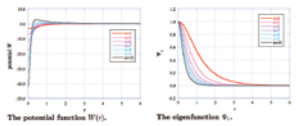
$$f(t,r) = f_0(r) + \varepsilon f_1(r)e^{i\omega t}, \quad (4)$$
$$\delta(t,r) = \delta_0(r) + \varepsilon \delta_1(r)e^{i\omega t}, \quad (5)$$
$$R(t,r) = R_0(r) + \varepsilon R_1(r)e^{i\omega t}, \quad (6)$$
$$\phi(t,r) = \phi_0(r) + \varepsilon \phi_1(r)e^{i\omega t}. \quad (7)$$

- Gauge invariant quantity $\psi_1 = R_0^{\frac{n-2}{2}}(\phi_1 - \frac{\phi_0'}{R_0'}R_1)$
- regularized variable $\Psi_1 = \mathcal{D}_+\psi_1$ where $\mathcal{D}_+ = \frac{d}{dr} - \frac{\bar\psi_1'}{\bar\psi_1}$, the master equation becomes

$$-\Psi_1'' + W(r)\Psi_1 = \omega^2\Psi_1, \quad (8)$$

where

$$W(r) = -\frac{1}{4R_0^2}\left[\frac{3(n-2)^2}{R_0^{2(n-3)}} - (n-4)(n-6)\right]. \quad (9)$$

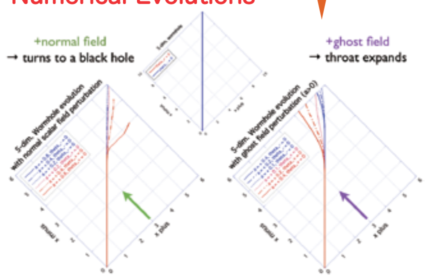


The potential function $W(r)$. The eigenfunction Ψ_1.

n	ω^2
4	−1.397052433715108
5	−2.984058930277901
6	−4.686520542994600
7	−6.462584141263180
8	−8.289759363062593
9	−10.153553045186070
10	−12.044265301474384

Obtained negative eigenvalues of ω^2. ⟹ Unstable.

Numerical Evolutions



Field Eqs.

Gauss-Bonnet gravity

- Action

$$S = \int_M d^{N+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{GB}\} + \mathcal{L}_{matter}\right] \quad (1)$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field eqs.

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu} \quad (2)$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- Features
  - □ has GR correction terms from String Theory.
  - □ has two solution branches (GR/non-GR).
  - □ is expected to have singularity avoidance feature. (but has never been demonstrated.)
  - □ new topic in numerical relativity. (S Golod & T Piran, PRD 85 (2012) 104015; F Izaurieta & E Rodriguez, 1207.1496; N Deppe+ 1208.5250)

Assumptions

□ 5-dim. Spherical Symmetry
□ Dual-null coordinate

$$ds^2 = -2e^{-f(x^+,x^-)}dx^+dx^- + r^2(x^+,x^-)d\Omega_3^2 \quad (3)$$

conformal factor	$\Omega = \frac{1}{r}$	(4)
expansions	$\vartheta_\pm = 3\partial_\pm r = -3\Omega^{-2}\partial_\pm\Omega$	(5)
inaffinities	$\nu_\pm = \partial_\pm f$	(6)
momenta of ϕ	$p_\pm = r\partial_\pm\phi = \Omega^{-1}\partial_\pm\phi$	(7)
momenta of ψ	$\pi_\pm = r\partial_\pm\psi = \Omega^{-1}\partial_\pm\psi$	(8)

□ matter = normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$T_{\mu\nu} = T^{\psi}_{\mu\nu} + T^{\phi}_{\mu\nu} = \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \quad (9)$$

