直角三角形と円と正方形を用いた標準正規分布の回転対称性

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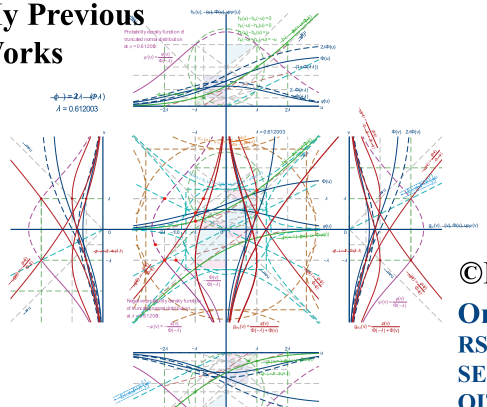
Rotationally Symmetric Relations of Standard Normal Distribution Using Right Triangle, Circle, and Square

First Author, Presenter, and Designer about these researches:
© Shingo NAKANISHI (Osaka Inst. of Tech.).

Co-author: Masamitsu OHNISHI (Osaka Univ.)

My Previous Works



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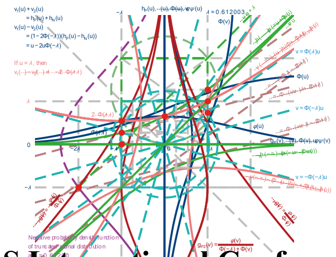

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Original References by the First Author
RSS2019 (ICC Belfast)
SETA2019（Osaka Univ.）
OIT Innovation Days 2019 (Osaka Inst. of Tech.)
RIMS2078-10 and These Modified Versions with comments on Nov. 14, 2018
TORSJ(55, 1-26, 2012), CIE2007
Presentations of ORSJ, EURO2013, EURO2016, and IFORS2014

The Replication of Posters:
http://www.oit.ac.jp/center/~nakanishi/RSS-OIT(A0x2)2019-11-23.pdf

Key Points:

The Aspect ratio which is 1.0 informs us of a lot of geometric characterizations and symmetric relations from now.

<u>During our presentation, please remember the following conditions.</u>

Karl Pearson’s finding probability point is

$\lambda = 0.612003.$

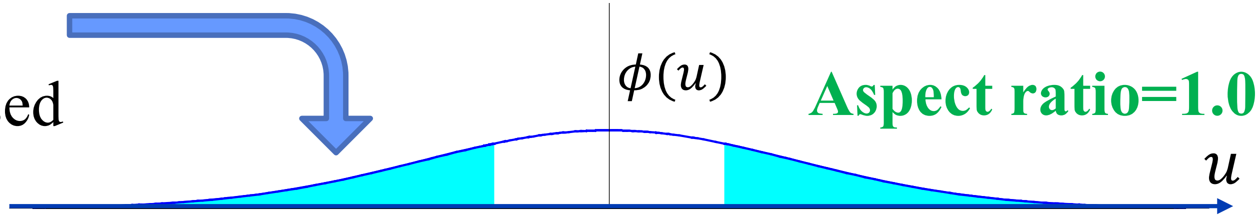
Its cumulative probability is

$\Phi(-\lambda) = 0.2702678.$

From these values, Kelley proposed

$\phi(\lambda) = 2\lambda\Phi(-\lambda) = 0.3308.$

Cox also confirmed the clustering about 3 groups of normal distribution on λ.



(URL: http://www.oit.ac.jp/center/~nakanishi/)

Probability Points:

1. $k = \pm 0.0 \because \Phi(-k) = \Phi(k) = 1/2$
2. $k = \pm 0.30263084 \because \phi(k)/\Phi(-k) = 1$
3. $k = \pm 0.506054 \because k = \phi(k)/\Phi(k)$
4. $k = \pm 0.612003 \because \phi(k) = 2k\Phi(-k)$
5. $k = \pm \mathbf{0.67449} \because \Phi(-k) = 1/4, \Phi(k) = 3/4$

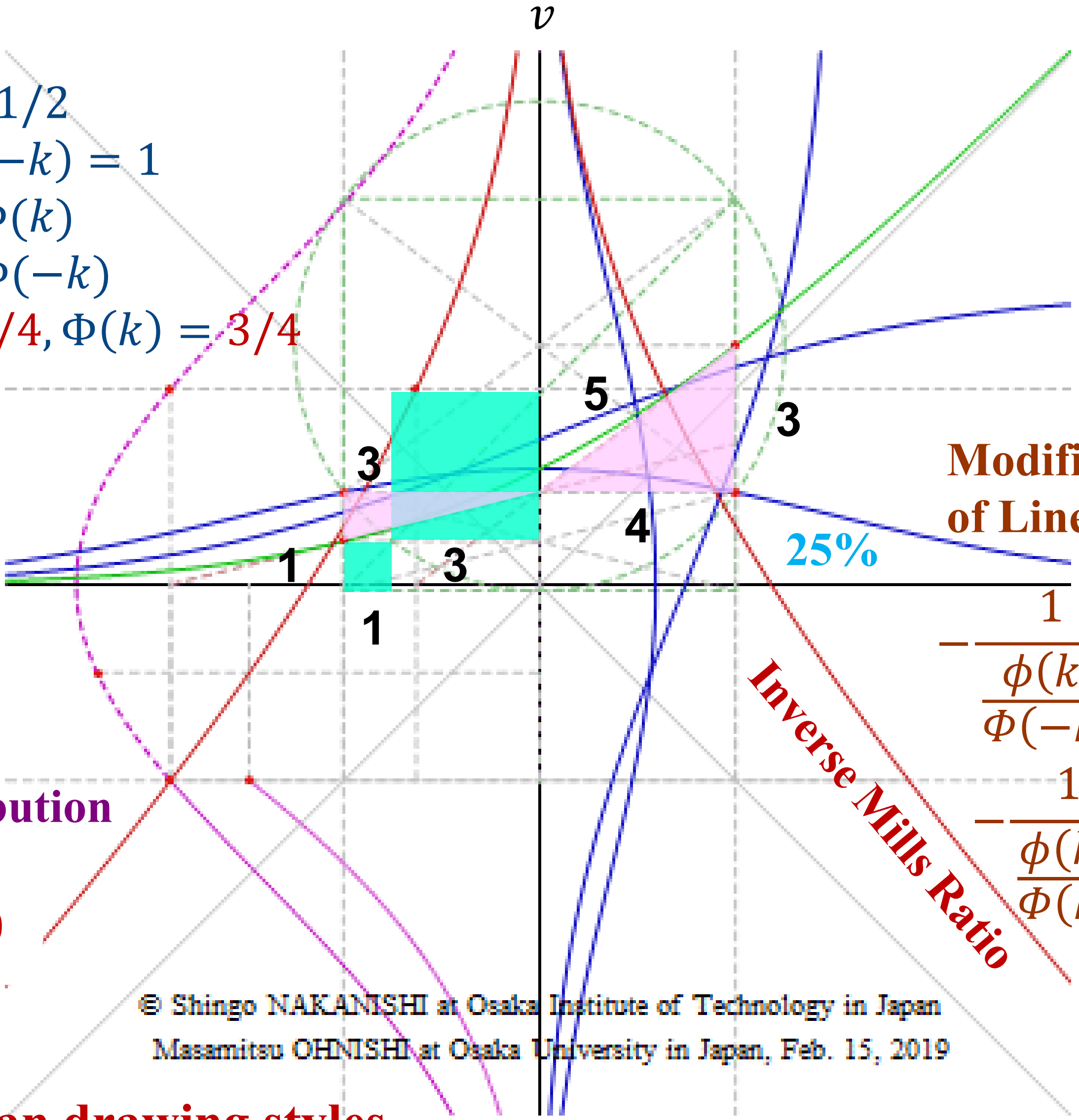
Integral form of cumulative distribution is $h_P(u) = \phi(u) + u\Phi(u)$,

$$\frac{d^2 h_P(u)}{du^2} + u\frac{dh_P(u)}{du} - h_P(u) = 0$$

25% : 75%=1/4 : 3/4

Negative Inverse Mills Ratio is

$$-g_P(u) = -\frac{\phi(u)}{\Phi(u)}$$



Modified Intercept Forms of Linear Equations:

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u + \frac{1}{\phi(k)}v = 1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Truncated Normal Distribution

$$\frac{dg_P(u)}{du} + ug_P(u) + g_P(u)^2 = 0$$

References
ORSJ
(Chiba Institute of Tech.)
(Tokyo Institute of Tech.)
SETA2019（Osaka Univ.）
RSS2019(ICC Belfast)

From the antient Egyptian drawing styles, we can create the harmonies between standard normal distribution, inverse Mills ratio, and Linear Intercept Forms based on the Greek Pythagorean theorem.

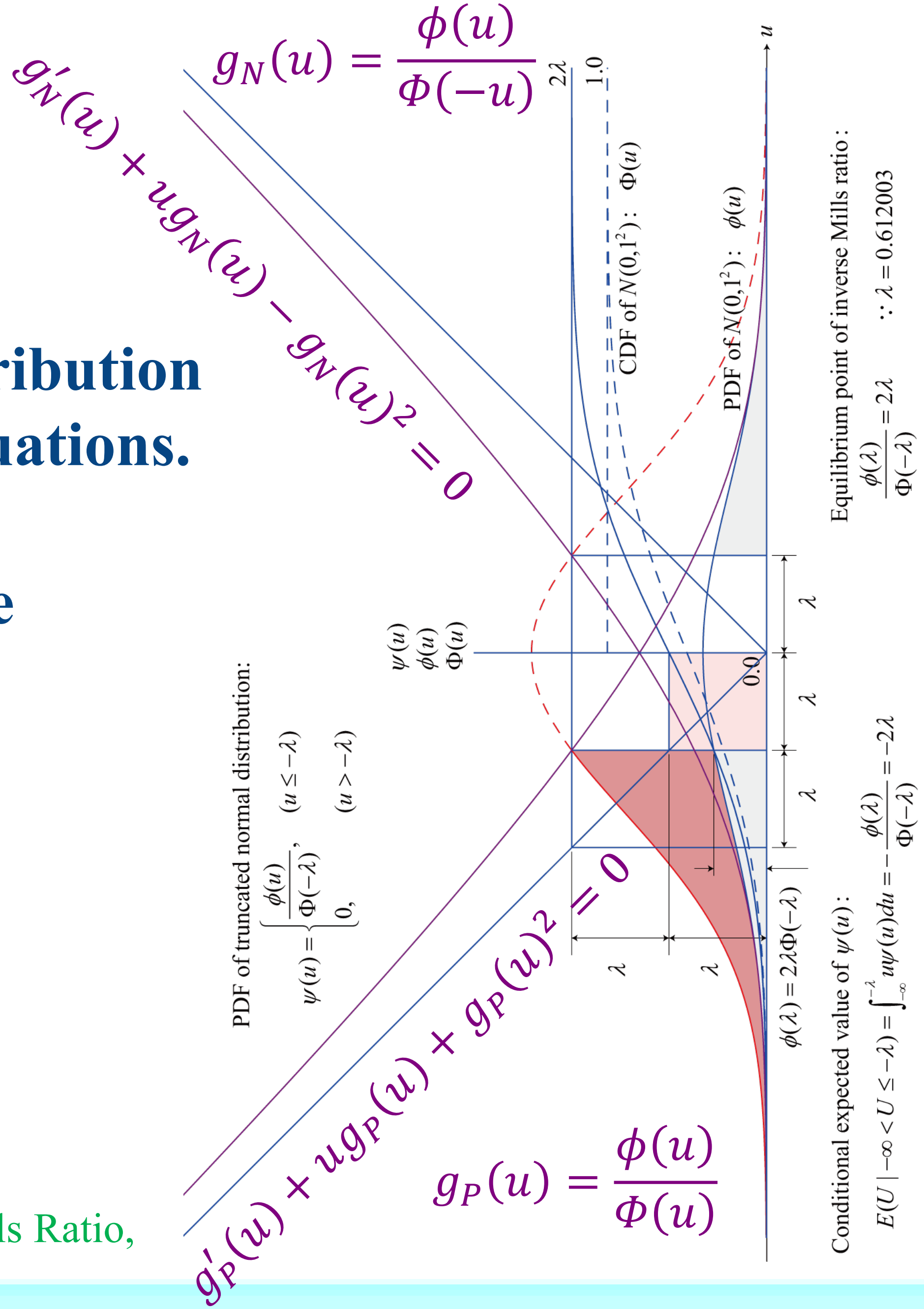
Symmetric Beauty between Squares and Standard Normal Distribution at 0.612003

: Each cumulative probability is about 27 percent.

: Truncated cumulative probability is 100 percent.

Relations of Squares about Truncated Normal Distribution With $\lambda = 0.612003$ and Bernoulli Differential Equations.

Differential Equations about Inverse Mills Ratio are Bernoulli Differential Equations.



References.

