Test of General Relativity from Gravitational Wave data:
Direct extraction of ring-down modes of binary black-hole mergers using auto-regressive method


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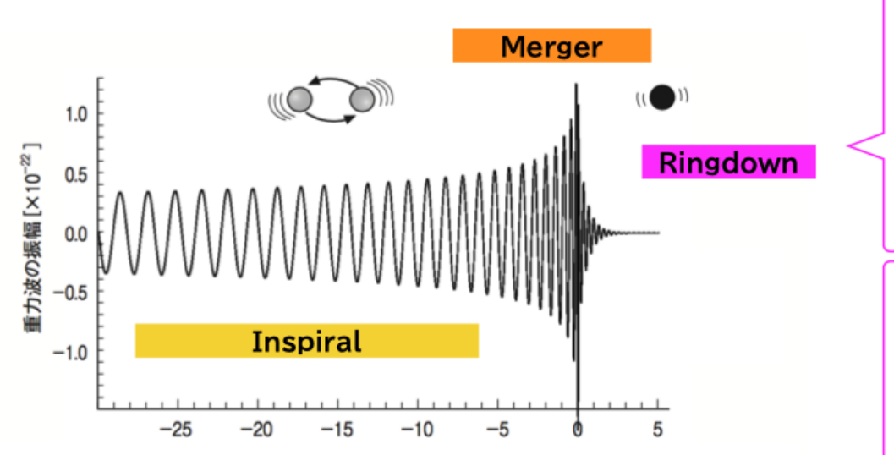
Outline & Summary

The ring-down part of gravitational waves in the final stage of merger of compact objects tells us the nature of strong gravity which can be used for testing the theories of gravity. The ring-down wave, however, fades out in a very short time with a few cycles, and hence it is challenging for gravitational wave data analysis to extract the ringdown frequency and its damping time scale.
We develop a new method, the autoregressive modeling (AR) approach, which extracts waveform by fitting a linear function from bare data. It works well for small number of data points, and does not require any templates. After obtaining the best parameters using mockdata, we applied this method for black-hole merger events of the LIGO/Virgo/KAGRA O3 catalog (GWTC-3). We find that for high SNR events, we can extract ring-down waves properly.
The identified ringdown modes are around those reported in GWTC-3, i.e. no significant deviations from the modes predicted by general relativity.
This method should work for extracting higher modes of ring-down waves, but we do not find them yet.

Motivation & data GWTC-3

Towards testing gravity theories ⟫⟫ Ringdown-part extraction is a key



BH quasi-normal modes
⬅ BH perturbation in GR
➡ (M, a)

Strongest Field ever
➡ suitable for testing GR

Can we see freq. & damping rate?
Can we see overtones?
Can we see higher modes?
… as is predicted by GR

Ringdown part decays quickly. (damping rate = 3.7 ms for 60 Msun, a=0.75)
We need to develop a technique.
➡ Auto-Regressive model
➡ Apply O3 data.

➡➡ When the ringdown starts?
➡➡ Overtones? Higher modes?
➡➡ Consistent with GR?

Transient Catalog O1/O2/O3a,b has 90 events

報告されたおもな重力波（2022 年 7 月現在）
連星の質量を M_1, M_2 としたときの，チャープ質量 $M_c = (M_1M_2)^{3/5}/(M_1+M_2)^{1/5}$，質量比（中央値の比）$M_2/M_1$，有効スピン $\chi_{\rm eff}$，最終的に形成された BH の質量 $M_{\rm final}$(NS を含む場合は全質量 $M_{全} = M_1 + M_2$)，距離，波源特定精度（平方度）$(\Delta\theta)^2$，シグナル・ノイズ比を示す．幅のある量は 90% の信頼区間．（種類ごとに日付順．BBH については，GW190521 と SNR が 17.3 より大きいもののみ．）

イベント (BBH)	$M_c(M_\odot)$	質量比	$\chi_{\rm eff}$	$M_{\rm final}(M_\odot)$	距離 (Mpc)	$(\Delta\theta)^2$	SNR		
GW150914	$28.6^{+1.7}_{-1.5}$	0.86	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.4}_{-3.0}$	440^{+150}_{-170}	182	24.4	✓	good event for test
GW190412	$13.3^{+0.5}_{-0.5}$	0.32	$0.21^{+0.12}_{-0.13}$	$35.6^{+4.8}_{-4.5}$	720^{+240}_{-220}	240	19.8	✓	
GW190521	$63.3^{+19.6}_{-14.6}$	0.58	$-0.14^{+0.5}_{-0.45}$	$147.4^{+40.0}_{-16.0}$	3310^{+2790}_{-1800}	1000	14.3	✓	too heavy, low freq.
GW190521_074359	$32.8^{+3.2}_{-2.8}$	0.77	$0.1^{+0.13}_{-0.13}$	$72.6^{+6.5}_{-5.4}$	1080^{+580}_{-530}	470	25.9	✓	
GW190814	$6.11^{+0.06}_{-0.05}$	0.11	$0^{+0.07}_{-0.07}$	$25.7^{+1.3}_{-1.3}$	230^{+40}_{-50}	22	25.3	✓	too light, high freq.
GW191109_010717	$47.5^{+9.6}_{-7.5}$	0.72	$-0.29^{+0.42}_{-0.31}$	$107^{+18.0}_{-15.0}$	1290^{+1130}_{-650}	1600	17.3	✓	too heavy, low freq.
GW191204_171526	$8.55^{+0.38}_{-0.27}$	0.69	$0.16^{+0.08}_{-0.05}$	$19.21^{+1.79}_{-0.95}$	650^{+190}_{-250}	350	17.5	✓	too light, high freq.
GW191216_213338	$8.33^{+0.22}_{-0.19}$	0.64	$0.11^{+0.13}_{-0.06}$	$18.87^{+2.8}_{-0.94}$	340^{+120}_{-130}	490	18.6	✓	too light, high freq.
GW200112_155838	$27.4^{+2.6}_{-2.1}$	0.79	$0.06^{+0.15}_{-0.15}$	$60.8^{+5.3}_{-4.3}$	1250^{+430}_{-460}	4300	19.8	✓	
GW200129_065458	$27.2^{+2.1}_{-2.3}$	0.84	$0.11^{+0.11}_{-0.16}$	$60.3^{+4.0}_{-3.3}$	900^{+290}_{-380}	130	26.8	✓	
GW200224_222234	$31.1^{+3.2}_{-2.6}$	0.81	$0.1^{+0.15}_{-0.15}$	$68.6^{+6.6}_{-4.7}$	1710^{+490}_{-640}	50.0	20	✓	
GW200311_115853	$26.6^{+2.4}_{-2.0}$	0.81	$-0.02^{+0.16}_{-0.2}$	$59^{+4.8}_{-3.9}$	1170^{+280}_{-400}	35	17.8	✓	
イベント (BNS)	$M_c(M_\odot)$	質量比	$\chi_{\rm eff}$	$M_{全}(M_\odot)$	距離 (Mpc)	$(\Delta\theta)^2$	SNR		
GW170817	$1.186^{+0.001}_{-0.001}$	0.87	$0^{+0.02}_{-0.01}$	–	$40^{+7.0}_{-15.0}$	16	33		
GW190425	$1.44^{+0.02}_{-0.02}$	0.62	$0.07^{+0.07}_{-0.05}$	$3.4^{+0.3}_{-0.1}$	150^{+80}_{-60}	8700	12.4		
イベント (NSBH)	$M_c(M_\odot)$	質量比	$\chi_{\rm eff}$	$M_{全}(M_\odot)$	距離 (Mpc)	$(\Delta\theta)^2$	SNR		
GW190917_114630	$3.7^{+0.2}_{-0.2}$	0.22	$-0.08^{+0.21}_{-0.43}$	$11.6^{+3.1}_{-2.9}$	720^{+300}_{-310}	2100	8.3		
GW200105_162426	$3.42^{+0.08}_{-0.08}$	0.21	$0.0^{+0.13}_{-0.18}$	$10.7^{+1.5}_{-1.4}$	270^{+120}_{-110}	7900	13.7		
GW200115_042309	$2.43^{+0.05}_{-0.07}$	0.24	$-0.15^{+0.24}_{-0.42}$	$7.2^{+1.8}_{-1.7}$	290^{+150}_{-100}	370	11.3		

