

Dynamics in n-dim Gauss-Bonnet gravity

I. Colliding Scalar Waves II. Wormhole evolutions

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Outline & Summary

We show how the dynamics depend on the dimensionality and how the higher-order curvature terms affect to singularity formation in two models: (I) colliding scalar pulses in planar space-time, and (II) perturbed wormhole in spherical symmetric space-time. Our numerical code uses dual-null formulation, and we compare the dynamics in 5, 6 and 7-dimensional Gauss-Bonnet gravity.

(1) For scalar wave collisions, we observe that curvature evolutions (Kretschmann invariant) are milder in the presence of Gauss-Bonnet term and/or in higher-dimensional space-time.

(2) For wormhole dynamics, we observe that the perturbed throat will be easily enhance in the presence of Gauss-Bonnet term. Both suggest that the thresholds for the singularity formations become higher in Gauss-Bonnet gravity, although it is not evitable.

1. Motivation

Dynamics in Gauss-Bonnet gravity?

- Action: $S = \int d^D x \sqrt{-g} \left[\frac{1}{2} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi) + \alpha_{GB} R^2 + \Lambda \right]$ where $\alpha_{GB} = R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$
- Field equation: $\alpha_{GB} \square^2 R + \alpha_{GB} \square^2 R_{\mu\nu} + \alpha_{GB} \square^2 R_{\mu\nu\rho\sigma} + \dots = -\lambda g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi$ where $\square = \nabla^\mu \nabla_\mu$

- has GR correction terms from String Theory
- has two solution branches (GR/non-GR).
- is expected to have singularity avoidance feature. (but has never been demonstrated.)

new topic in numerical relativity.
S. Gold & T. Pirsa, PRD 85 (2012) 104015
N. Deppe, PRD 86 (2012) 104011
F. Izaurieta & E. Rodriguez, 1207.1496

much attentions in WH community
H. Shinkai & H. Niikawa, PRD 79 (2009) 034003
P. Kanti, B. Kleihaus & J. Kunz, PRL 107 (2011) 271101
P. Kanti, B. Kleihaus & J. Kunz, PRD 85 (2012) 044007

Field Eqs.

Field Equations (1)
Formulation for evolution [dual null]

Metric: $ds^2 = -2e^{2\psi} du dv + e^{2\chi} dx^2 + e^{2\gamma} dy^2$

Variables: ψ Conformal factor, χ expansion, f lapse function, α_{GB} infinity (kth)

Parameters: n dimension, Λ cosmological constant

For simplicity we define:
 $\phi = (n-2)\psi$ scalar field (normal)
 $\pi_x = \partial_x \psi$ scalar momentum
 $\pi_z = \partial_z \psi$ scalar momentum

Field Equations (2)
matter variables

normal field $\psi(x, v)$ and/or ghost field $\phi(x, v)$

$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(g)}$
 $T_{\mu\nu}^{(m)} = \partial_\mu \psi \partial_\nu \psi - \frac{1}{2} g_{\mu\nu} (\partial^\alpha \psi \partial_\alpha \psi)$
 $T_{\mu\nu}^{(g)} = -\frac{1}{2} g_{\mu\nu} (\partial^\alpha \phi \partial_\alpha \phi)$

this derives Klein-Gordon equations:
 $\square \psi = \frac{\partial^2 \psi}{\partial v^2} - \frac{\partial^2 \psi}{\partial u^2} + \dots$

Klein-Gordon eqs.
 $\square \phi = \frac{\partial^2 \phi}{\partial v^2} - \frac{\partial^2 \phi}{\partial u^2} + \dots$

Energy-momentum tensor:
 $T_{\mu\nu} = \partial_\mu \psi \partial_\nu \psi - \frac{1}{2} g_{\mu\nu} (\partial^\alpha \psi \partial_\alpha \psi)$
 $T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial^\alpha \phi \partial_\alpha \phi)$

Field Equations (3)
evolution equations (1)

Equations for ψ direction:

$$\partial_u \psi = -\frac{1}{2} \partial_v \psi^2$$

$$\partial_u \chi = -\frac{1}{2} \partial_v \chi^2$$

$$\partial_u \alpha_{GB} = -\frac{1}{2} \partial_v \alpha_{GB}^2$$

Field Equations (4)
evolution equations (2)