My Doctoral Thesis, March 2015, No.17777 (Osaka Univ.) about Square
ORSJ 2016 Spring (Keio Univ.) about Square
EURO2016(Poznan Univ. of Tech.) about Square
ORSJ 2017 Fall (Kansai Univ.) about Square
RIMS2078（Kyoto Univ.） about Bernoulli Differential Equations
SETA2019（Osaka Univ.） about Bernoulli Differential Equations
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Refs.
Hald, A., 1949, about the Bernoulli differential equation of inverse Mills Ratio,
Isa, K., 2011 in Japan about illustrations of inverse Mills ratio

Second order linear differential equations and these tangent lines

$h_N''(u) + uh_N'(u) - h_N(u) = 0$

$h_N(u) = \phi(u) - u\Phi(-u)$

$h_N'(u) = -\Phi(-u)$

$h_N''(u) = \phi(u)$

$h_N(u), h_P(u), v$

$\phi(u), \Phi(u), \psi(u)$

$h_P''(u) + uh_P'(u) - h_P(u) = 0$

$h_P(u) = \phi(u) + u\Phi(u)$

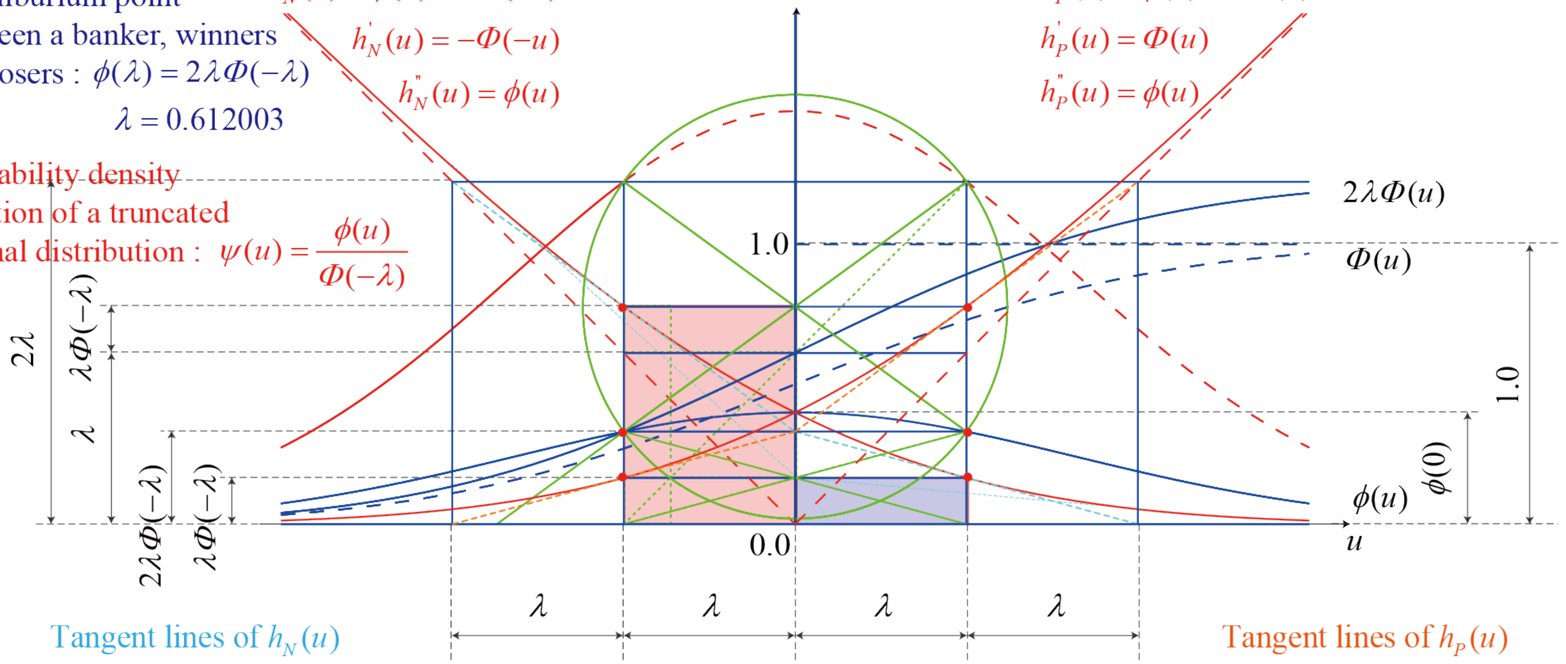
$h_P'(u) = \Phi(u)$

$h_P''(u) = \phi(u)$

Equiliburium point between a banker, winners and losers : $\phi(\lambda) = 2\lambda\Phi(-\lambda)$

$\lambda = 0.612003$

Probability density function of a truncated normal distribution : $\psi(u) = \dfrac{\phi(u)}{\Phi(-\lambda)}$



Tangent lines of $h_N(u)$

$v = -\Phi(\lambda)u + \phi(\lambda) = -\Phi(\lambda)u + 2\lambda\Phi(-\lambda) \quad (u = -\lambda)$

$v = -\Phi(-\lambda)u + \phi(\lambda) = -\Phi(-\lambda)u + 2\lambda\Phi(-\lambda) \quad (u = \lambda)$

Tangent lines of $h_P(u)$

$v = \Phi(-\lambda)u + \phi(\lambda) = \Phi(-\lambda)u + 2\lambda\Phi(-\lambda) \quad (u = -\lambda)$

$v = \Phi(\lambda)u + \phi(\lambda) = \Phi(\lambda)u + 2\lambda\Phi(-\lambda) \quad (u = \lambda)$

First derivative is the probability function: $\Phi(u)$.
Second derivative is the density probability function: $\phi(u)$.
These curves are combinations of both them.

Refs. in Japan by
Fumio Hashimoto *et al.*, 1985 about integrals of CDF,
And Takahiro Nagashima, 2005 about ordinally differential equations.

Refs.
RIMS2078（2017）
as the First Presentation
(Kyoto Univ.)
RIMS Modified Version2078-10
（2018）as Second Comments
at my OIT website
(Osaka Inst. of Tech.)
ORSJ (Kansai Univ.)
(National Grad. Inst. for Policy Studies)
(Chiba Inst. of Tech.)
(Tokyo Inst. of Tech.)
SETA2019 (Osaka Univ.)
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$$-\frac{g_P'(u)}{g_P(u)} = \frac{h_P(u)}{h_P'(u)} = \frac{m_P'(u)}{m_P(u)}$$

$$h_P''(u) + uh_P'(u) - h_P(u) = 0$$

$$m_P''(u) - um_P'(u) - m_P(u) = 0$$

$$g_P'(u) + ug_P(u) + g_P(u)^2 = 0$$

Several important probability points of our previous works.

Ex. $x = 0.0$, $x = 0.30263084$, $x = 0.506054$, and $\cdots$.

Refs.
ORSJ (National Grad. Inst. for Policy Studies)
(Chiba Inst. of Tech.)
(Tokyo Inst. of Tech.)
SETA2019 (Osaka Univ.)
RSS2019
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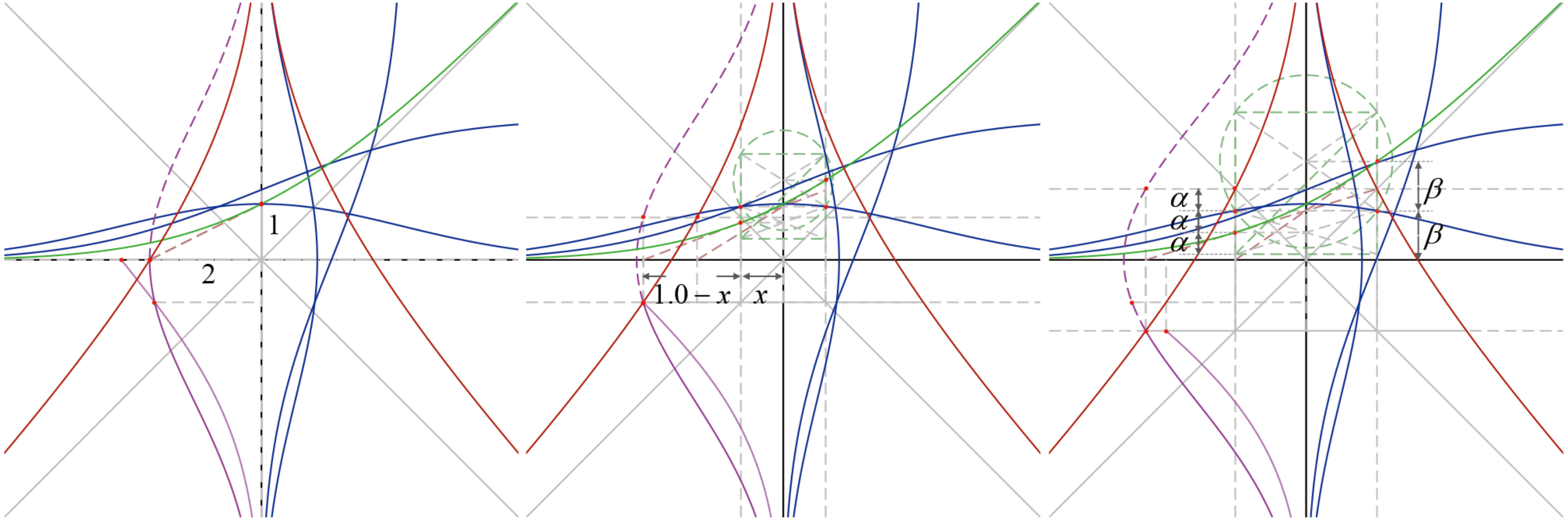


Probability point: $x = 0$

Probability point: $x = 0.30263084 (= \eta)$
$(\phi(x) = \Phi(-x))$
$\phi(x) / \Phi(-x) = 1$

Probability point: $x = 0.506054$ $(= \phi(x) / \Phi(x))$
$\Phi(x) = 0.693591$
$\Phi(-x) = 0.306409$
$\alpha = x\Phi(-x)$
$\beta = x\Phi(x) = \phi(x)$
$x = \phi(x) / \Phi(x) = x = \alpha + \beta$

Probability Points:

1. $k = \pm 0.0 \because \Phi(-k) = \Phi(k) = 1/2$
2. $k = \pm 0.30263084 \because \phi(k)/\Phi(-k) = 1$
3. $k = \pm 0.506054 \because k = \phi(k)/\Phi(k)$
4. $k = \pm 0.612003 \because \phi(k) = 2k\Phi(-k)$
5. $k = \pm 0.67449 \because \Phi(-k) = 1/4, \Phi(k) = 3/4$

The circle is widely spreading little by little according to the probability point from 0 to ∞.
Of course, 0.612003 is also the most important probability point.

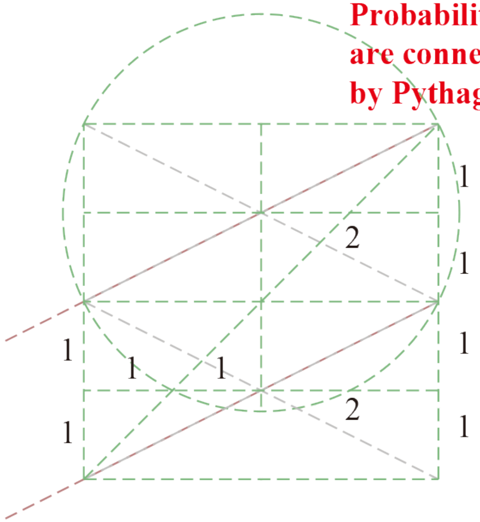
Half

Quantile =Half × Half

What are ½ and ¼ about standard normal distribution? Are only half and quantile points?

All points whithin the circle and square are converging at one point practically because their pobability points are 0.

Probabilities of standard normal distribution are connected to the right triangle by Pythagorean theorm geometrically.



Probability point : $x = 0$

$\Phi(x) = 1 - \Phi(-x) = 0.5$

$\Phi(-x) = 0.5$

Proportion of right triangle=1:2:$\sqrt{5}$

$\Phi(-x) : \Phi(x) = 0.5 : 0.5 = 1 : 1$

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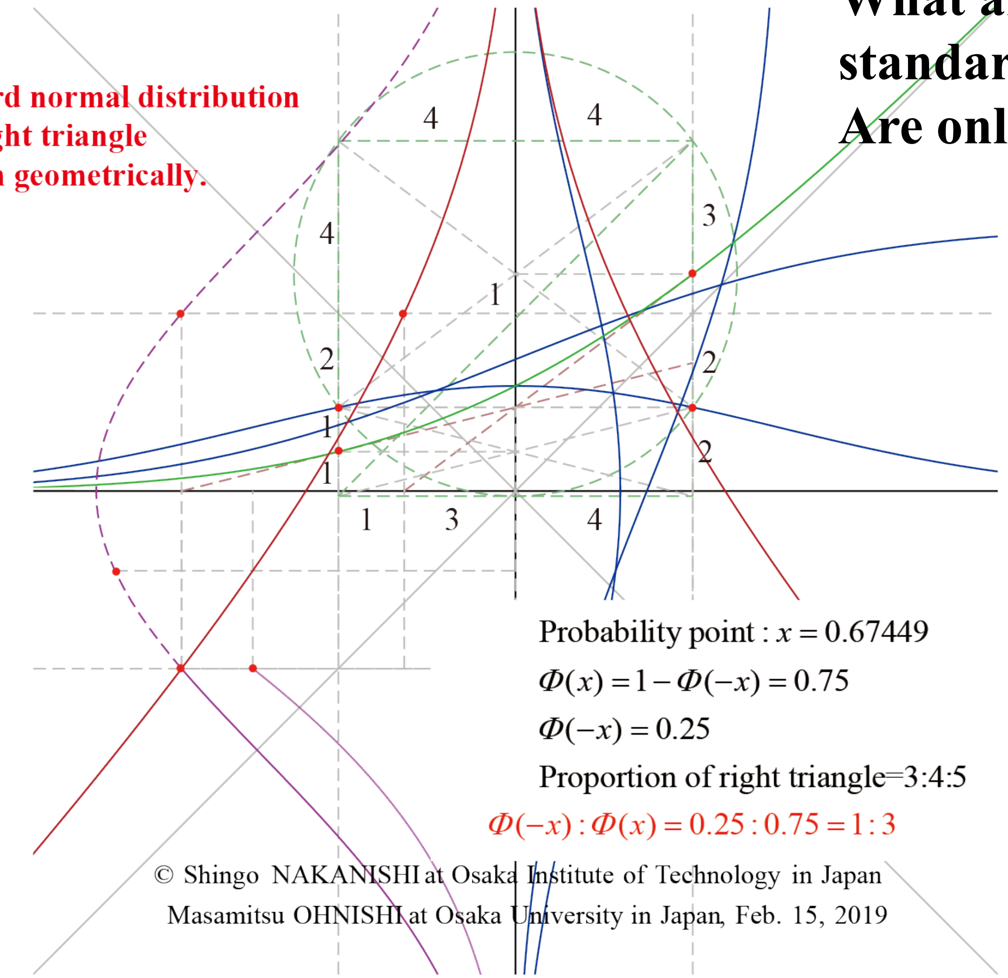
Masamitsu OHNISHI at Osaka University in Japan, Feb. 15, 2019



Probability point : $x = 0.67449$

$\Phi(x) = 1 - \Phi(-x) = 0.75$

$\Phi(-x) = 0.25$

Proportion of right triangle=3:4:5

$\Phi(-x) : \Phi(x) = 0.25 : 0.75 = 1 : 3$

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Masamitsu OHNISHI at Osaka University in Japan, Feb. 15, 2019

Refs.
RIMS2078（2017）
as the First Presentation (Kyoto Univ.)
RIMS Modified Version2078-10（2018）as Second Comments at my OIT website (Osaka Inst. of Tech.)
ORSJ (National Grad. Inst. for Policy Studies) (Chiba Inst. of Tech.) (Tokyo Inst. of Tech.) (Higashi Hiroshima)
SETA2019 (Osaka Univ. & Osaka Inst. of Tech.)