Mockdata Comparison

Phys. Rev. D 99, 124032 (2019) [arXiv:1811.06443]

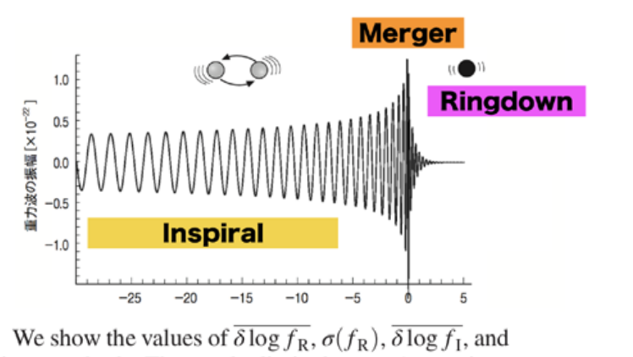
Nakano+, PRD99 (2019) 124032

mockdata-challenge comparison

PHYSICAL REVIEW D 99, 124032 (2019)

Comparison of various methods to extract ringdown frequency from gravitational wave data

Hiroyuki Nakano, Tatsuya Narikawa, Ken-ichi Oohara, Kazuki Sakai, Hisa-aki Shinkai, Hirotaka Takahashi, Takahiro Tanaka, Nami Uchikata, Shun Yamamoto, and Takahiro S. Yamamoto



ringdown search
60 mockdata

TABLE III. We show the values of $\overline{\delta\log f_R}$, $\sigma(f_R)$, $\overline{\delta\log f_I}$, and $\sigma(f_I)$ for various methods. The results limited to set A are given on the first law of each method, while those limited to set B are on the second.

			$\overline{\delta\log f_R}(\%)$	$\sigma(f_R)(\%)$	$\overline{\delta\log f_I}(\%)$	$\sigma(f_I)(\%)$
matched filtering	MF-R	A	−12.88	28.36	−71.51	97.79
		B	−0.82	27.53	−46.11	75.48
	MF-MR	A	6.25 Very good	17.27 Good	−12.62 Very good	37.9 Very good
		B	2.47	10.41	7.18	27.61
Hilbert-Huan Transformation	HHT	A	−13.38	21.91	−44.11	61.58
		B	−8.08	19.81	−28.78	49.61
Auto-Regression Method	AR	A	0.2 Excellent	9.93 Excellent	4.88 Excellent	38.75 Good
		B	1.91	8.57	6.2	34.64
Neural Network method	NN	A	−6.64 Good	16.48 Very good	−15.23 Good	33.96 Excellent
		B	−6.65	11.97	9.96	23.76

https://gw-genesis.scphys.kyoto-u.ac.jp/ilias/goto_root_fold_669.html
http://www.oit.ac.jp/is/shinkai/mockdatachallenge/

Method

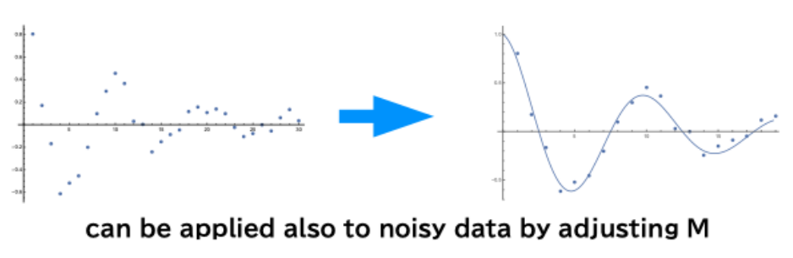
Auto-Regressive model (Method, general)

Fitting data with linear func.

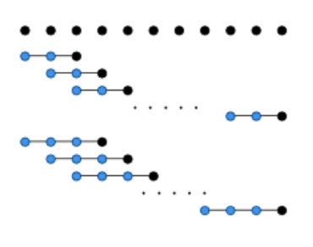
$$x_n = a_1x_{n-1} + a_2x_{n-2} + \cdots + a_Mx_{n-M} + \varepsilon = \sum_{j=1}^{M} a_jx_{n-j} + \varepsilon$$

e.g. $x_n = Ae^{-rn\Delta t}\cos(\omega n\Delta t)$

$$Z_1 = e^{-(r-j\omega)\Delta t},\quad Z_2 = e^{-(r+j\omega)\Delta t} \rightarrow x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1+Z_2)x_{n-1} - Z_1Z_2x_{n-2}$$



can be applied also to noisy data by adjusting M



- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

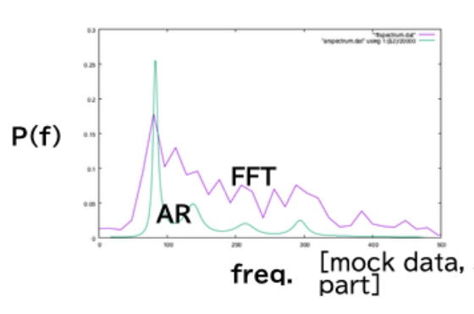
power spectrum

$$p(f) = \frac{\sigma^2}{\left|1 - \sum_{j=1}^{M} a_je^{-I2\pi jf\Delta t}\right|^2}$$

characteristic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_jz^j = 0$$

$|z_k|$ says amplitude, $\arg(z_k)$ says frequency.



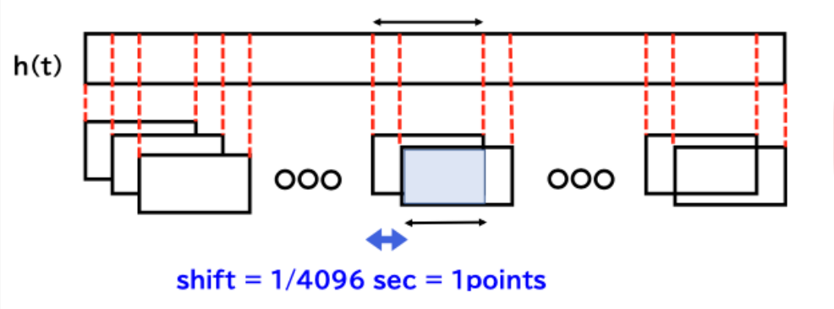
The order M can be fixed at 2~8.

Even for short segment, AR model shows precise power-spectrum.