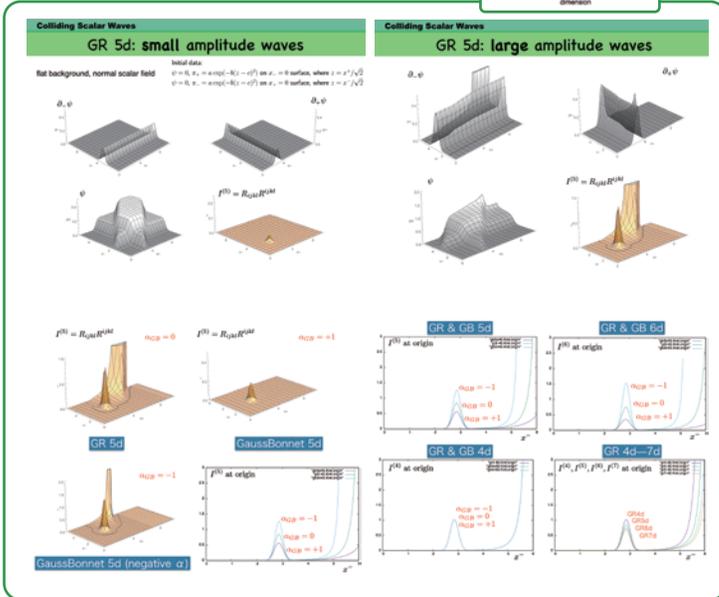
Equations for ϕ direction:

$$\partial_u \phi = -\frac{1}{2} \partial_v \phi^2$$

$$\partial_u \chi = -\frac{1}{2} \partial_v \chi^2$$

$$\partial_u \alpha_{GB} = -\frac{1}{2} \partial_v \alpha_{GB}^2$$

Colliding Scalar Waves



Wormhole Evolutions

1. Motivation

Why Wormhole?

They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.

They are very similar to black holes --both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

BH & WH are interconvertible?
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar --both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH). Only the causal nature of the TH differs, whether this evolve in plus / minus density which is given locally.

	Black Hole	Wormhole
Location	Achiral (spatial) null surface TH	Temporal (timelike) null TH
Travelability	1-way traversable	2-way traversable
Content	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Occurrence	Occur naturally	Unlikely to occur naturally, but constructible??

initial data

- Static condition: $\partial_t \psi = 0, \partial_t \chi = 0 \Rightarrow \partial_t \psi = 0, \partial_t \chi = 0$
- $\partial_t \psi = 0, \partial_t \chi = 0 \Rightarrow \partial_t \psi = 0, \partial_t \chi = 0$
- $\partial_t \psi = 0, \partial_t \chi = 0 \Rightarrow \partial_t \psi = 0, \partial_t \chi = 0$
- $\partial_t \psi = 0, \partial_t \chi = 0 \Rightarrow \partial_t \psi = 0, \partial_t \chi = 0$

Solve ψ and χ equations with the starting condition at the throat

$\psi = \psi_0 + \dots$
 $\chi = \chi_0 + \dots$

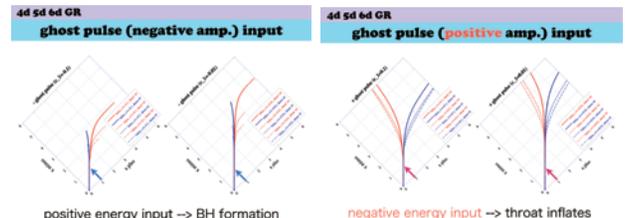
If we assume only ghost field ϕ , then

$\phi = \phi_0 + \dots$

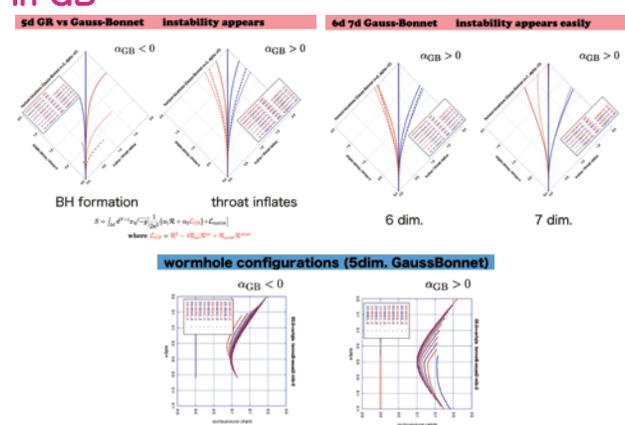
add perturbation

$\psi(x^+, x^-, t) = \psi_0(x^+, x^-, t) + \alpha_{GB} \delta \psi(x^+, x^-, t)$

in GR



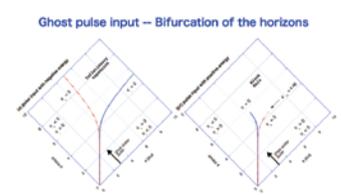
in GB



Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons
 \rightarrow go to a Black Hole or inflationary expansion



ワームホールを通過できるか

Time Machine & Science of Space-time (HS, 2011)

Figure 3. Bifurcation diagrams, $\alpha_{GB} = 0$, for perturbed wormhole. Fig.(a) is the case we regularize the ghost field, $\alpha_{GB} = 0.1$, and Fig.(b) and Fig.(c) are when we regularize the field, $\alpha_{GB} = 0.1$ and $\alpha_{GB} = 0.01$. Shaded lines and solid lines are $\psi = 0$ and $\chi = 0$ respectively. In all cases, the pair lines for wormhole throat at $(\psi = 0, \chi = 0)$. A $\psi = 0$ maximum/minimum relation of the figure corresponds to a partial throat diagram.