Probability point is **0.0** as ½.

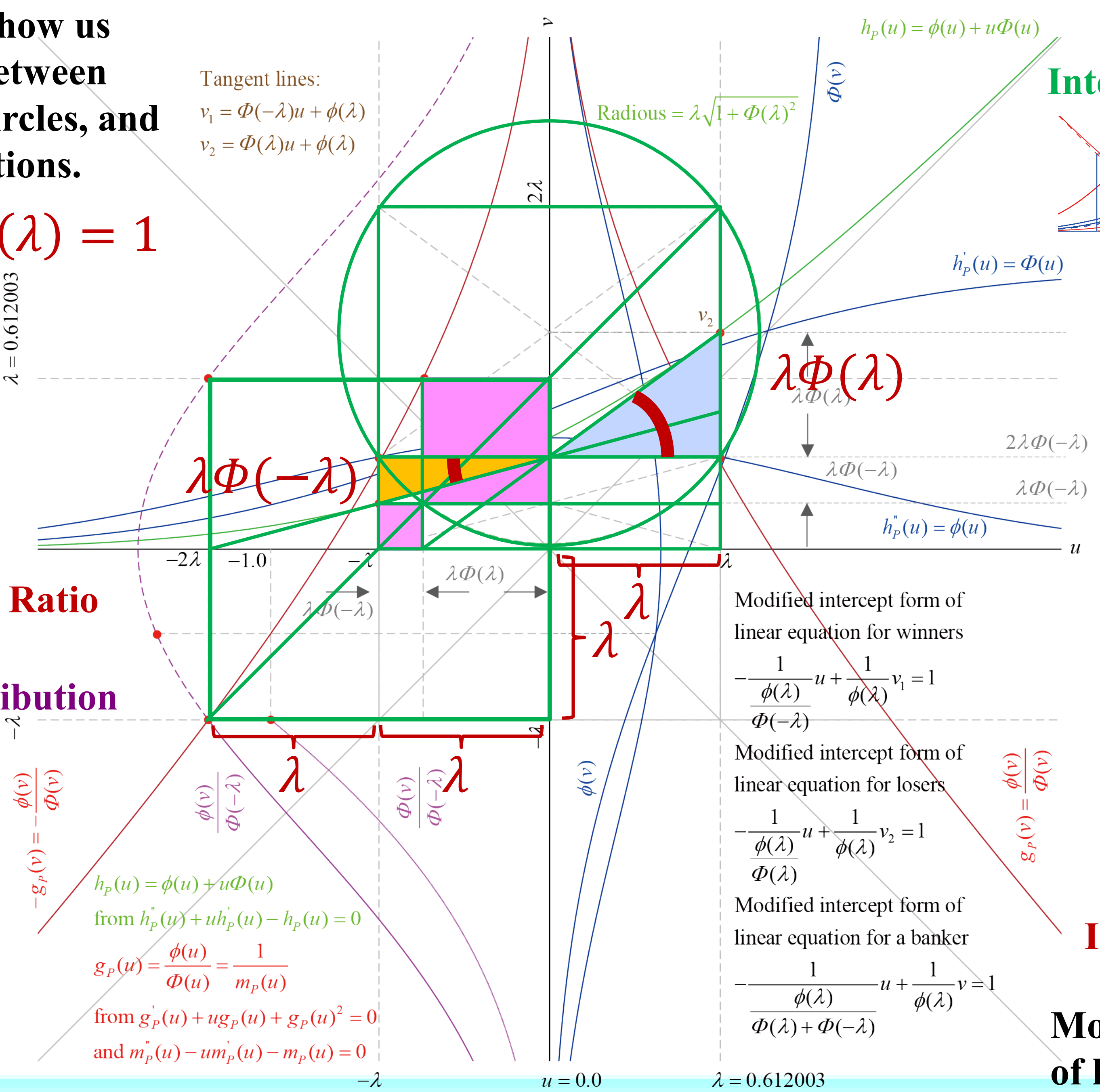
The idea about the folds of distance
from the advice by Prof. Kohji Kamejima at OIT

Probability point is **0.67449** as ¼ .

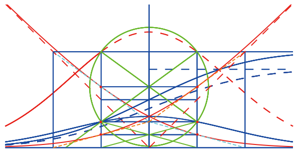
The idea about the small difference with beauty
from the advice by Prof. Hidemasa Yoshimura at OIT

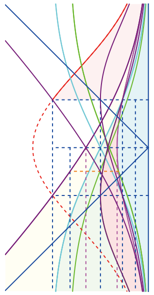
Egyptian drawing styles show us the symmetric relations between right triangles, squares, circles, and standard normal distributions.

$$\Phi(-\lambda) + \Phi(\lambda) = 1$$



Integrals of CDF





Negative Inverse Mills Ratio

Truncated Normal Distribution

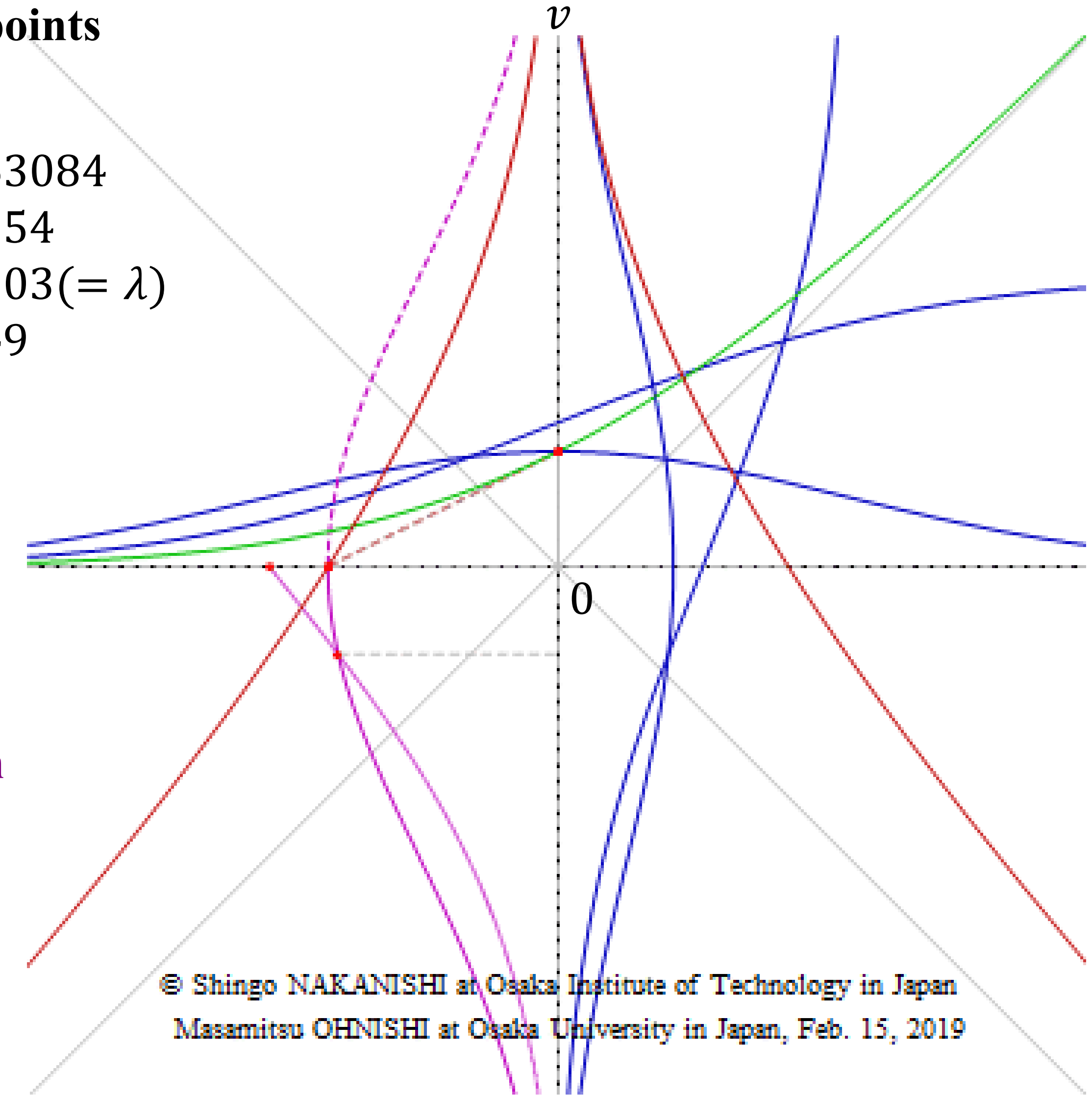
Refs.
ORSJ (Chiba Inst. of Tech.)
(Tokyo Inst. of Tech.)
SETA2019 (Osaka Univ.)
RSS2019 (ICC Belfast, UK)

Inverse Mills Ratio

Modified Intercept forms of linear equations

k =**Probability points**

1. $k = \pm 0.0$
2. $k = \pm 0.30263084$
3. $k = \pm 0.506054$
4. $k = \pm 0.612003 (= \lambda)$
5. $k = \pm 0.67449$



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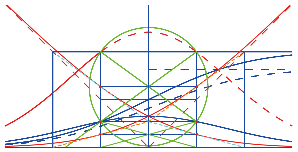
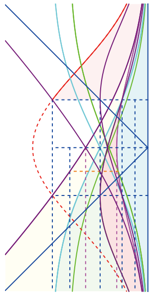


Modified intercept forms of linear equations according to k

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u + \frac{1}{\phi(k)}v = 1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Inverse Mills Ratio



Negative Inverse Mills Ratio

Truncated Normal Distribution

Refs.
ORSJ (Chiba Inst. of Tech.)
(Tokyo Inst. of Tech.)
SETA2019 (Osaka Univ.)
RSS2019 (ICC Belfast, UK)

With two tangent lines of the green solid lines on two probability points.

k =**Probability points**

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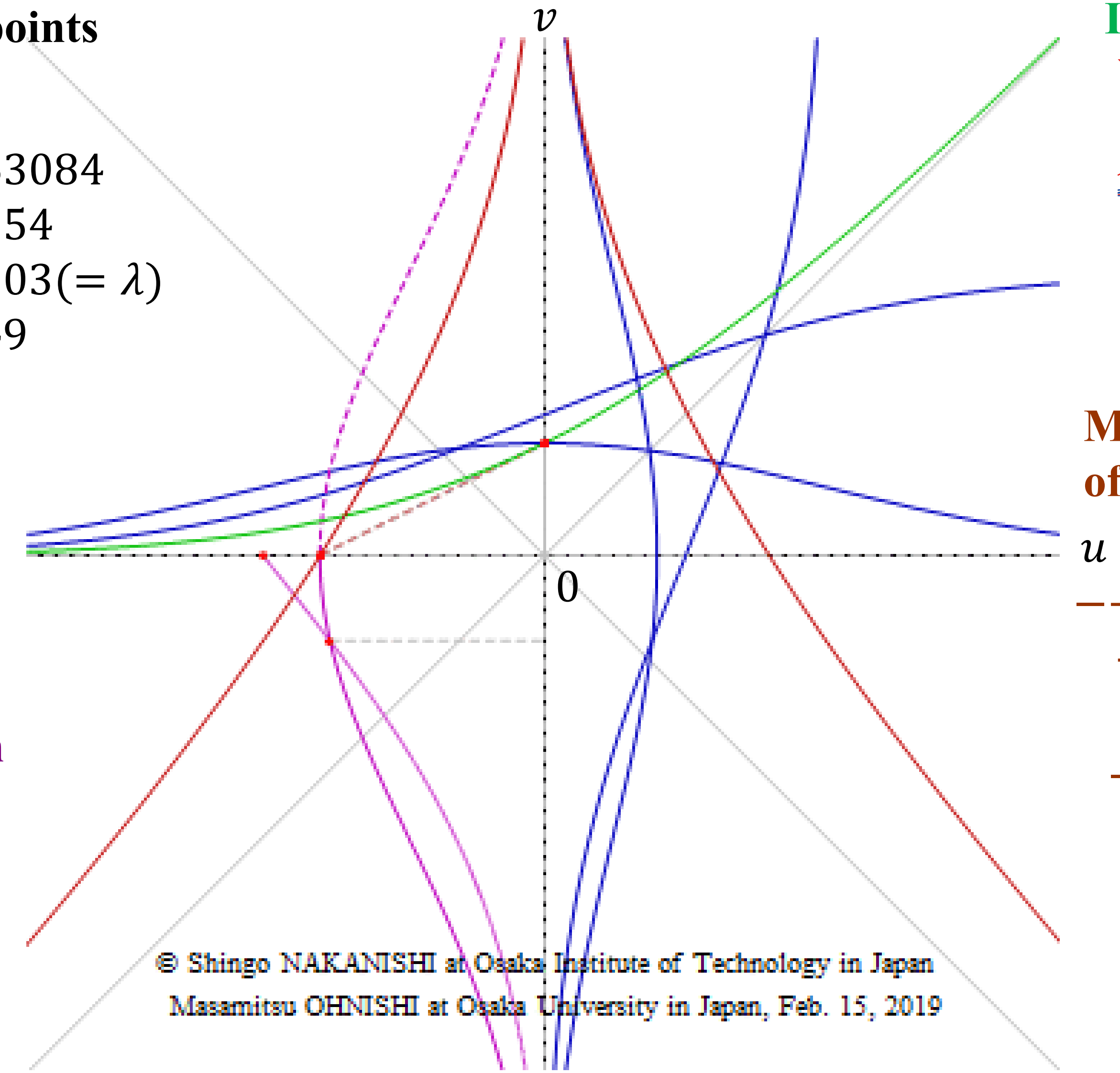
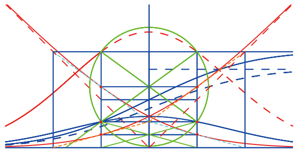