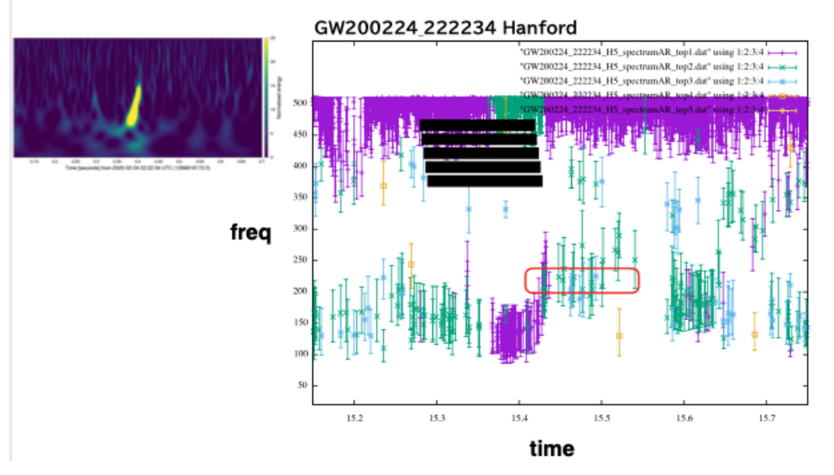
[mock data, SNR=40, inspiral part]

Method of Auto-Regressive model for extracting GW signal

sampling rate=4096　segment = 1/16 sec = 256 points
or = 1/64 sec = 64 points



shift = 1/4096 sec = 1points



We analyse the (whitened) data around the event reported.
Using AR method, we extract frequencies $f_{\rm real}$, and damping rate τ (= $f_{\rm imag}$).
No Prior Models, Only from Data **Independent to each detector**

We search constant-frequency signals (ring-down mode !?)
The variance of $f_{\rm real}$ < 1-sigma, $\tau(= f_{\rm imag})$ < 1.25-sigma
We can extract the existing period for each mode

We can pick up several such (f, τ) in a single segment
Can extract multipul modes simalteneously

Refer the LVK catalog data (M_f, a_f) + redshift z
Derive (f, τ) and let them the GR predictions

$$f_R = f_1 + f_2(1-a)^{f_3},\quad Q \equiv \frac{f_R}{2f_I} = q_1 + q_2(1-a)^{q_3}$$

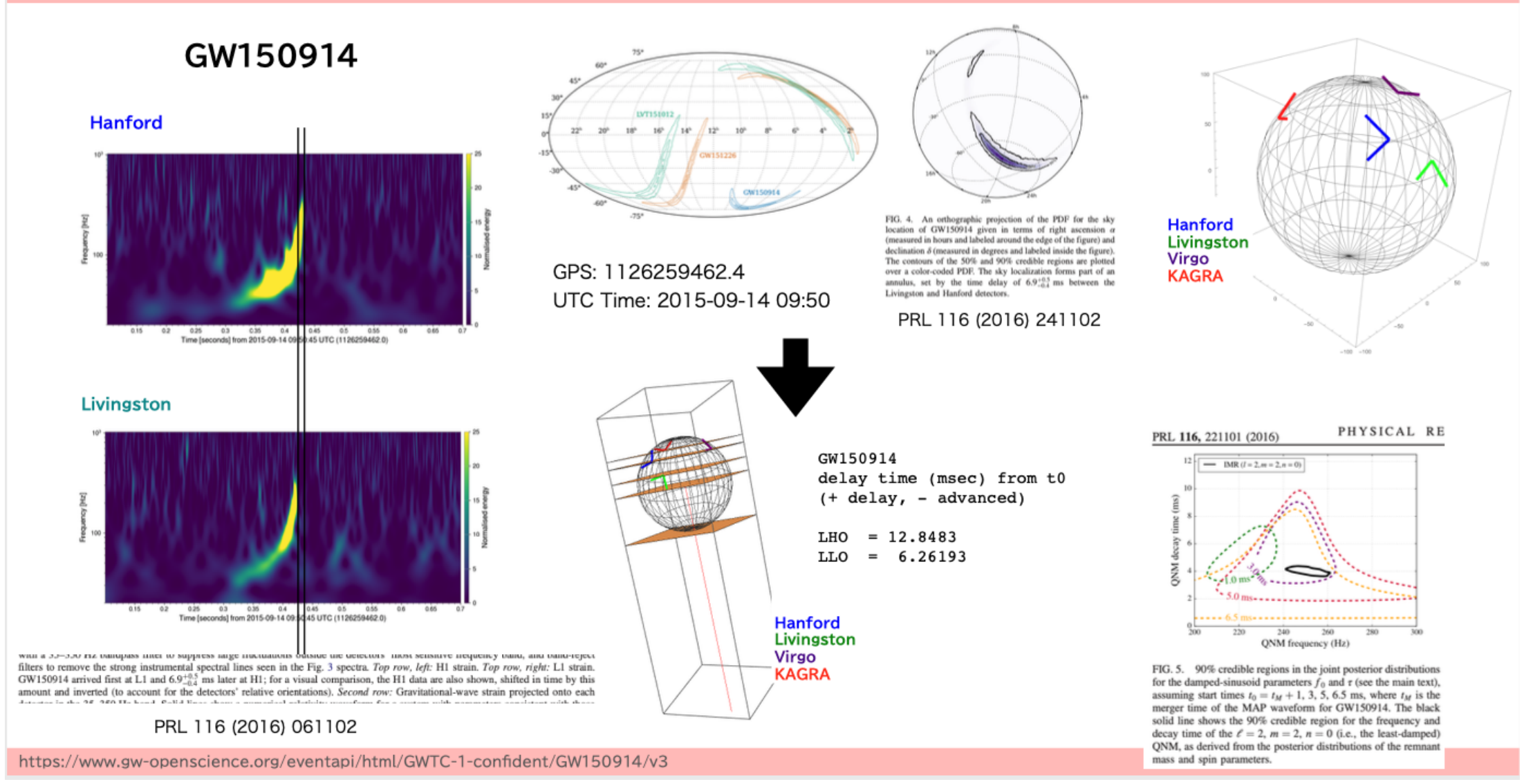
$$f_{\rm qnm}[{\rm Hz}] = \frac{c^3}{2\pi GM}f_R \sim 32314.1\left(\frac{M_\odot}{M}\right)f_R,\quad a = 1 - \left(\frac{Q-q_1}{q_2}\right)^{1/q_3},\quad M[M_\odot] = 32314.1 \times \frac{f_1 + f_2(1-a)^{f_3}}{f_{\rm qnm}[{\rm Hz}]}$$

Berti, Cardoso & Will PRD 73, 064030 (2006).

➡ Does AR extract ringdown mode properly?
Consistent values from 3 detectors (Hanford, Livingston, Virgo) ?
Consistent values with LVK catalog

➡➡ When the ringdown starts?
Ovetones? Higher modes?
Consistent with GR?

