ワームホールの不安定性

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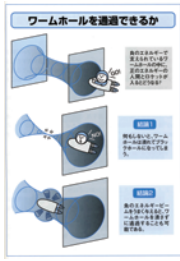
Outline & Summary

タイムマシンの原理としても有名な Morris-Thorne のワームホール解（Ellis WH 解）はゴーストスカラー場で構成される仮想的な一般相対論の解である．静的な仮定で導かれた解だが，著者らは動的に不安定であることを以前報告した【Shinkai & Hayward, Phys, Rev. D 66 (2002) 044005】.
今回は，高次元に拡張したり，宇宙項を入れたとしても同様に不安定であることを報告する．

我々は，n 次元に拡張した時空で Ellis WH 解を求め，摂動に対して不安定であることを見つけた【Torii & Shinkai, PRD88 (2013) 064023 】. 数値計算により，不安定であることを確かめ，WH の喉 (throat) は，与えるゆらぎのエネルギーの正負によって，ブラックホール (BH) に転じるか，あるいはインフレーション的に膨張することがわかった．BH に転じる場合，与えるゆらぎは小さくても形成される BH 質量には最小値が存在し，次元が上がれば最小値は大きくなる．宇宙項の正負によっても WH は BH または拡大する．Gauss-Bonnet 重力理論では，BH 形成は抑えられるようだが，定常解ではない．
総じて，このような単純なワームホールは不安定であると結論でき，観測するのは難しそうである．

Time Machine & Science of Space-time (HS, 2011)



Motivations

Why wormholes?

- They make great science fiction – short cuts between otherwise distant regions. Morris & Thorne 1988, Sagan "Contact" etc
- They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.
- They are very similar to black holes –both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole ≡ Hypersurface foliated by marginally trapped surfaces

- BH and WH are interconvertible?
  New duality?

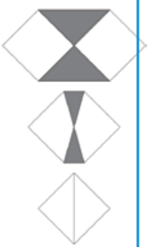
[1] How the stability changes in 5-d GR?
[2] How the stability changes in Gauss-Bonnet gravity?

BH and WH are interconvertible ? (New Duality?)

S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

- They are very similar – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)
- Only the causal nature of the THs differs, whether THs evolve in plus / minus density.

	Black Hole	Wormhole
Locally defined by	Achronal(spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally. but constructible ???



N-dim. Ellis Wormhole sol.

A Wormhole Solution (n-Dim, massless ghost scalar)

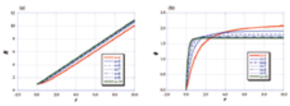
- spherical symmetry.

$$ds^2 = -f(t,r)e^{-2\delta(t,r)}dt^2 + f(t,r)^{-1}dr^2 + R(t,r)^2 d\Omega^2 \qquad (1)$$

- with massless ghost scalar field ϕ.
- static, $f \equiv 1$, and throat radius $R(0) = a$; Just solve

$$\frac{d^2R}{dr^2} = \frac{(n-3)a^{2(n-3)}}{R^{2n-5}} \qquad (2)$$

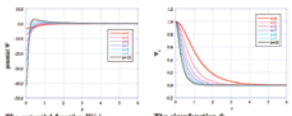
$$\frac{d\phi}{dr} = \sqrt{(n-2)(n-3)}\frac{a^{n-3}}{R^{n-2}} \qquad (3)$$



The n-dimensional wormhole solutions. The circumference radius R (a) and the scalar field ϕ (b) are plotted as a function of radial coordinate r. The cases of $n = 4$–10 are shown.

Torii & HS, PRD88 (2013), 064023

Perturbation Analysis



The potential function $W(r)$. The eigenfunction Φ_1.

n	ω^2
4	−1.397032433715108
5	−2.984358930277901
6	−4.686020542904600
7	−6.462584141263180
8	−8.289759363062103
9	−10.15355304518670
10	−12.04426501474384

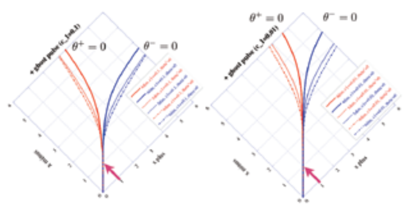
Obtained negative eigenvalues of ω^2. ⟹ Unstable.

摂動に対して不安定なモードが必ず存在する

WH evolution in 4, 5, 6-dim. GR

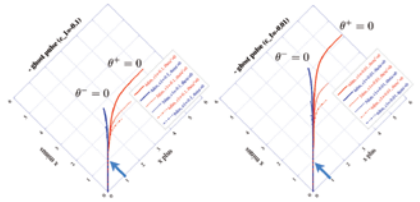
in prep.