Integrals of CDF



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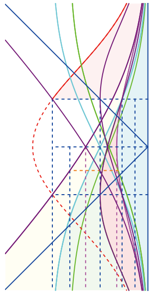
Inverse Mills Ratio



Negative Inverse Mills Ratio

Truncated Normal Distribution

Refs.
ORSJ (Chiba Inst. of Tech.)
(Tokyo Inst. of Tech.)
SETA2019 (Osaka Univ.)
RSS2019 (ICC Belfast, UK)

With two groups of parallel lines between golden dashed lines and silver dashed lines based on the circle.

k =**Probability points**

1. $k = \pm 0.0$
2. $k = \pm 0.30263084$
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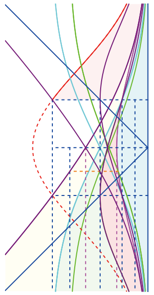
The proportion : $\Phi(-k) : \Phi(k)$



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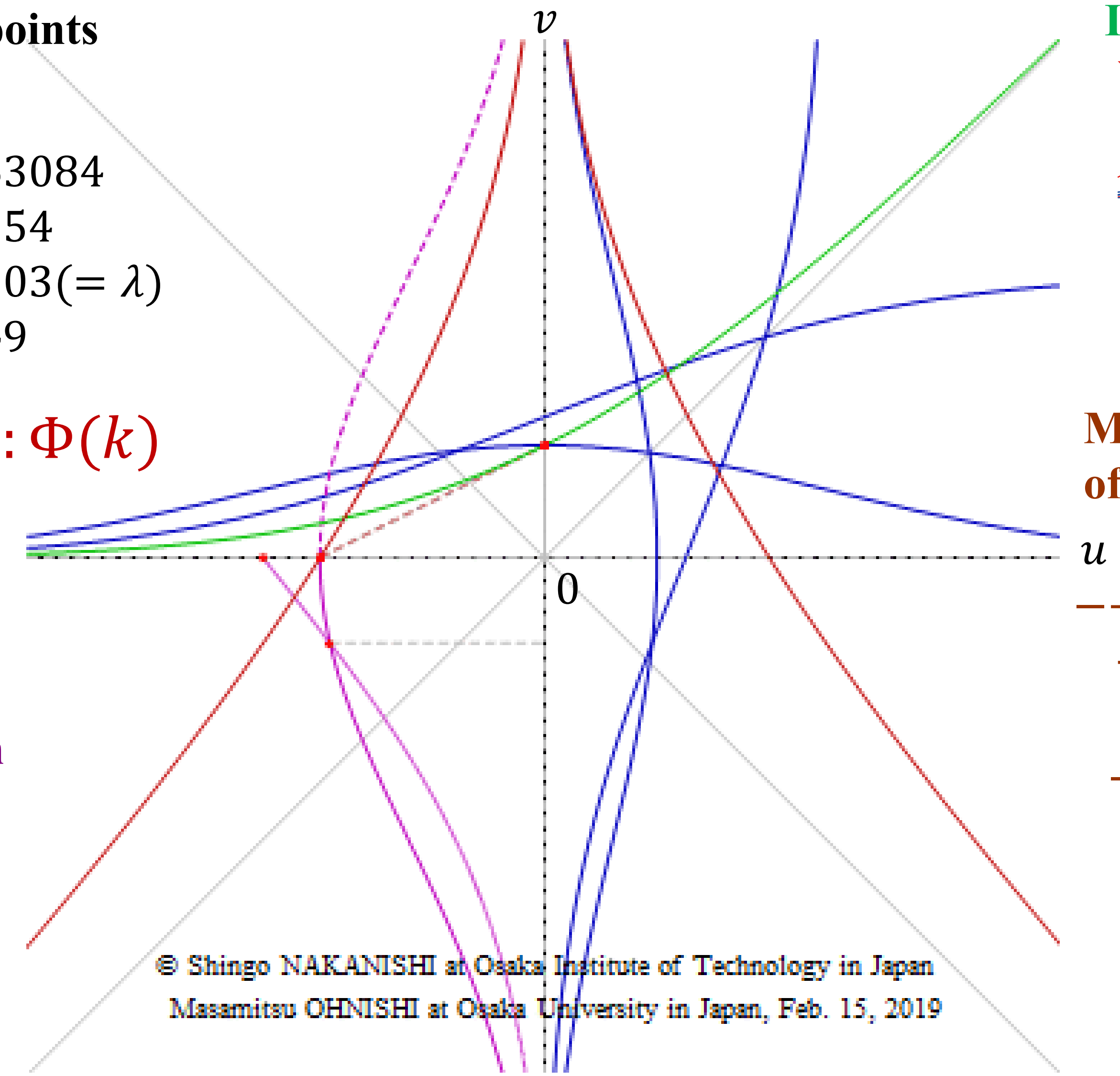
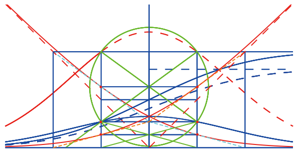


Integrals of CDF



Modified intercept forms of linear equations according to k

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u + \frac{1}{\phi(k)}v = 1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Inverse Mills Ratio

With a circle and a square. The bottom line is located on the horizontal axis at the probability point $k = 0.612003$.

k =Probability points

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4. $k = \pm 0.612003 (= \lambda)$
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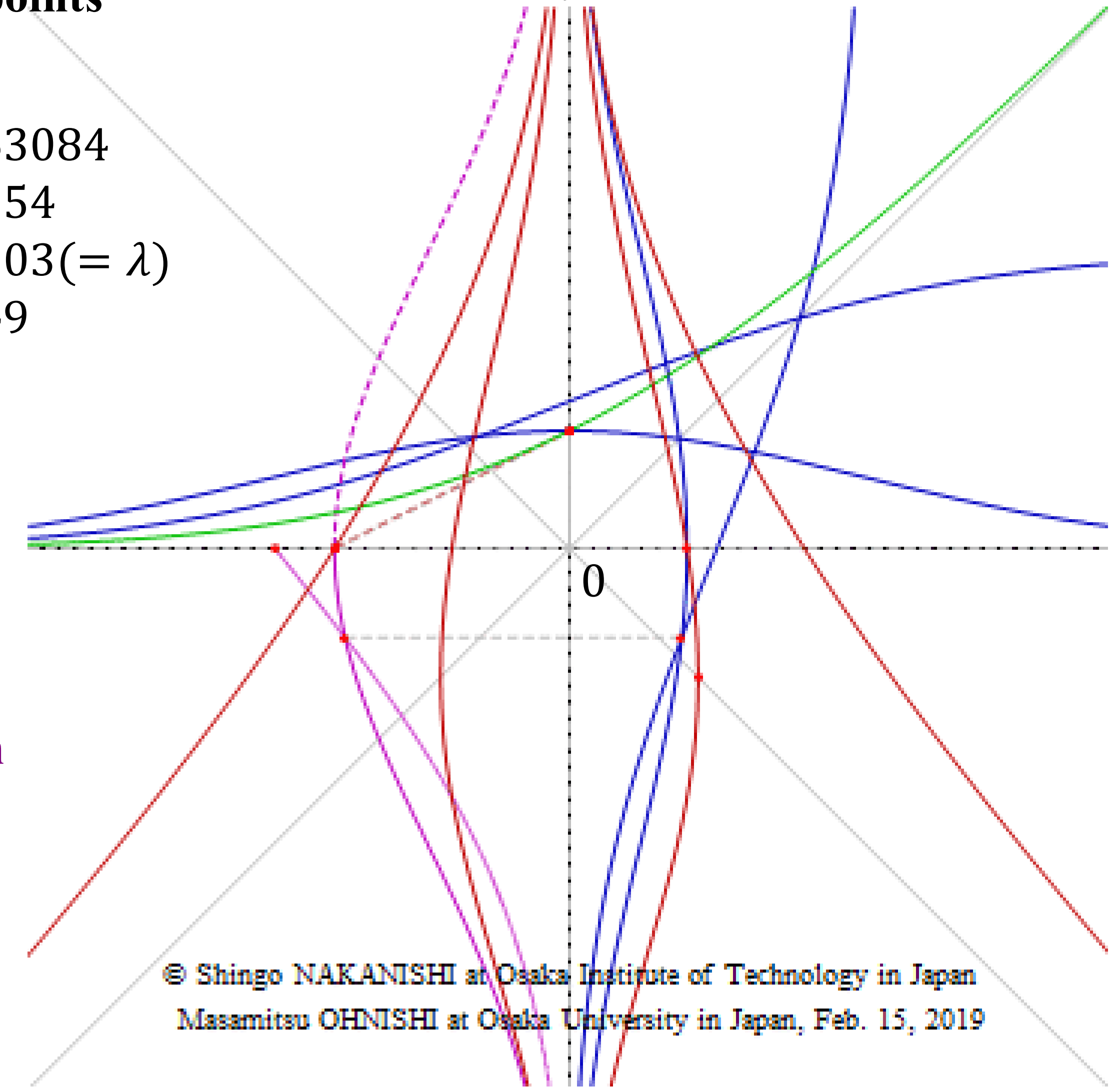

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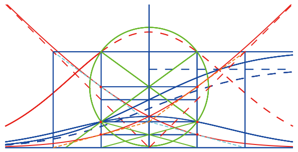

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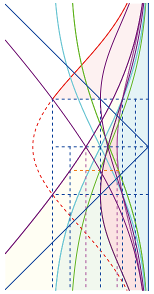
Inverse Mills Ratio



Negative Inverse Mills Ratio

Truncated Normal Distribution

Refs.
ORSJ (Chiba Inst. of Tech.)
(Tokyo Inst. of Tech.)
SETA2019 (Osaka Univ.)
RSS2019 (ICC Belfast, UK)