Calibration of transient time of GW at each detector



GPS: 1126259462.4
UTC Time: 2015-09-14 09:50

PRL 116 (2016) 241102

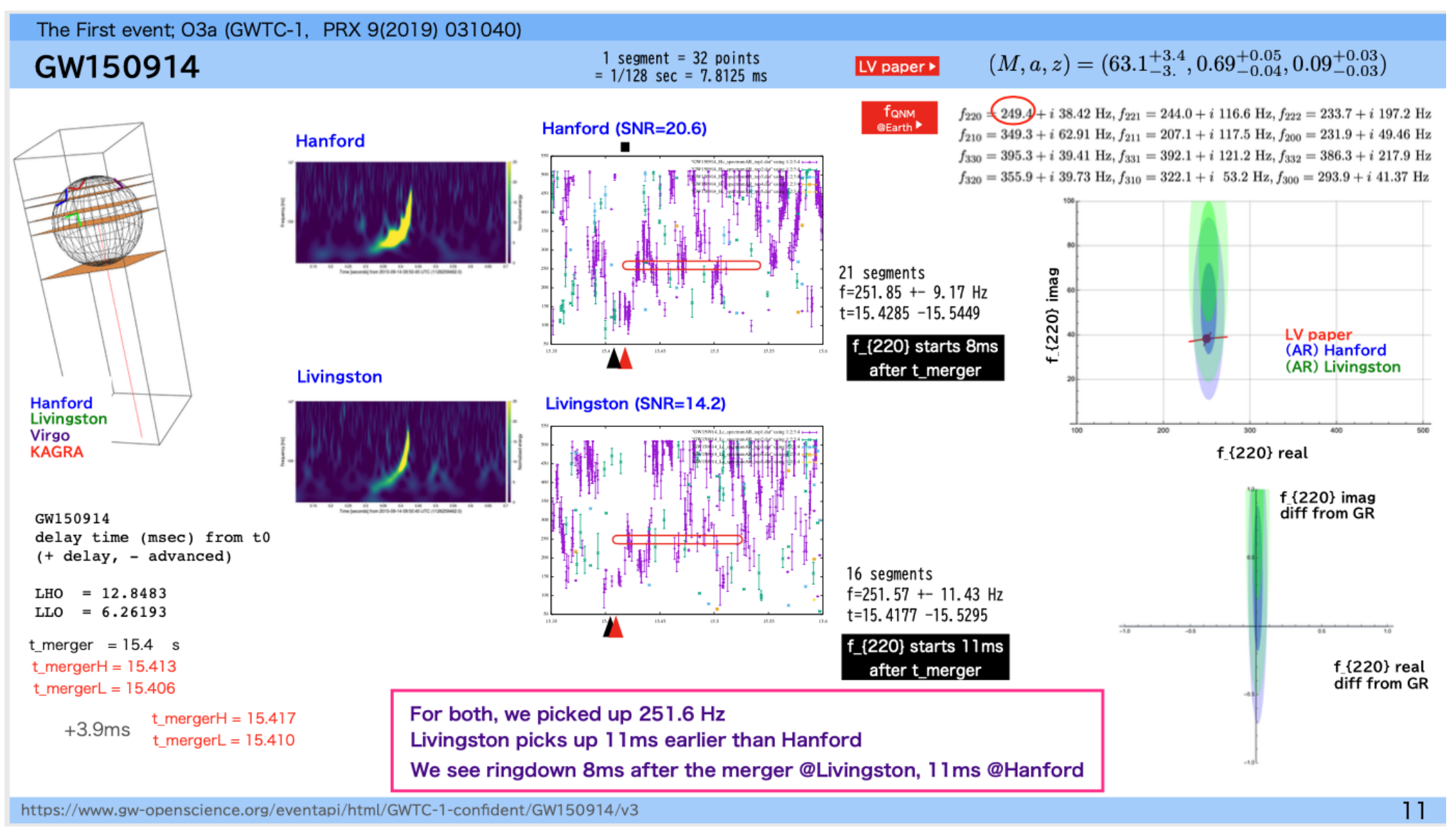
```
GW150914
delay time (msec) from t0
(+ delay, - advanced)

LHO  = 12.8483
LLO  =  6.26193
```

PRL 116 (2016) 061102

PRL 116, 221101 (2016)

https://www.gw-openscience.org/eventapi/html/GWTC-1-confident/GW150914/v3

Results



The First event; O3a (GWTC-1, PRX 9(2019) 031040)

GW150914　1 segment = 32 points = 1/128 sec = 7.8125 ms　LV paper　$(M, a, z) = (63.1^{+3.4}_{-3.}, 0.69^{+0.05}_{-0.04}, 0.09^{+0.03}_{-0.03})$

$f_{220} = 249.4 + i\,38.42$ Hz, $f_{221} = 244.0 + i\,116.6$ Hz, $f_{222} = 233.7 + i\,197.2$ Hz
$f_{210} = 349.3 + i\,62.91$ Hz, $f_{211} = 207.1 + i\,117.5$ Hz, $f_{200} = 231.9 + i\,49.46$ Hz
$f_{330} = 395.3 + i\,39.41$ Hz, $f_{331} = 392.1 + i\,121.2$ Hz, $f_{332} = 386.3 + i\,217.9$ Hz
$f_{320} = 355.9 + i\,39.73$ Hz, $f_{310} = 322.1 + i\,53.2$ Hz, $f_{300} = 293.9 + i\,41.37$ Hz

Hanford (SNR=20.6): 21 segments, f=251.85 +- 9.17 Hz, t=15.4285 -15.5449 — f_{220} starts 8ms after t_merger

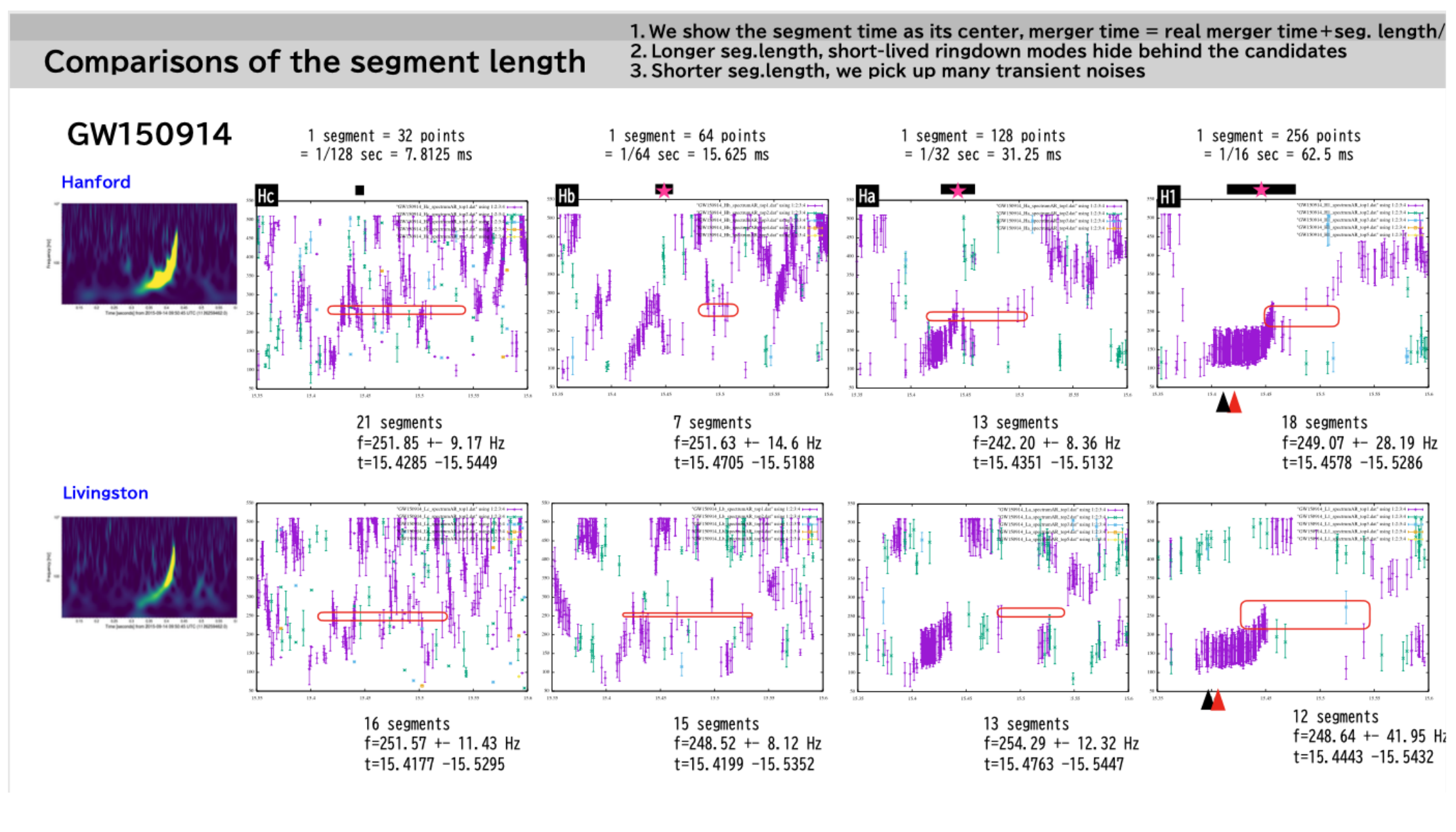
Livingston (SNR=14.2): 16 segments, f=251.57 +- 11.43 Hz, t=15.4177 -15.5295 — f_{220} starts 11ms after t_merger

```
GW150914
delay time (msec) from t0
(+ delay, - advanced)

LHO  = 12.8483
LLO  = 6.26193
```

t_merger = 15.4 s
t_mergerH = 15.413
t_mergerL = 15.406
+3.9ms　t_mergerH = 15.417　t_mergerL = 15.410