4d 5d 6d GR
ghost pulse (additional amp.) input



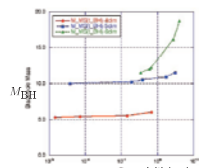
negative energy input --> throat inflates

4d 5d 6d GR
ghost pulse (subtract amp.) input



positive energy input --> BH formation

Minimum Mass of Black Hole



Quasi-Local Energy (Misner-Sharp mass in N-dim.)

Using the area of the unit n-dimensional space of constant curvature, V_n, (or the associated area $A_n = V_n r^n$, E_n may be defined as (cf Maeda & Nozawa, 2008)

$$E_n = \frac{(n-2)A_{n-2}}{2\kappa_n^2}\Omega\left[-\frac{1}{\Omega^2}\hat{k} + \left(k + \frac{2}{(n-2)^2}e^{f}\vartheta_+\vartheta_-\right)\right]$$

Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons
-- go to a Black Hole or Inflationary expansion

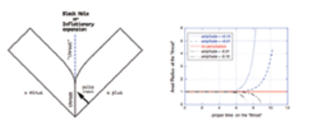


Figure 4: Partial Penrose diagram of the evolved space-time.
Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Ghost pulse input -- Bifurcation of the horizons

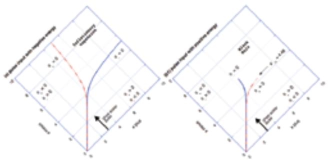


Figure 3: Horizon locations, $\vartheta_\pm = 0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

Travel through a Wormhole
-- with Maintenance Operations!

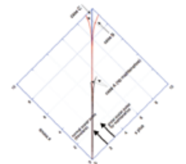


Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. The travellers pulse are commonly expressed with a normal scalar field pulse, $(\beta_c, \beta_s, \beta_r) = (+0.1, 6.0, 2.0)$. Horizon locations $\vartheta_+ = 0$ are plotted for three cases:
(A) no maintenance case [results in a black hole],
(B) with maintenance pulse of $(c_a, c_b, c_c) = (0.02300, 6.0, 3.0)$ [results in an inflationary expansion],
(C) with maintenance pulse of $(c_a, c_b, c_c) = (0.02295, 6.0, 3.0)$ [keep stationary structure upto the end of this range].

Field Eqs.

Gauss-Bonnet gravity

- Action

$$S = \int_M d^{N+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\right] \qquad (1)$$

where $\mathcal{L}_{\rm GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field eqs.

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu} \qquad (2)$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^\alpha{}_\nu - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_\mu{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{\rm GB}$

Assumptions

- ☐ 5-dim. Spherical Symmetry
- ☐ Dual-null coordinate

$$ds^2 = -2e^{-f(x^+,x^-)}dx^+dx^- + r^2(x^+,x^-)d\Omega_3^2 \qquad (3)$$

conformal factor	$\Omega = \frac{1}{r}$	(4)
expansions	$\vartheta_\pm = 3\partial_\pm r = -3\Omega^{-2}\partial_\pm\Omega$	(5)
inaffinities	$\nu_\pm = \partial_\pm f$	(6)
momenta of ϕ	$\varphi_\pm = r\partial_\pm\phi = \Omega^{-1}\partial_\pm\phi$	(7)
momenta of ψ	$\pi_\pm = r\partial_\pm\psi = \Omega^{-1}\partial_\pm\psi$	(8)

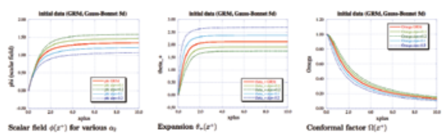
- ☐ matter = normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$T_{\mu\nu} = T^{\psi}_{\mu\nu} + T^{\phi}_{\mu\nu} = \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \qquad (9)$$

WH evolution in 5-dim. GB

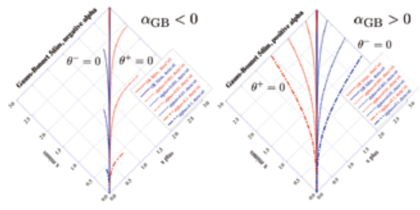
in prep.

Wormholes in Gauss-Bonnet gravity (initial data on x^+)



Scalar field $\phi(x^+)$ for various α_2 Expansion $\vartheta_+(x^+)$ Conformal factor $\Omega(x^+)$

5d GR vs Gauss-Bonnet instability appears



BH formation throat inflates

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