$\phi(k) = \min \Phi(k)h_P(u) + \Phi(-k)h_N(u)$

k =**Probability points**

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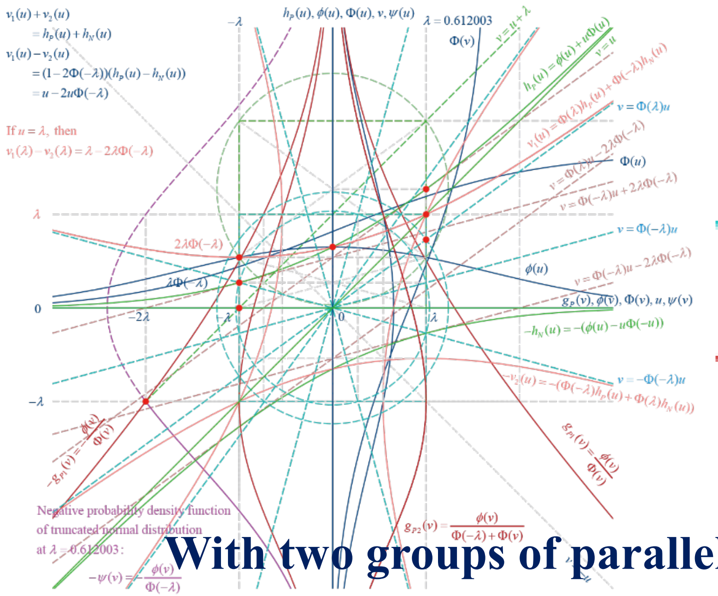
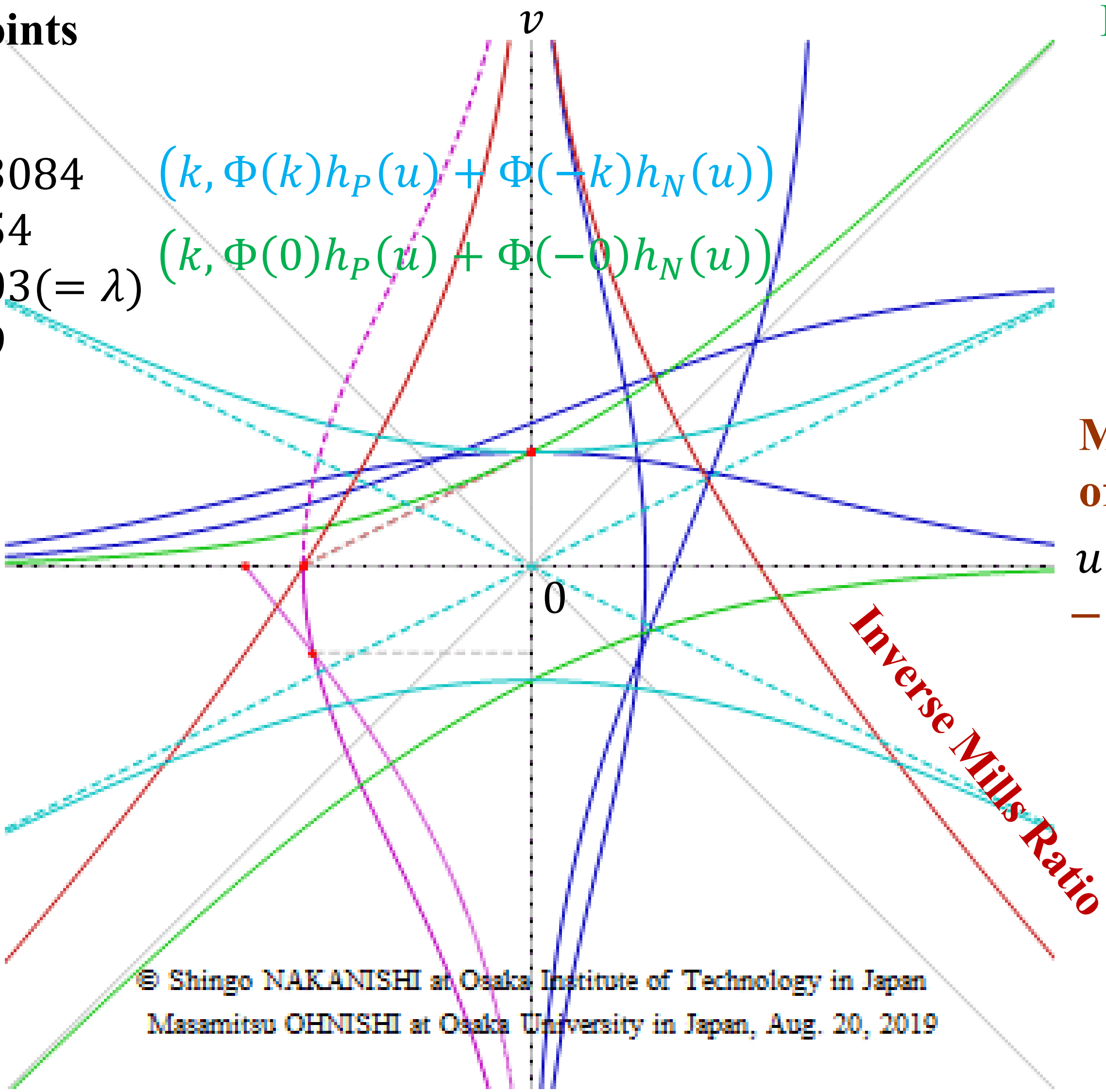
5. $k = \pm 0.67449$

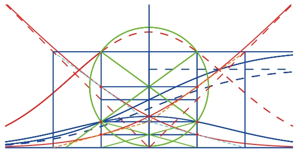
Ref. RSS2019(ICC Belfast)

Special case as the geometric characterizations and rotationally symmetric relations between winners, losers, and their banker based on the condition: $\lambda = 0.612003$ and $\Phi(-\lambda) = 0.2702678$.





Integrals of CDF



Modified intercept forms of linear equations according to k

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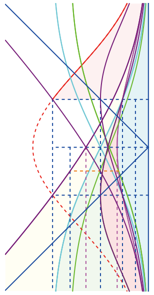
$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Refs.

ORSJ (Higashi Hiroshima.)

RSS2019 (ICC Belfast, UK)

With two groups of parallel lines between cyan, golden, and silver dashed lines based on the circle.



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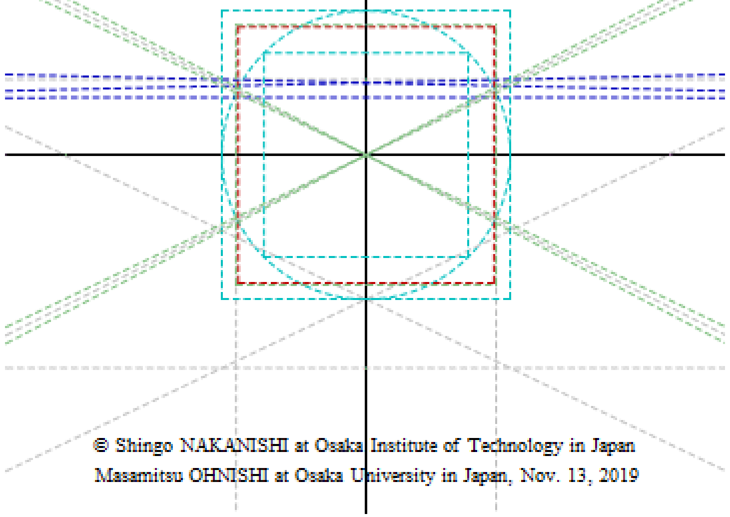
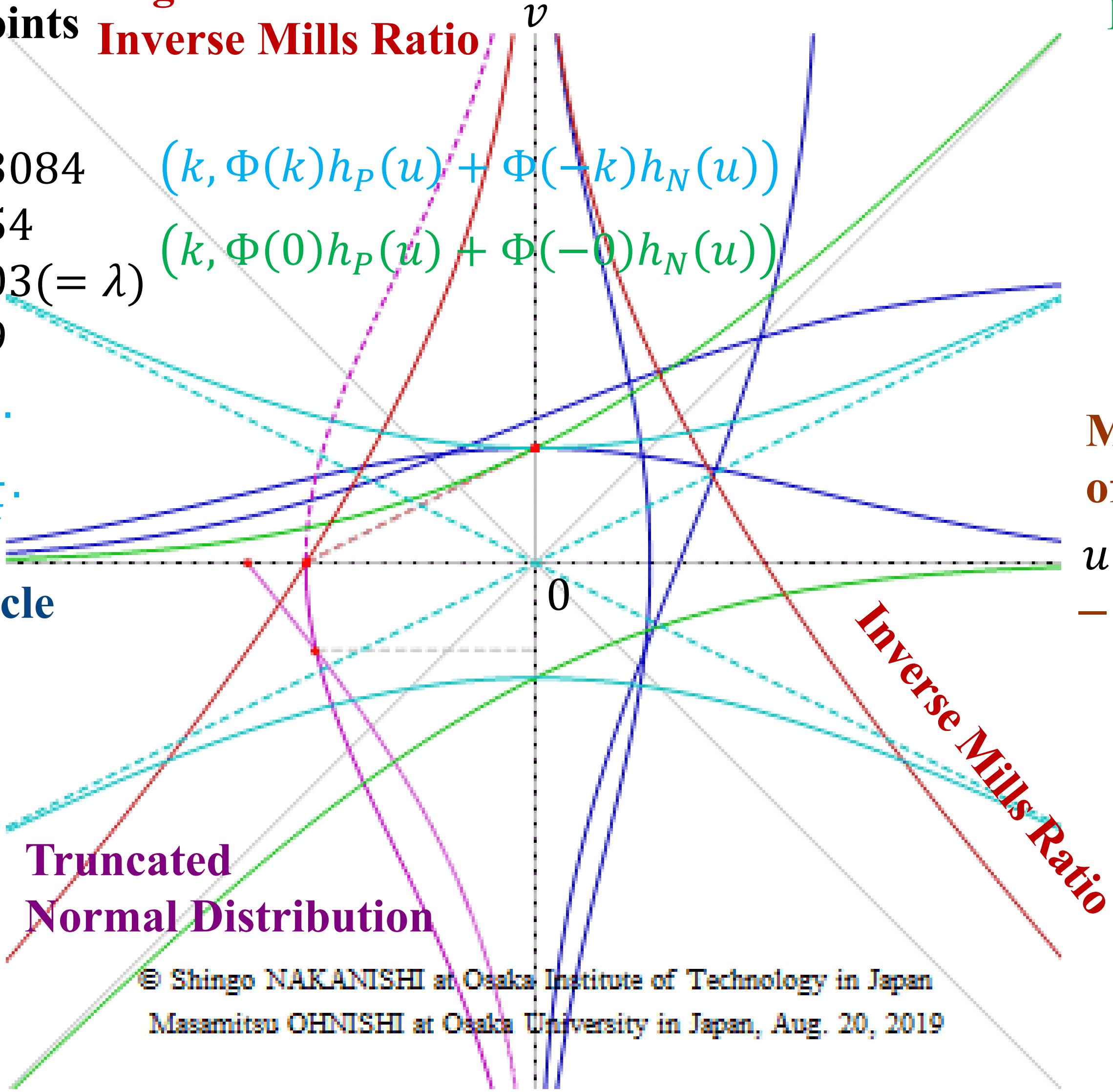
4. $k = \pm 0.612003 (= \lambda)$
5. $k = \pm 0.67449$

The area of the large cyan square is $\frac{4}{\pi}$.

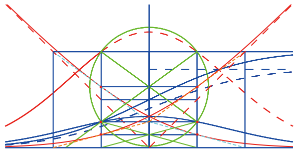
The area of the small cyan square is $\frac{2}{\pi}$.

The idea based on squaring the circle from $\pi/4$ to $\pi/2$.





Integrals of CDF



Modified intercept forms of linear equations according to k

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u + \frac{1}{\phi(k)}v = 1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Refs.

ORSJ (Higashi Hiroshima.)

RSS2019 (ICC Belfast, UK)

The probability points based on k get the optimal values $\phi(k)$.
And these tendencies are related to the Squaring the Circle.

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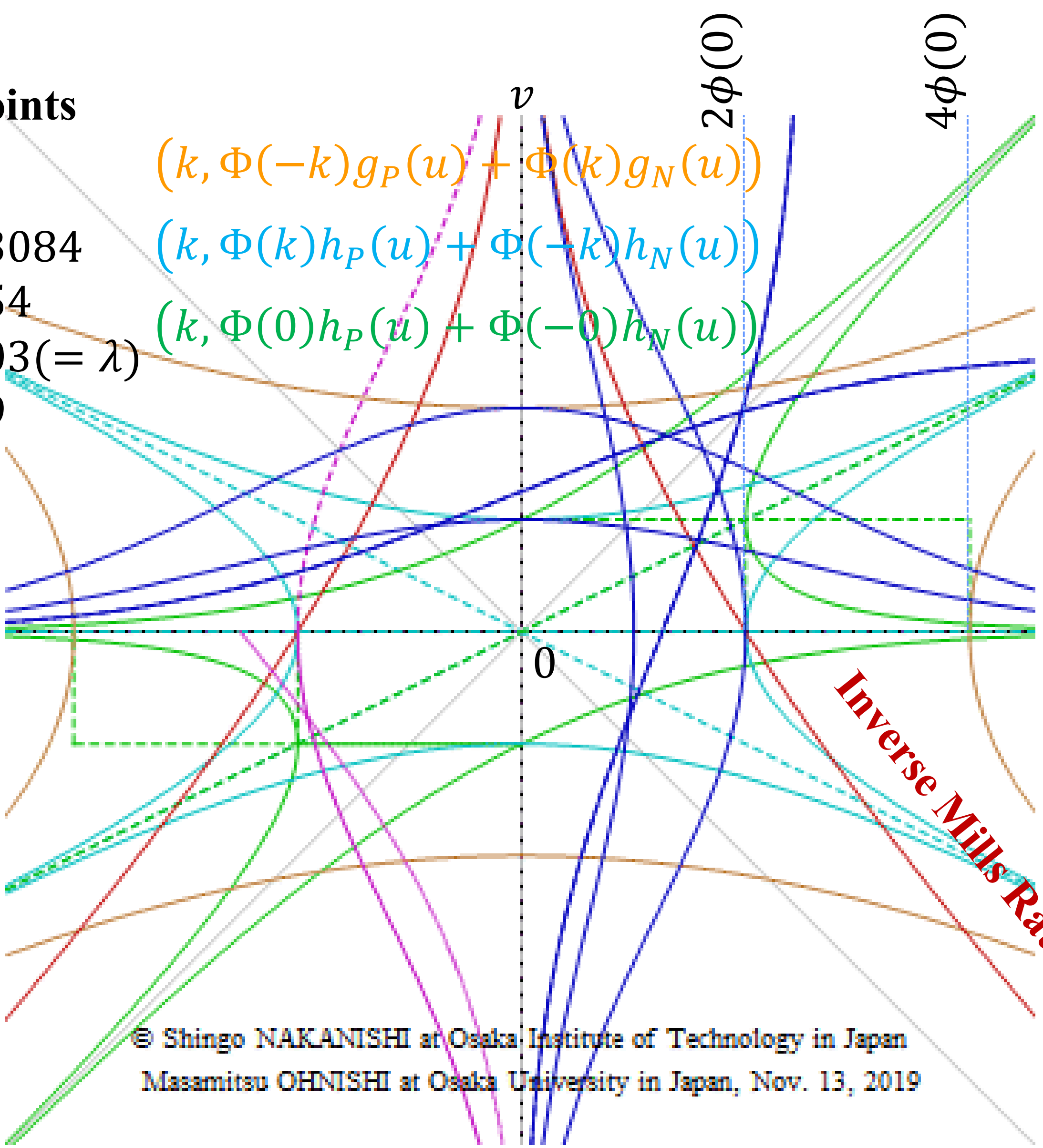
1. $k = \pm 0.0$
2. $k = \pm 0.30263084$
3. $k = \pm 0.506054$
4. $k = \pm 0.612003 (= \lambda)$
5. $k = \pm 0.67449$

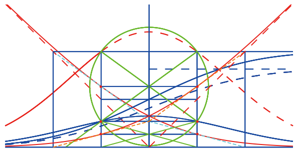
$\left(g_P(\mathrm{u}) + g_N(\mathrm{u}), \Phi(-k)g_P(u) - \Phi(k)g_N(u)\right)$

$\left(h_P(u) + h_N(u), \Phi(k)h_P(u) - \Phi(-k)h_N(u)\right)$

$\left(h_P(u) + h_N(u), h_P(u)\right)$

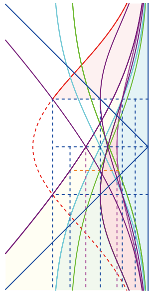


Integrals of CDF



Negative Inverse Mills Ratio

Truncated Normal Distribution



Modified intercept forms of linear equations according to k

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u + \frac{1}{\phi(k)}v = 1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Refs.