For both, we picked up 251.6 Hz
Livingston picks up 11ms earlier than Hanford
We see ringdown 8ms after the merger @Livingston, 11ms @Hanford

https://www.gw-openscience.org/eventapi/html/GWTC-1-confident/GW150914/v3　11

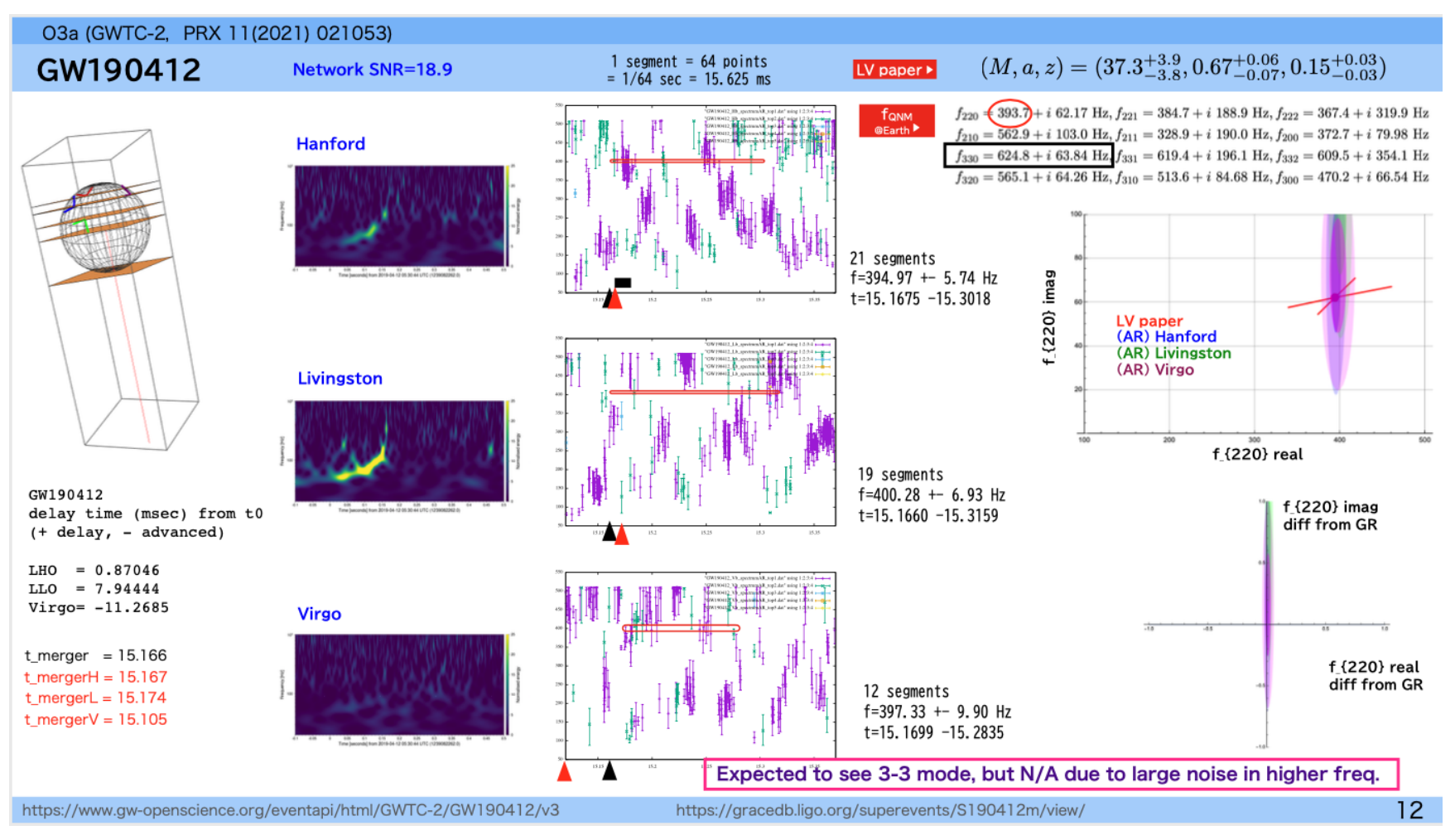


Comparisons of the segment length

1. We show the segment time as its center, merger time = real merger time+seg. length/
2. Longer seg.length, short-lived ringdown modes hide behind the candidates
3. Shorter seg.length, we pick up many transient noises

GW150914

	1 segment = 32 points = 1/128 sec = 7.8125 ms	1 segment = 64 points = 1/64 sec = 15.625 ms	1 segment = 128 points = 1/32 sec = 31.25 ms	1 segment = 256 points = 1/16 sec = 62.5 ms
Hanford	21 segments f=251.85 +- 9.17 Hz t=15.4285 -15.5449	7 segments f=251.63 +- 14.6 Hz t=15.4705 -15.5188	13 segments f=242.20 +- 8.36 Hz t=15.4351 -15.5132	18 segments f=249.07 +- 28.19 Hz t=15.4578 -15.5286
Livingston	16 segments f=251.57 +- 11.43 Hz t=15.4177 -15.5295	15 segments f=248.52 +- 8.12 Hz t=15.4199 -15.5352	13 segments f=254.29 +- 12.32 Hz t=15.4763 -15.5447	12 segments f=248.64 +- 41.95 Hz t=15.4443 -15.5432



O3a (GWTC-2, PRX 11(2021) 021053)

GW190412　Network SNR=18.9　1 segment = 64 points = 1/64 sec = 15.625 ms　LV paper　$(M, a, z) = (37.3^{+3.9}_{-3.8}, 0.67^{+0.06}_{-0.07}, 0.15^{+0.03}_{-0.03})$

$f_{220} = 393.7 + i\,62.17$ Hz, $f_{221} = 384.7 + i\,188.9$ Hz, $f_{222} = 367.4 + i\,319.9$ Hz
$f_{210} = 562.9 + i\,103.0$ Hz, $f_{211} = 328.9 + i\,190.0$ Hz, $f_{200} = 372.7 + i\,79.98$ Hz
$f_{330} = 624.8 + i\,63.84$ Hz, $f_{331} = 619.4 + i\,196.1$ Hz, $f_{332} = 609.5 + i\,354.1$ Hz
$f_{320} = 565.1 + i\,64.26$ Hz, $f_{310} = 513.6 + i\,84.68$ Hz, $f_{300} = 470.2 + i\,66.54$ Hz