Field Equations in 5-Dim. Gauss-Bonnet gravity

$$\eta = \Omega^2\left(e^{-f} + \frac{2}{9}\vartheta_+\vartheta_-\right), \quad \tilde A = (\alpha_1 + 4\alpha_2\eta e^f), \quad B = \kappa^2 T_{+-} + e^{-f}\Lambda$$

x^+-direction ∂_+ | x^--direction ∂_-

$$\partial_\pm\Omega = -\frac{1}{3}\vartheta_\pm\Omega^2 \quad (1)$$
$$\partial_\pm\vartheta_\pm = -\nu_\pm\vartheta_\pm - \frac{1}{\tilde A\Omega}\kappa^2 T_{\pm\pm} \quad (2)$$
$$\partial_\pm\vartheta_\mp = \frac{1}{\tilde A\Omega}(-3\alpha_1\eta + B) \quad (3)$$
$$\partial_\pm f = \nu_\pm \quad (4)$$
$$\partial_\pm\nu_\mp = \frac{\alpha_1}{\tilde A}\left\{\eta - \frac{4(3\alpha_1\eta - B)}{3\tilde A}\right\} + \frac{(\kappa^2 T_{zz}\Omega^2 - \Lambda)}{\tilde A e^f} + \frac{8\alpha_2}{9\tilde A^3}\left\{e^f(3\alpha_1\eta - B)^2 - \kappa^4 T_{++}T_{--}\right\} \quad (5)$$
$$\partial_\pm\psi = \Omega\pi_\pm \quad (6)$$
$$\partial_\pm\phi = \Omega p_\pm \quad (7)$$
$$\partial_\pm\pi_\mp = -\frac{1}{6}\Omega\vartheta_\pm\pi_\mp - \frac{1}{2}\Omega\vartheta_\mp\pi_\pm - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \quad (8)$$
$$\partial_\pm p_\mp = -\frac{1}{6}\Omega\vartheta_\pm p_\mp - \frac{1}{2}\Omega\vartheta_\mp p_\pm - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \quad (9)$$

Energy-momentum tensor

$$T_{++} = \Omega^2(\pi_+^2 - p_+^2) \quad (10)$$
$$T_{--} = \Omega^2(\pi_-^2 - p_-^2) \quad (11)$$
$$T_{+-} = -e^{-f}(V_1(\psi) + V_2(\phi)) \quad (12)$$
$$T_{zz} = e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}(V_1(\psi) - V_2(\phi)) \quad (13)$$

Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons -- go to a Black Hole or Inflationary expansion

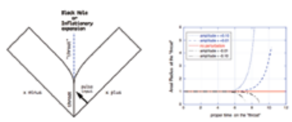


Figure 4: Partial Penrose diagram of the evolved space-time.
Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Ghost pulse input -- Bifurcation of the horizons

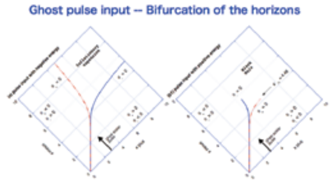


Figure 3: Horizon locations, $\vartheta_\pm = 0$, for perturbed wormhole. [...] Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

Travel through a Wormhole -- with Maintenance Operations!

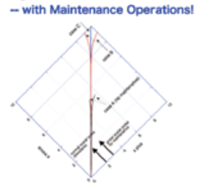
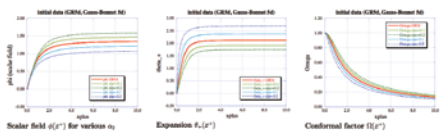


Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. [...]
(A) no maintenance case (results in a black hole).
(B) with maintenance pulse [...] (results in an inflationary expansion).
(C) with maintenance pulse [...] (keep stationary structure upto the end of this range).

WH evolution in 5-dim. GB

in prep.

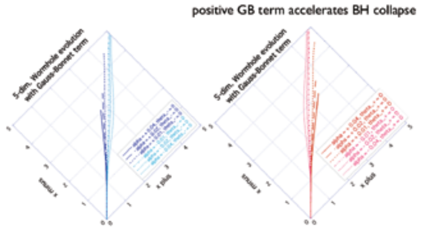
Wormholes in Gauss-Bonnet gravity (initial data on x^+)



Scalar field $\phi(x^+)$ for various α_2 | Expansion $\vartheta_+(x^+)$ | Conformal factor $\Omega(x^+)$

WH evolution in 5D Gauss-Bonnet gravity

positive GB term accelerates BH collapse



$$S = \int_M d^{N+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{GB}\} + \mathcal{L}_{matter}\right]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$