ORSJ (Higashi Hiroshima.)

RSS2019 (ICC Belfast, UK)

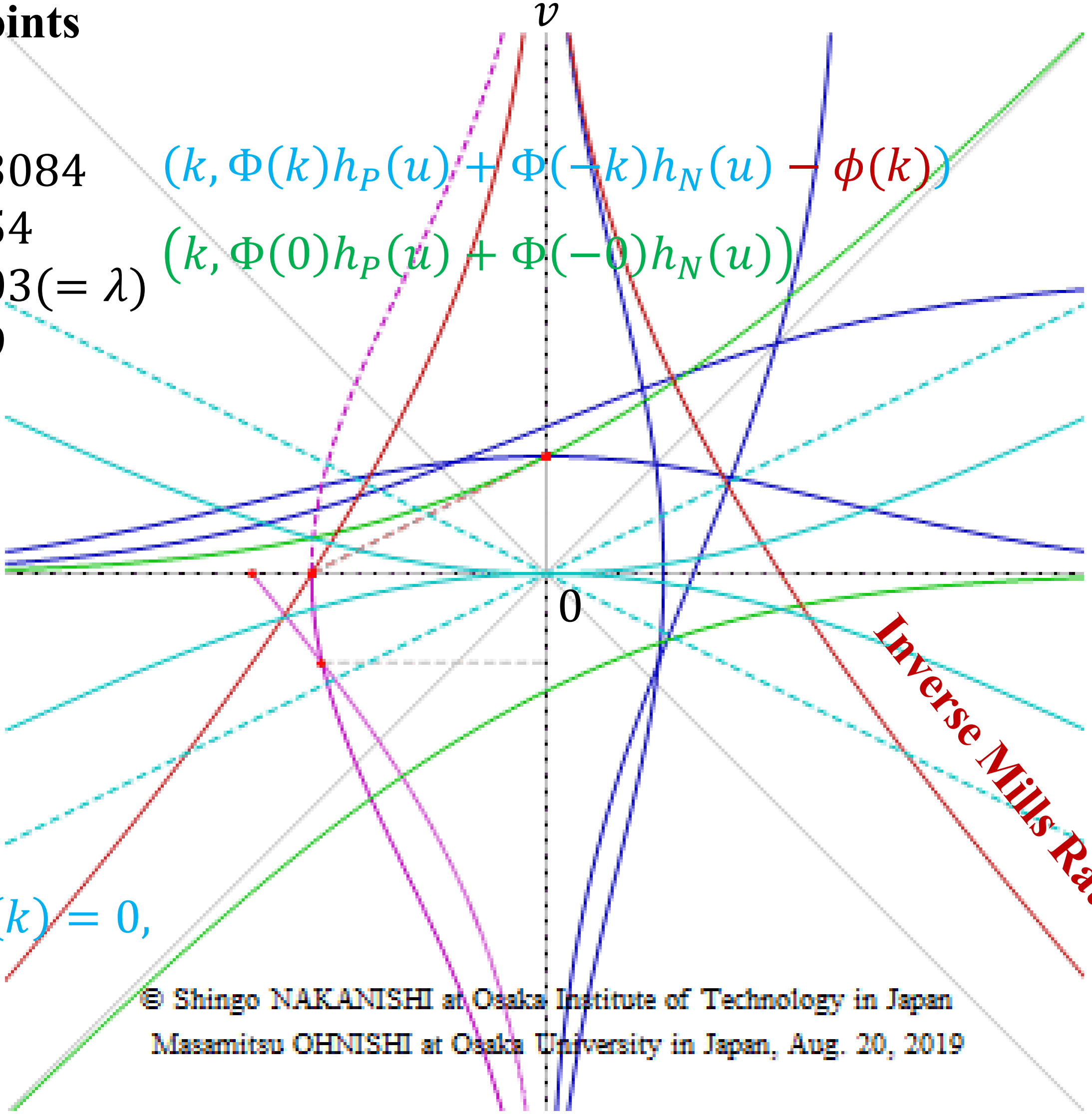
The important probability points k and 0 are shown the Rotationally Symmetric Relation:

$$\Phi(-k)g_P(k) + \Phi(k)g_N(k) = 2\left(\Phi(k)h_P(k) + \Phi(-k)h_N(k)\right).$$

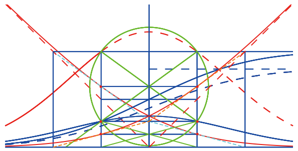
The important probability point k is shown the Rotationally Symmetric Relation.

k =**Probability points**

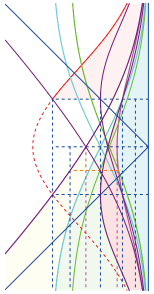
1. $k = \pm 0.0$
2. $k = \pm 0.30263084$
3. $k = \pm 0.506054$
4. $k = \pm 0.612003 (= \lambda)$
5. $k = \pm 0.67449$



Integrals of CDF



The right terminal point of k is 0.



Negative Inverse Mills Ratio

Truncated Normal Distribution

Modified intercept forms of linear equations according to k

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u + \frac{1}{\phi(k)}v = 1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u + \frac{1}{\phi(k)}v = 1$$

Boundary conditions:

$$\Phi(k)h_P(x) + \Phi(-k)h_N(x) - \phi(k) = 0,$$
$$\Phi(k)h'_P(x) + \Phi(-k)h'_N(x) = 0$$

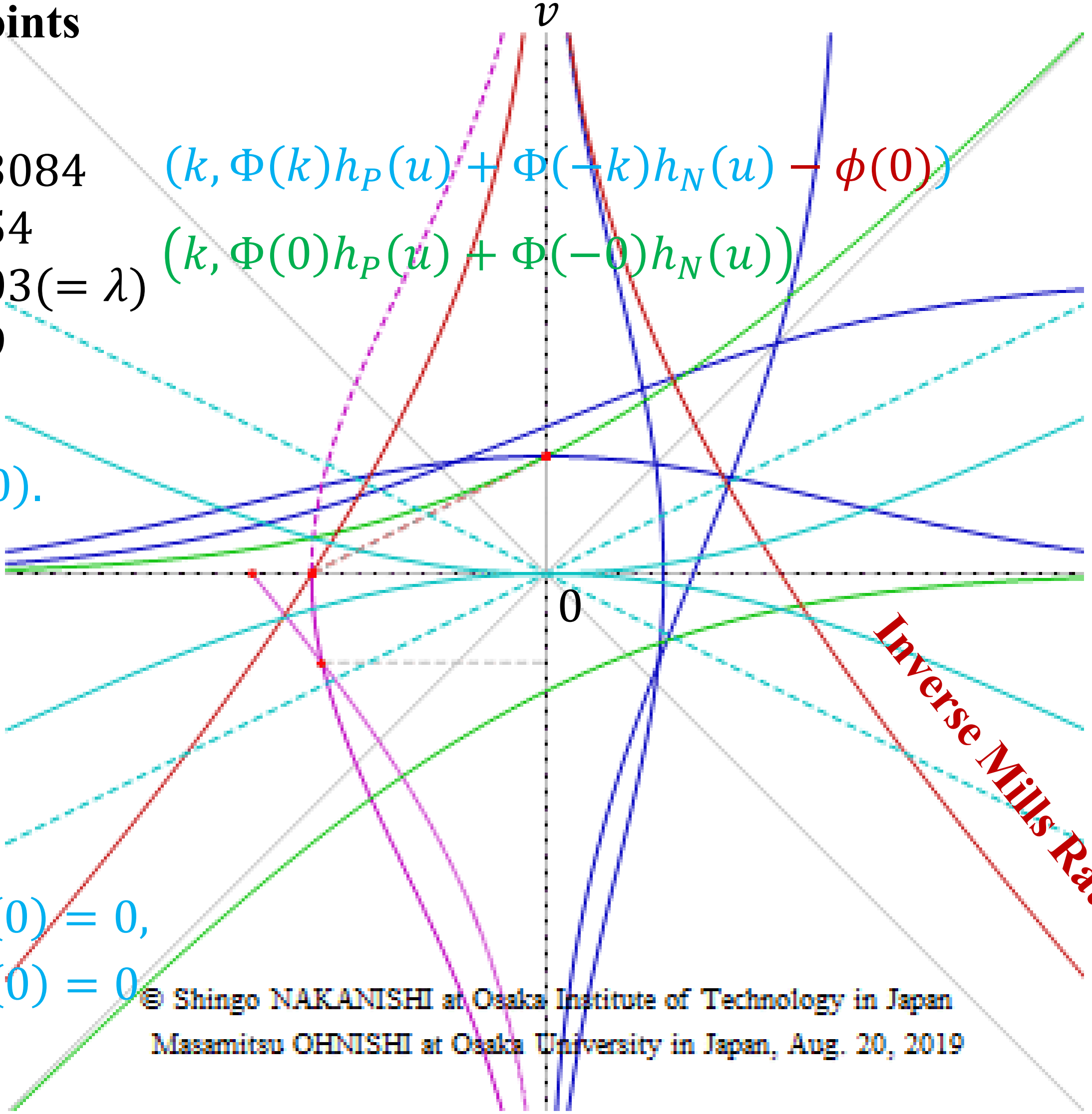
Refs.
ORSJ (Higashi Hiroshima.)
RSS2019 (ICC Belfast, UK)

$$\phi\left(-\frac{\phi(k)}{\Phi(k)}\right) - \frac{\phi(k)}{\Phi(k)}\Phi\left(-\frac{\phi(k)}{\Phi(k)}\right) = \Phi(k)\left(\phi\left(-\frac{\phi(k)}{\Phi(k)}\right) - \frac{\phi(k)}{\Phi(k)}\Phi\left(-\frac{\phi(k)}{\Phi(k)}\right)\right) + \Phi(-k)\left(\phi\left(-\frac{\phi(k)}{\Phi(k)}\right) + \frac{\phi(k)}{\Phi(k)}\left(1 - \Phi\left(-\frac{\phi(k)}{\Phi(k)}\right)\right)\right) - \phi(k)$$

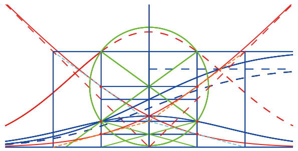
The important probability points k and 0 are shown the Rotationally Symmetric Relation.

k = **Probability points**

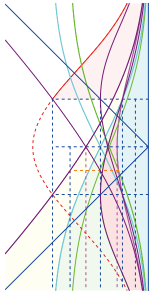
1. $k=\pm 0.0$
2. $k=\pm 0.30263084$
3. $k=\pm 0.506054$
4. $k=\pm 0.612003(=\lambda)$
5. $k=\pm 0.67449$



Integrals of CDF



The right terminal point of k is $\phi(0)$.