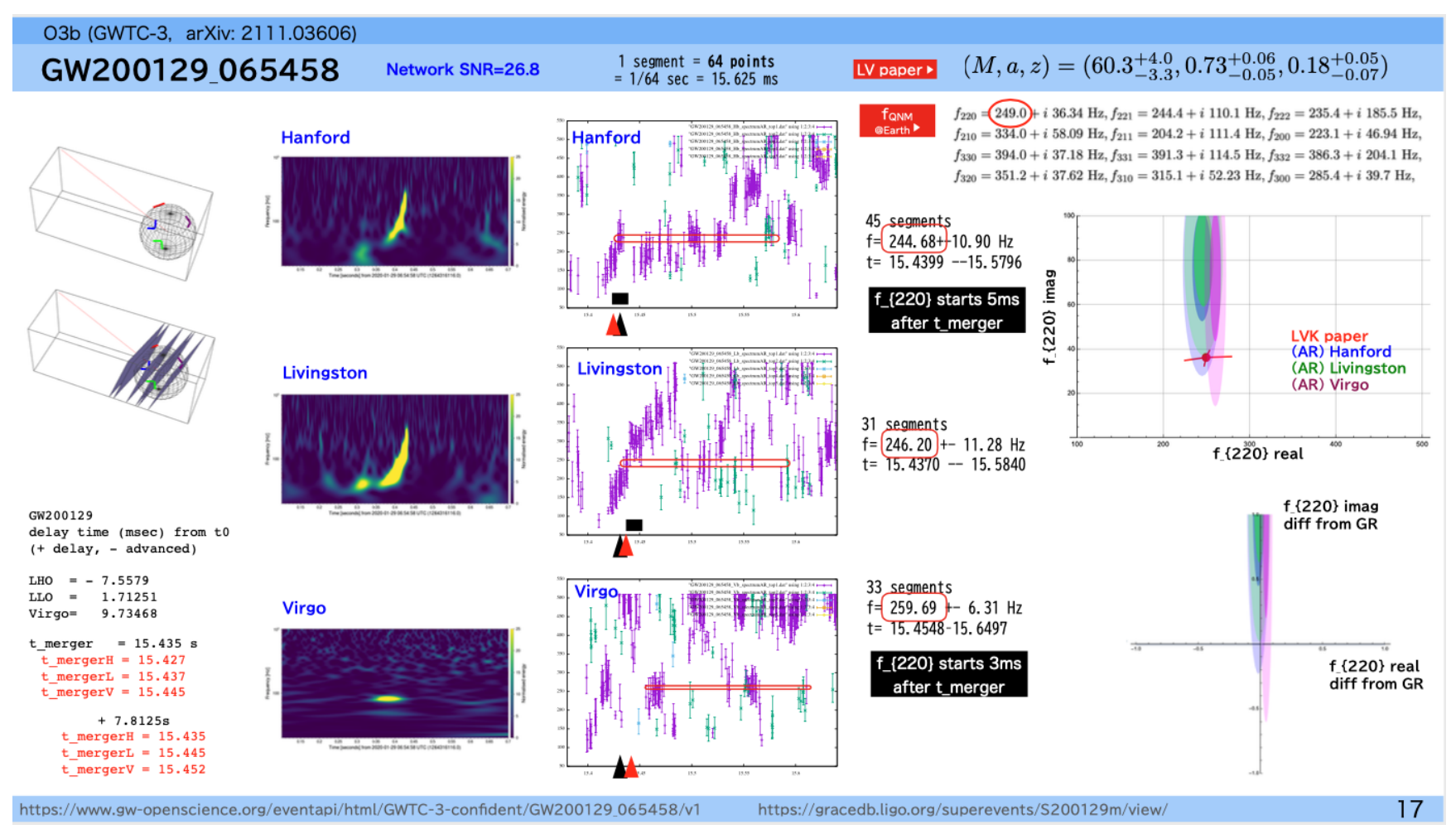
Hanford: 21 segments, f=394.97 +- 5.74 Hz, t=15.1675 -15.3018
Livingston: 19 segments, f=400.28 +- 6.93 Hz, t=15.1660 -15.3159
Virgo: 12 segments, f=397.33 +- 9.90 Hz, t=15.1699 -15.2835

```
GW190412
delay time (msec) from t0
(+ delay, - advanced)

LHO  = 0.87046
LLO  = 7.94444
Virgo= -11.2685
```

t_merger = 15.166
t_mergerH = 15.167
t_mergerL = 15.174
t_mergerV = 15.105

Expected to see 3-3 mode, but N/A due to large noise in higher freq.

https://www.gw-openscience.org/eventapi/html/GWTC-2/GW190412/v3　https://gracedb.ligo.org/superevents/S190412m/view/　12



O3b (GWTC-3, arXiv: 2111.03606)

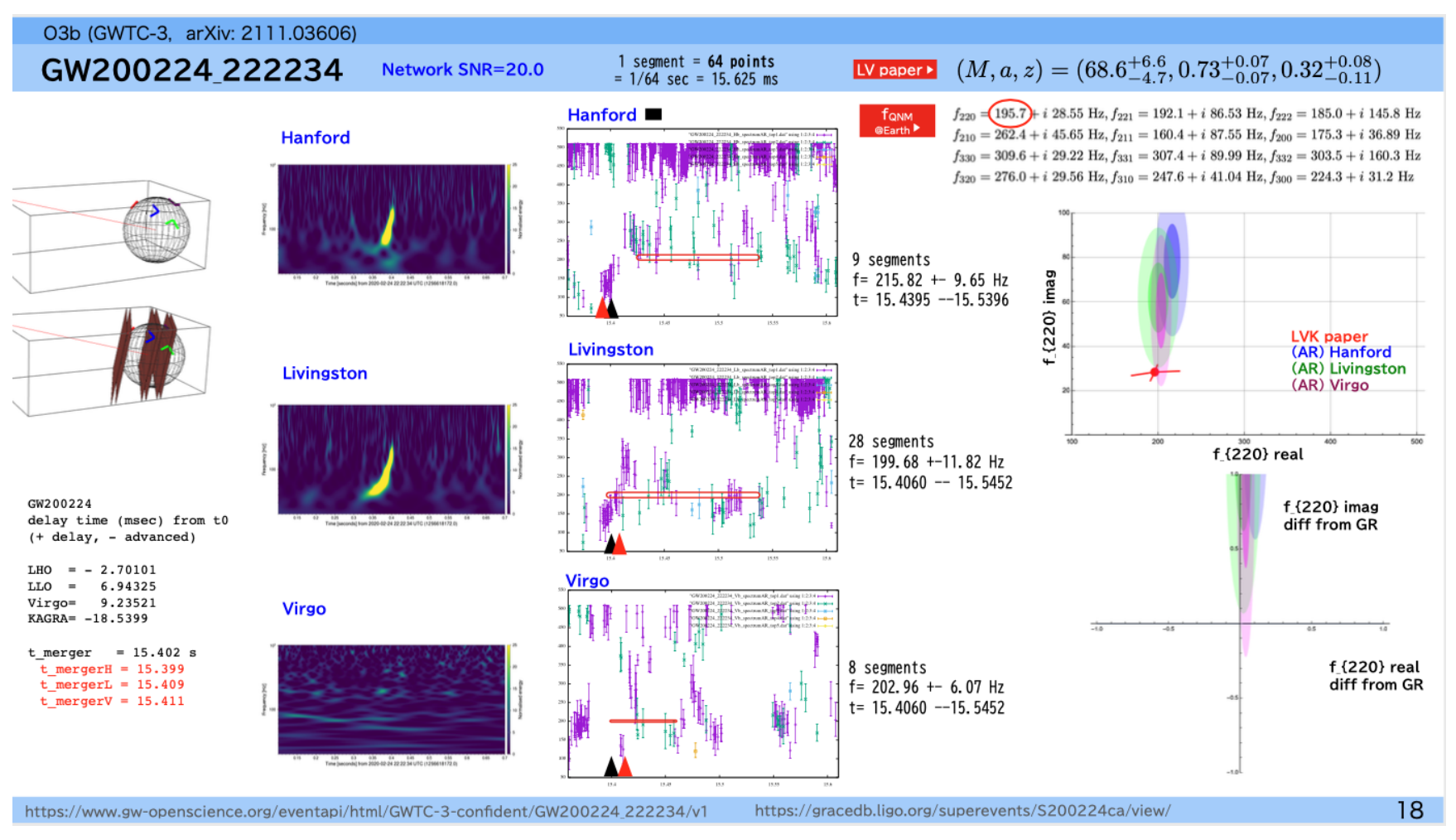
GW200129_065458　Network SNR=26.8　1 segment = 64 points = 1/64 sec = 15.625 ms　LV paper　$(M, a, z) = (60.3^{+4.0}_{-3.3}, 0.73^{+0.06}_{-0.05}, 0.18^{+0.05}_{-0.07})$