Negative Inverse Mills Ratio

Truncated Normal Distribution

Modified intercept forms of linear equations according to k

$$-\frac{1}{\frac{\phi(k)}{\Phi(-k)}}u+\frac{1}{\phi(k)}v=1$$

$$-\frac{1}{\frac{\phi(k)}{\Phi(k)}}u+\frac{1}{\phi(k)}v=1$$

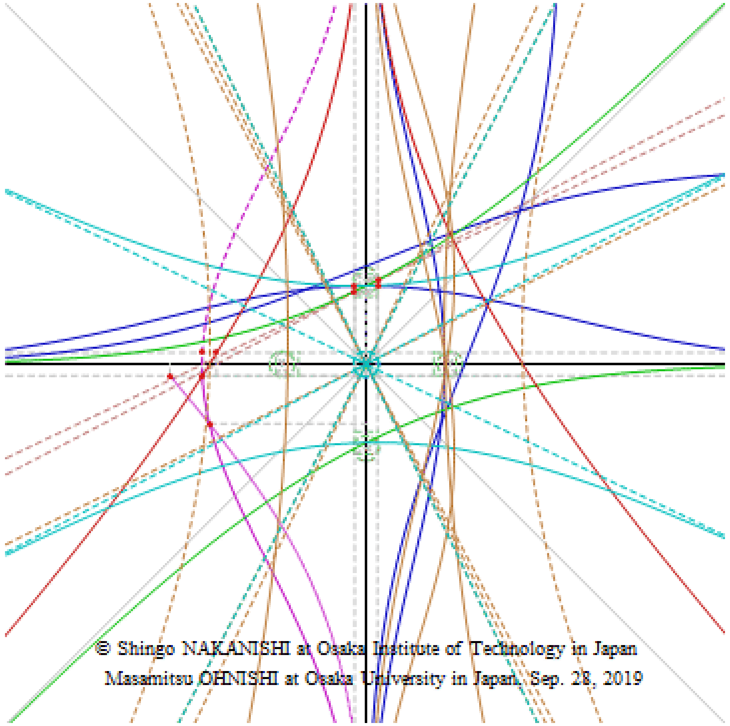
Boundary conditions:

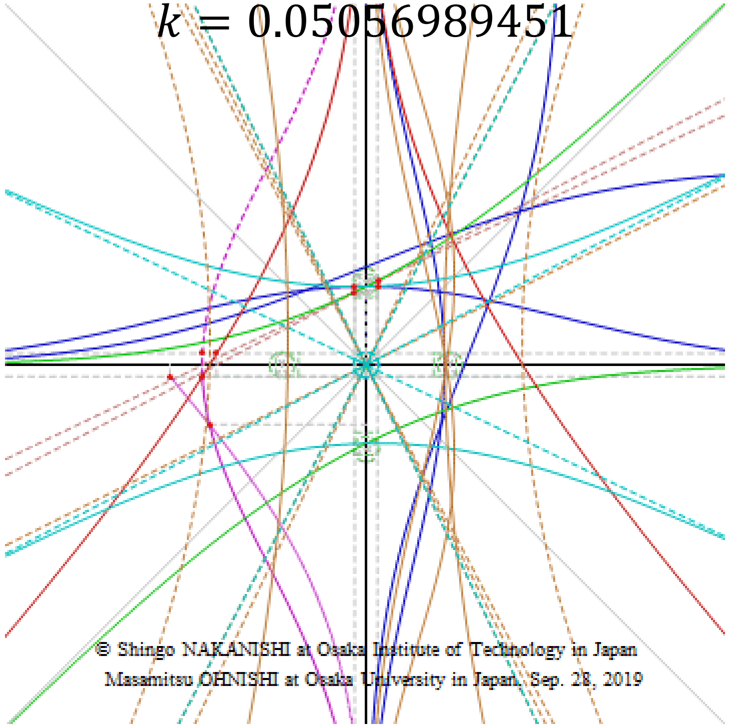
$$\Phi(k)h_P(0)+\Phi(-k)h_N(0)-\phi(0)=0,$$
$$\Phi(k)h_P(x)+\Phi(-k)h_N(x)-\phi(0)=0$$

Refs.
ORSJ (Higashi Hiroshima.)
RSS2019 (ICC Belfast, UK)

$$\phi\left(-\frac{\phi(0)}{\Phi(k)}\right)-\frac{\phi(0)}{\Phi(k)}\Phi\left(-\frac{\phi(0)}{\Phi(k)}\right)=\Phi(k)\left(\phi\left(-\frac{\phi(0)}{\Phi(k)}\right)-\frac{\phi(0)}{\Phi(k)}\Phi\left(-\frac{\phi(0)}{\Phi(k)}\right)\right)+\Phi(-k)\left(\phi\left(-\frac{\phi(0)}{\Phi(k)}\right)+\frac{\phi(0)}{\Phi(k)}\left(1-\Phi\left(-\frac{\phi(0)}{\Phi(k)}\right)\right)\right)-\phi(0)$$

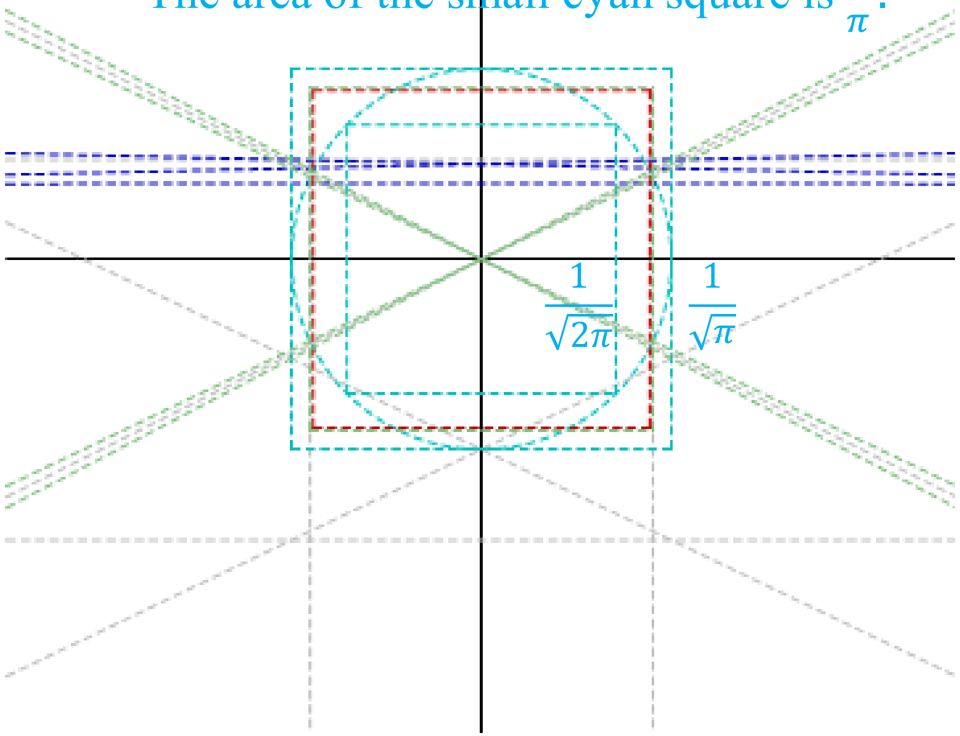
From the concept based on squaring the circle and fixed length of the radius of circle, we can show you the two slopes as cumulative probabilities at the probability points.



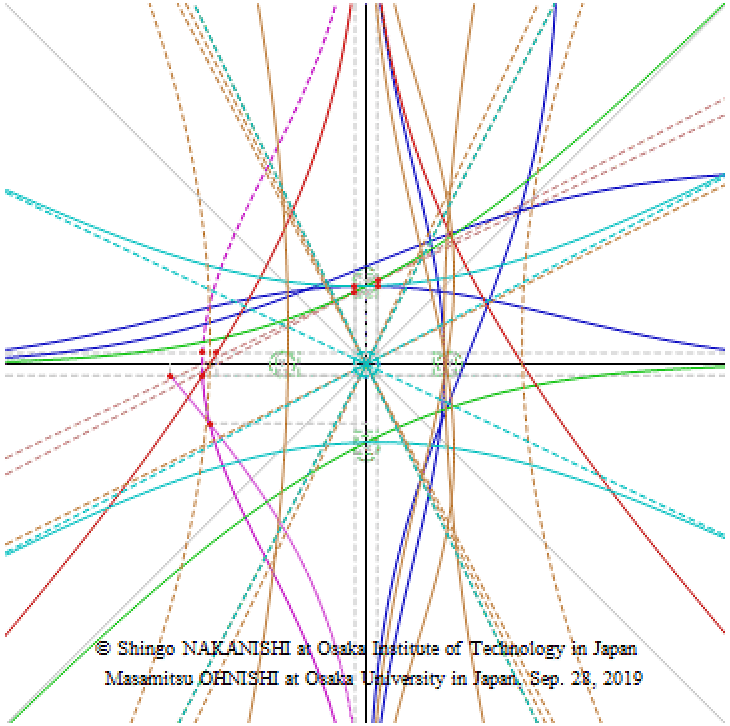


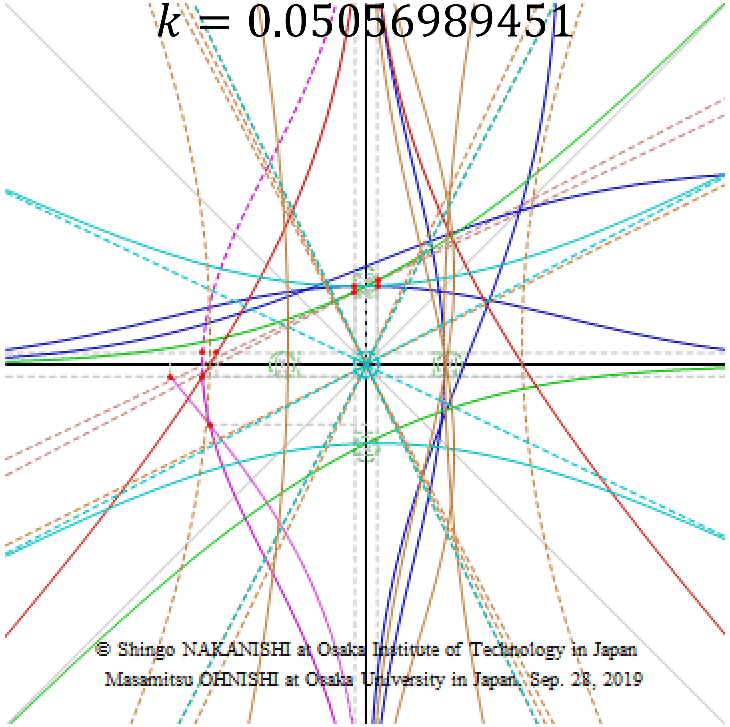
The area of the large cyan square is $\frac{4}{\pi}$.

The area of the small cyan square is $\frac{2}{\pi}$.



From the cross as squaring the circle at $k = 0.05056989451$
to the equilateral triangles at $k = 0.6435087$ as diamonds,
we can imagine a cross, a flower, a four-leaf clover,
and five RIMS or Olympic Track
according to the probability points k.
And from the visual animation,
we can also imagine the fire works in the sky.





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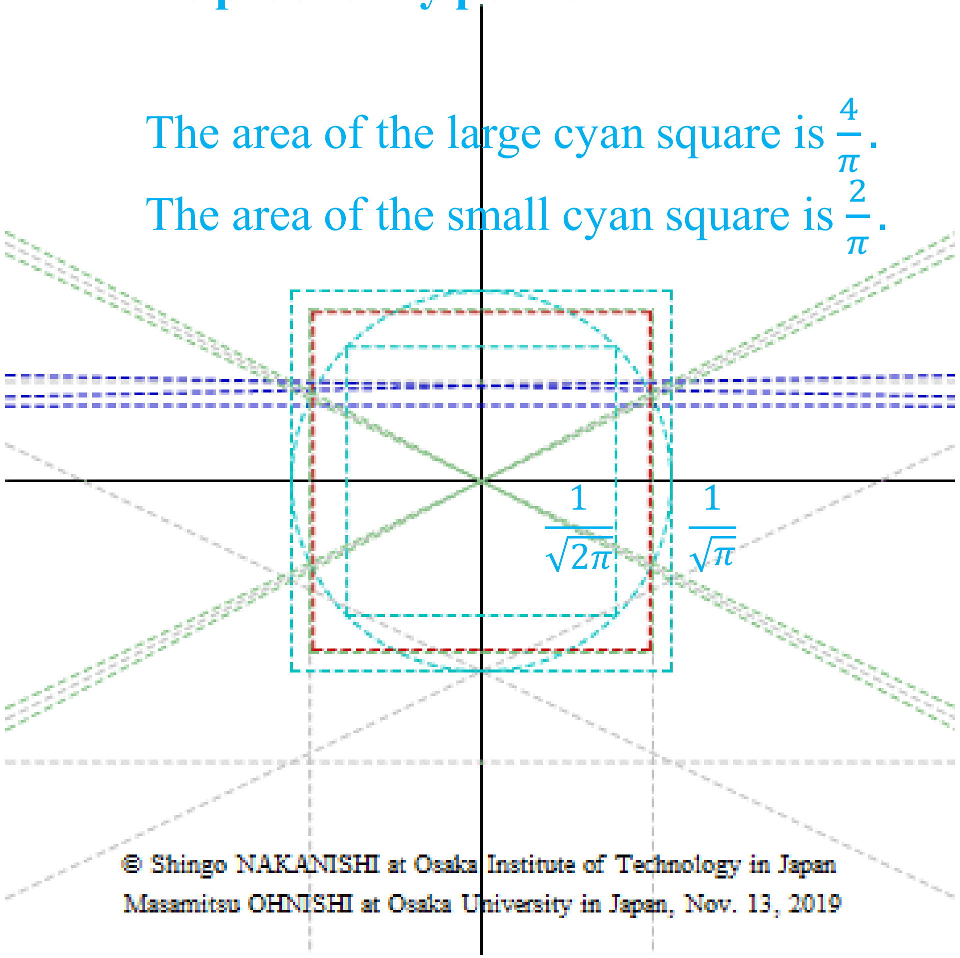
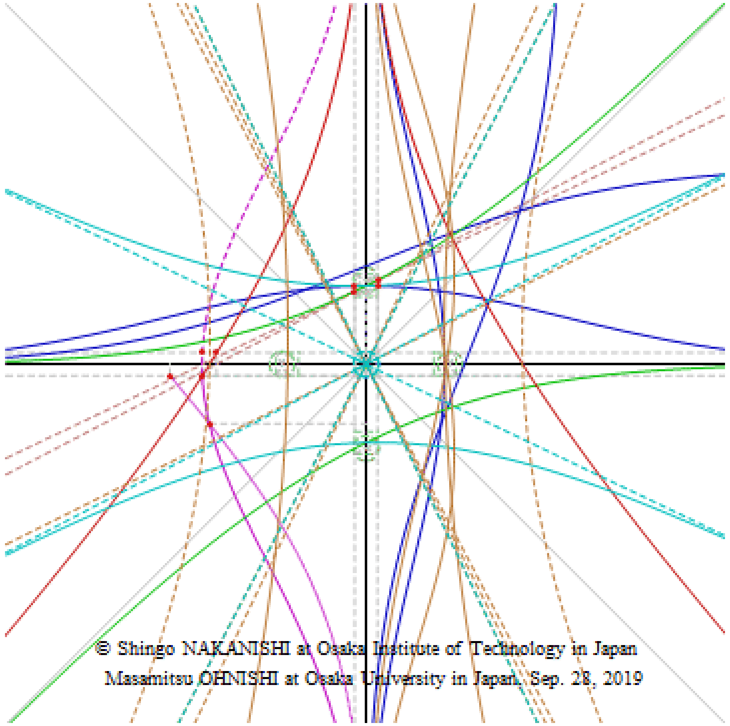