$f_{220} = 249.0 + i\,36.34$ Hz, $f_{221} = 244.4 + i\,110.1$ Hz, $f_{222} = 235.4 + i\,185.5$ Hz,
$f_{210} = 334.0 + i\,58.09$ Hz, $f_{211} = 204.2 + i\,111.4$ Hz, $f_{200} = 223.1 + i\,46.94$ Hz,
$f_{330} = 394.0 + i\,37.18$ Hz, $f_{331} = 391.3 + i\,114.5$ Hz, $f_{332} = 386.3 + i\,204.1$ Hz,
$f_{320} = 351.2 + i\,37.62$ Hz, $f_{310} = 315.1 + i\,52.23$ Hz, $f_{300} = 285.4 + i\,39.7$ Hz,

Hanford: 45 segments, f= 244.68+- 10.90 Hz, t= 15.4399 --15.5796 — f_{220} starts 5ms after t_merger
Livingston: 31 segments, f= 246.20 +- 11.28 Hz, t= 15.4370 -- 15.5840
Virgo: 33 segments, f= 259.69 +- 6.31 Hz, t= 15.4548-15.6497 — f_{220} starts 3ms after t_merger

```
GW200129
delay time (msec) from t0
(+ delay, - advanced)

LHO  = - 7.5579
LLO  =   1.71251
Virgo=   9.73468

t_merger   = 15.435 s
 t_mergerH = 15.427
 t_mergerL = 15.437
 t_mergerV = 15.445

   + 7.8125s
 t_mergerH = 15.435
 t_mergerL = 15.445
 t_mergerV = 15.452
```

https://www.gw-openscience.org/eventapi/html/GWTC-3-confident/GW200129_065458/v1　https://gracedb.ligo.org/superevents/S200129m/view/　17



O3b (GWTC-3, arXiv: 2111.03606)

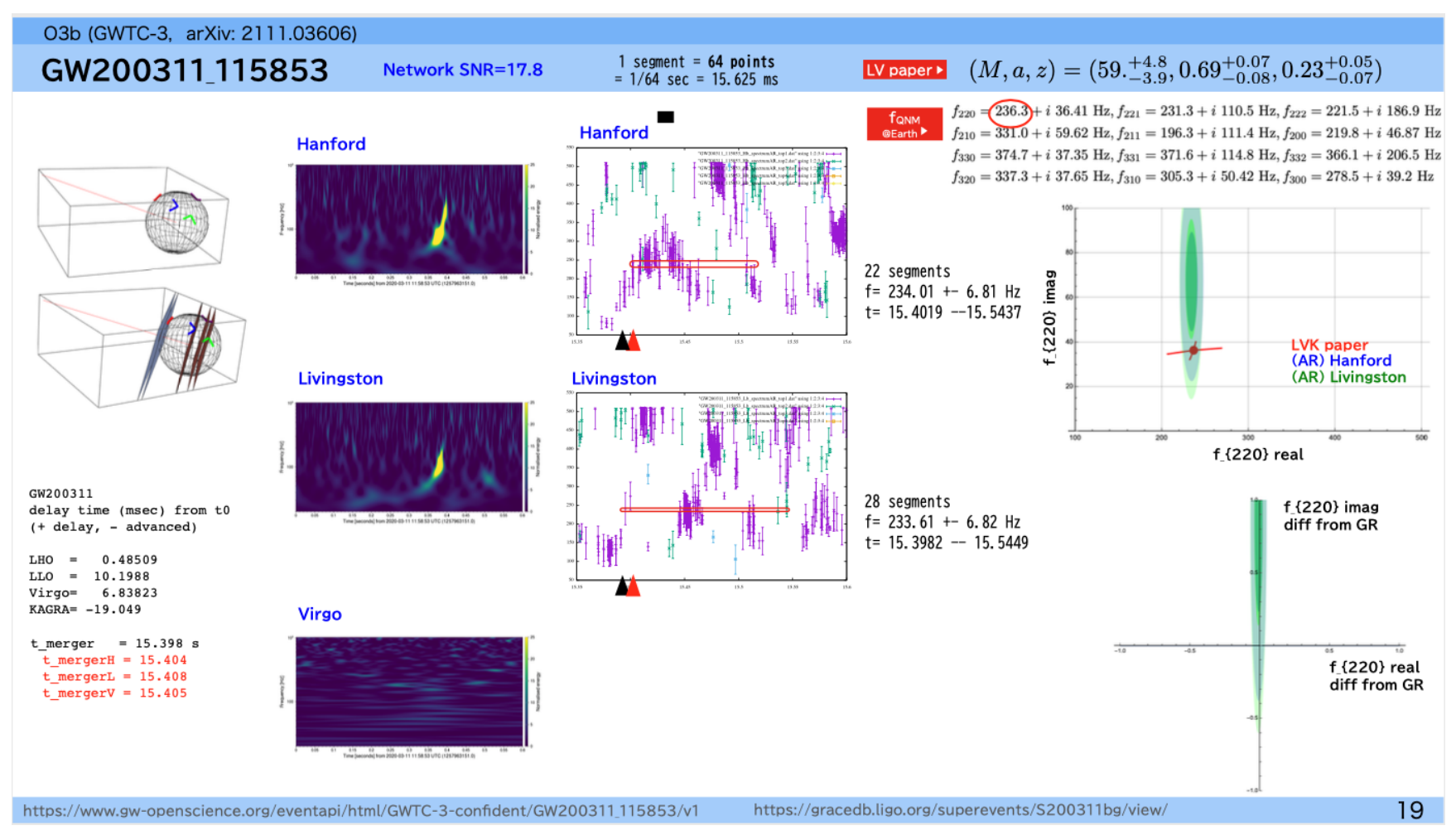
GW200224_222234　Network SNR=20.0　1 segment = 64 points = 1/64 sec = 15.625 ms　LV paper　$(M, a, z) = (68.6^{+6.6}_{-4.7}, 0.73^{+0.07}_{-0.07}, 0.32^{+0.08}_{-0.11})$