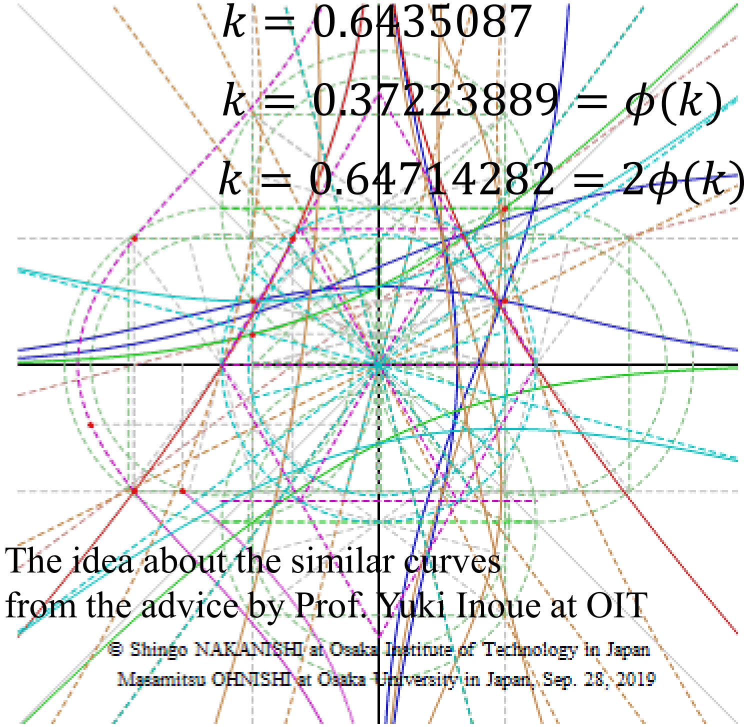
From the cross as squaring the circle at $k = 0.05056989451$ to the equilateral triangles at $k = 0.6435087$ as diamonds, we can imagine a cross, a flower, a four-leaf clover, and five RIMS or Olympic Track according to the probability points k.
And from the visual animation, we can also imagine the fire works in the sky.



$k = 0.6435087$

$k = 0.37223889 = \phi(k)$

$k = 0.64714282 = 2\phi(k)$



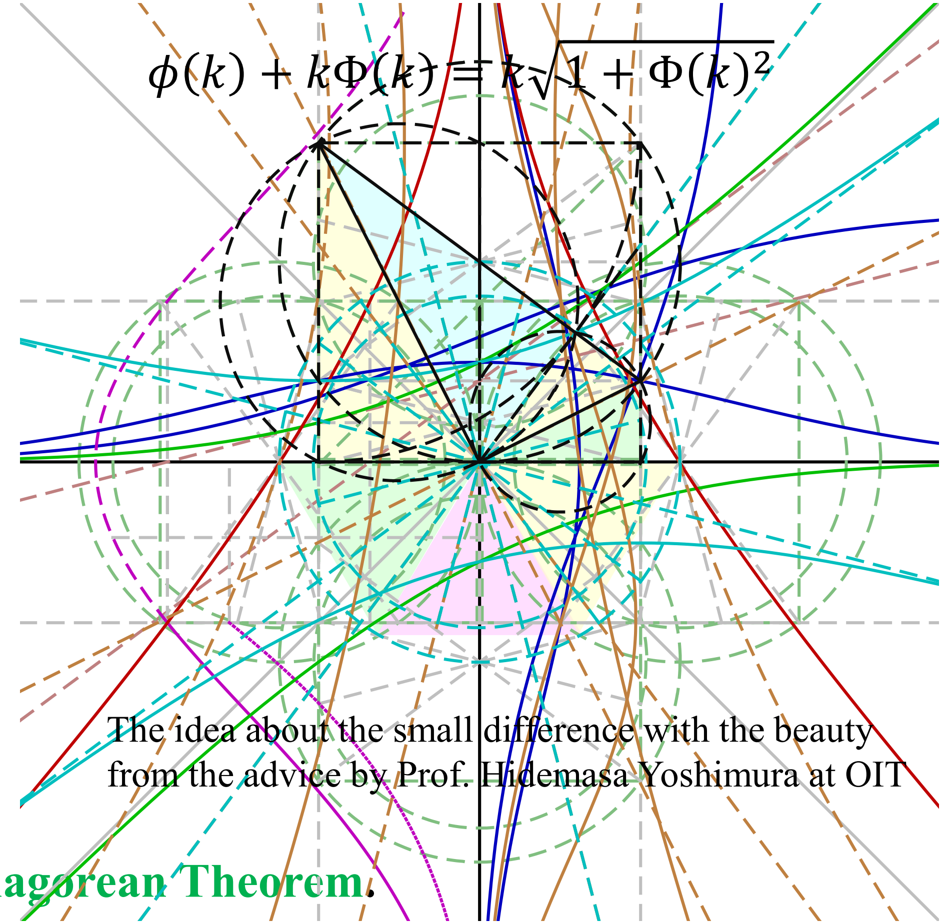
The idea about the similar curves
from the advice by Prof. Yuki Inoue at OIT

$k = \sqrt{\phi(k)\left(\phi(k) + 2k\Phi(k)\right)}$

$\because k = 0.6435087$

such as the concepts of Geometric Mean,
Regular Hexagon,
and Pythagorean Theorem

$\phi(k) + k\Phi(k) = k\sqrt{1 + \Phi(k)^2}$



The idea about the small difference with the beauty
from the advice by Prof. Hidemasa Yoshimura at OIT

From the cross as squaring the circle at $k = 0.05056989451$
to the equilateral triangles at $k = 0.6435087$ as diamonds,
we can imagine a cross, a flower, a four-leaf clover,
and five RIMS or Olympic Track
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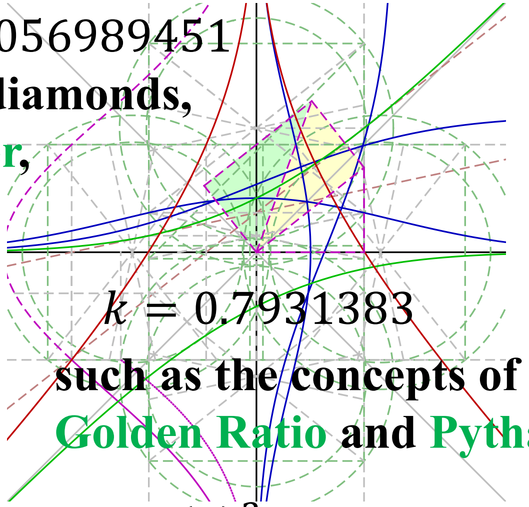


$k = 0.7931383$

such as the concepts of
Golden Ratio and Pythagorean Theorem.

$\Phi(k)\sqrt{1 + \Phi(k)^2} = 1, \because k = 0.7931383$ and $1 + \Phi(k)^2 =$ olden Ratio $= 1.6180339, \Phi(k)^{-2} = 1.6180339$

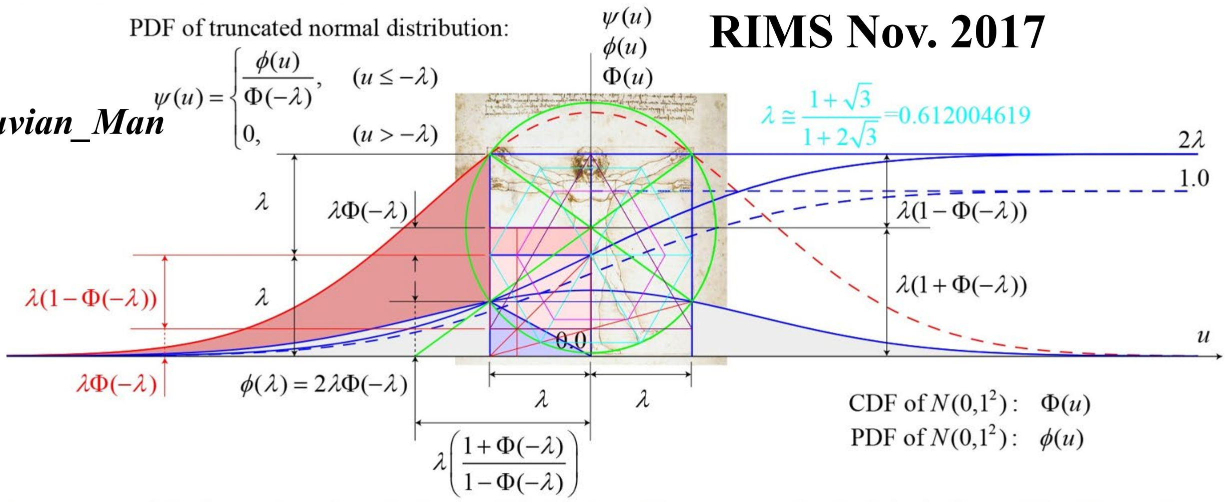
Concluding Remarks about my Researches

1. **Several rotationally weighted balances of integrals of Standard Normal Distribution are much more important than we thought.**
2. **So their inverse Mills ratios are.**
3. **Many geometrically interesting probability points can be found and firstly illustrated.**
4. **There might be some of the most emphasized facts between historically, geometrically, and mathematically attractive truth and beauty on earth in the future. …???**
5. **Finally, Right Triangles, Squares, and Circles are certainly related to the Standard Normal Distribution.**

Original Ref: ***Vitruvian Man***
https://en.wikipedia.org/wiki/Vitruvian_Man

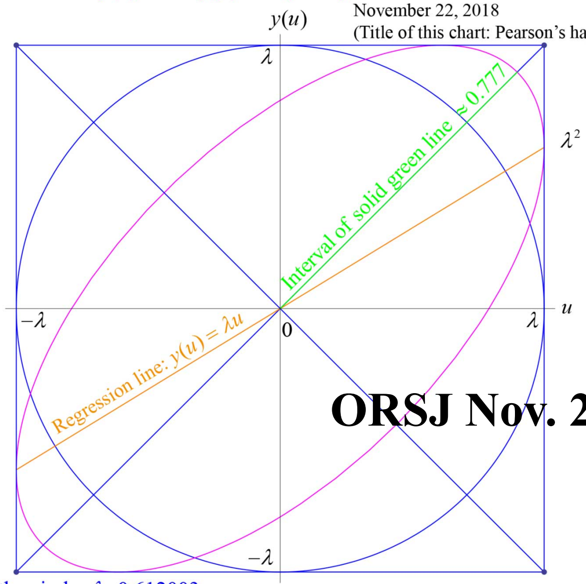
CIE2007 in Egypt

RIMS Nov. 2017



If you consider the correlation coefficient ρ as Pearson's another finding value λ(=0.612003), the interval of solid grren line $\approx$ 0.777 under the condition: $u^2+y(u)^2-2\lambda uy(u)=\lambda^2(1-\lambda^2)$

©Shingo NAKANISHI at Osaka Institute of Technology, JAPAN and Tetsuya TAKINE, Masamitsu OHNISHI at Osaka University, JAPAN November 22, 2018 (Title of this chart: Pearson's happy 777).



ORSJ Nov. 2018

Radius of the circle=λ=0.612003
Correlation cefficient of the ellipse: ρ=λ=0.612003

Nakanishi's Website (Nov. 2019):
http://www.oit.ac.jp/center/~nakanishi/english/

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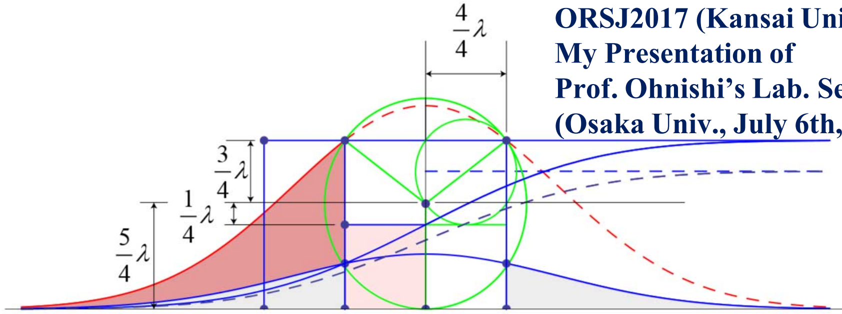

ORSJ2017 (Kansai Univ.) & My Presentation of Prof. Ohnishi's Lab. Seminar (Osaka Univ., July 6th, 2017)

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Title: From Mistakes to Modifications

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大阪大学(Osaka Univ.)，谷崎久志 先生，竹内惠行 先生，太田亘 先生，福重元嗣 先生，特に在学中の経済学研究科教職員，大学院生の皆様，
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高校時代の恩師の先生方，Mathematica, Illustrator, Excelの思考ツール，Amazon, Googleの検索ツールやApple(NeXTstation) やICTの歴史的環境変化の偶然, 2019年9月にベルファストでお世話になったRSS事務局の皆様，1年前にRIMS2078-10の論文の誤記について，小生のWebにて修正版を掲載することをご承諾くださったRIMS関係者の皆様，
本日の成果を予想すらできないところから専門分野を転身し，10年以上の歳月を費やしましたが，本日発表を収めました.
以上の他にも心温まるご支援を多くの皆様から頂戴したことを付記し，心より感謝申し上げます．最後に家族の愛娘にも感謝します．