$f_{220} = 195.7 + i\,28.55$ Hz, $f_{221} = 192.1 + i\,86.53$ Hz, $f_{222} = 185.0 + i\,145.8$ Hz
$f_{210} = 262.4 + i\,45.65$ Hz, $f_{211} = 160.4 + i\,87.55$ Hz, $f_{200} = 175.3 + i\,36.89$ Hz
$f_{330} = 309.6 + i\,29.22$ Hz, $f_{331} = 307.4 + i\,89.99$ Hz, $f_{332} = 303.5 + i\,160.3$ Hz
$f_{320} = 276.0 + i\,29.56$ Hz, $f_{310} = 247.6 + i\,41.04$ Hz, $f_{300} = 224.3 + i\,31.2$ Hz

Hanford: 9 segments, f= 215.82 +- 9.65 Hz, t= 15.4395 --15.5396
Livingston: 28 segments, f= 199.68 +-11.82 Hz, t= 15.4060 -- 15.5452
Virgo: 8 segments, f= 202.96 +- 6.07 Hz, t= 15.4060 --15.5452

```
GW200224
delay time (msec) from t0
(+ delay, - advanced)

LHO  = - 2.70101
LLO  =   6.94325
Virgo=   9.23521
KAGRA= -18.5399

t_merger   = 15.402 s
 t_mergerH = 15.399
 t_mergerL = 15.409
 t_mergerV = 15.411
```

https://www.gw-openscience.org/eventapi/html/GWTC-3-confident/GW200224_222234/v1　https://gracedb.ligo.org/superevents/S200224ca/view/　18



O3b (GWTC-3, arXiv: 2111.03606)

GW200311_115853　Network SNR=17.8　1 segment = 64 points = 1/64 sec = 15.625 ms　LV paper　$(M, a, z) = (59.^{+4.8}_{-3.9}, 0.69^{+0.07}_{-0.08}, 0.23^{+0.05}_{-0.07})$

$f_{220} = 236.3 + i\,36.41$ Hz, $f_{221} = 231.3 + i\,110.5$ Hz, $f_{222} = 221.5 + i\,186.9$ Hz
$f_{210} = 331.0 + i\,59.62$ Hz, $f_{211} = 196.3 + i\,111.4$ Hz, $f_{200} = 219.8 + i\,46.87$ Hz
$f_{330} = 374.7 + i\,37.35$ Hz, $f_{331} = 371.6 + i\,114.8$ Hz, $f_{332} = 366.1 + i\,206.5$ Hz
$f_{320} = 337.3 + i\,37.65$ Hz, $f_{310} = 305.3 + i\,50.42$ Hz, $f_{300} = 278.5 + i\,39.2$ Hz

Hanford: 22 segments, f= 234.01 +- 6.81 Hz, t= 15.4019 --15.5437
Livingston: 28 segments, f= 233.61 +- 6.82 Hz, t= 15.3982 -- 15.5449

```
GW200311
delay time (msec) from t0
(+ delay, - advanced)

LHO  =   0.48509
LLO  =  10.1988
Virgo=   6.83823
KAGRA= -19.049

t_merger   = 15.398 s
 t_mergerH = 15.404
 t_mergerL = 15.408
 t_mergerV = 15.405
```

https://www.gw-openscience.org/eventapi/html/GWTC-3-confident/GW200311_115853/v1　https://gracedb.ligo.org/superevents/S200311bg/view/　19

Summary & Outlook

LV O1/O2/O3a & LVK O3b

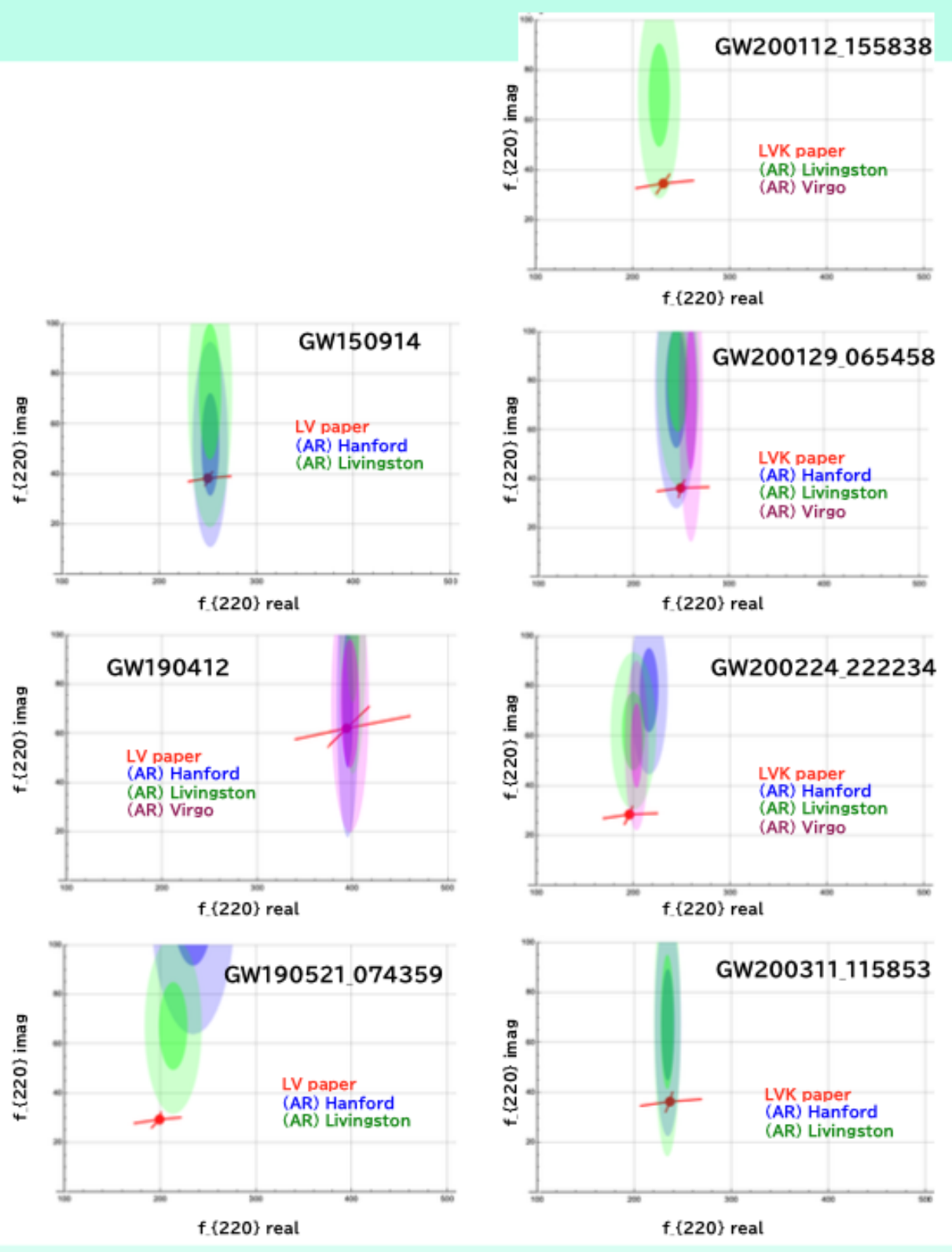
For events of S/N >= 15 (total S/N), AR picks up ringdown modes.
(We used the same parameters [merger time + 200 ms],
Band Filterings [20-600Hz] for all analysis)

➡ Can AR extract ring-down modes? **Yes**
Consistent between 3 IFs(Hanford, Livingston, Virgo) ? **Yes**
Consistent with the LVK catalog **Almost Yes**

➡ **Not perfect for f_imag (damping rate), estimating larger.**

➡ When the ring-down starts? **3ms-5ms after merger**
Overtones? Higher modes? **Not Yet**
Consistent with GR? **Consistent, so far.**

➡ **Consistency checks with other methods & parameter trials.**
➡ **Waiting good signals (large S/N)**



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