Dynamics in n-dimensional Gauss-Bonnet gravity (II)

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論点

＊4dim, 5dim, 6dim, … ダイナミクスはどう変化するか

＊Gauss-Bonnet項は，ダイナミクスにどう影響するか

Part I　Field Equations (dual-null formulation)

Part II　平面対称時空：Colliding Scalar Waves

<u>Part III　球対称時空：Wormhole-BH transition</u>

http://www.oit.ac.jp/is/~shinkai/　　2016年3月20日　物理学会＠東北学院大

Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\}+\mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **has minimum mass for static spherical BH solution**
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated in full gravity.)

- **new topic in numerical relativity.**
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496

- **much attentions in WH community**
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

Formulation for evolution [dual null]

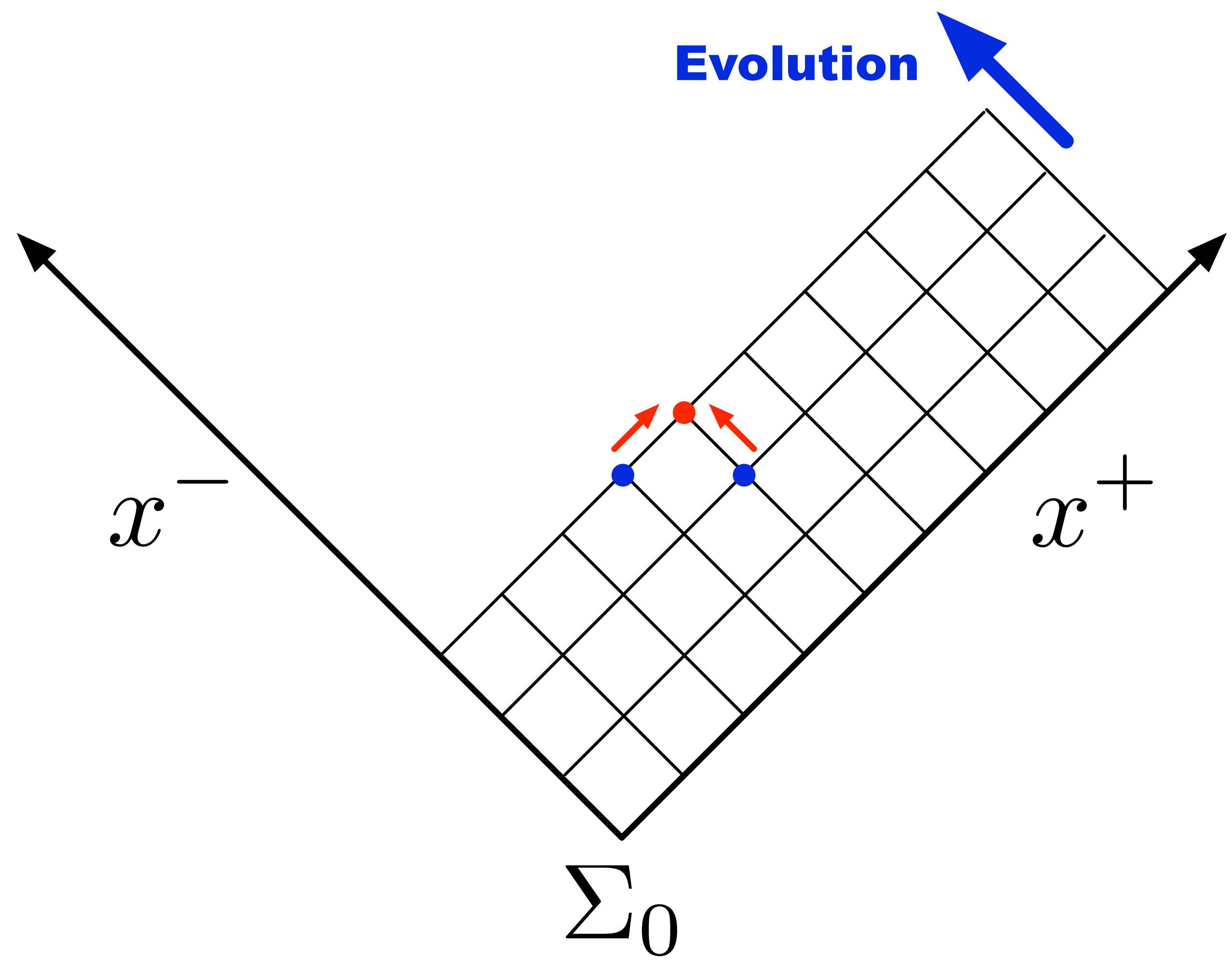
Metric n-dimensional, dual-null coordinate, $2+(n-2)$ decomposition

$$ds^2 = -2e^{-f(x^+,x^-)}\,dx^+\,dx^- + r^2(x^+,x^-)\gamma_{ij}dx^i dx^j \quad (1)$$

Variables

$\Omega = \frac{1}{r}$	Conformal factor
$\vartheta_\pm = (n-2)\partial_\pm r$	expansion
f	lapse function
$\nu_\pm = \partial_\pm f$	inaffinity (shift)

ψ	scalar field (normal)
$\pi_\pm = r\partial_\pm \psi$	scalar momentum
ϕ	scalar field (ghost)
$p_\pm = r\partial_\pm \phi$	scalar momentum



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Parameters

n	dimension
k	curvature
Λ	cosmological constant

For simplicity, we define

$$\tilde{\alpha} = (n-3)(n-4)\alpha_2, \tag{2}$$

$$A = \alpha_1 + 2\tilde{\alpha}\Omega^2 Z, \tag{3}$$

$$W = \frac{2e^f}{(n-2)^2}\vartheta_+\vartheta_-, \tag{4}$$

$$Z = k + W, \tag{5}$$

$$\eta = \Omega^2\frac{(n-2)(n-3)}{2}e^{-f}Z, \tag{6}$$

Field Equations (2)

matter variables

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) \\ &= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \end{aligned}$$

this derives Klein-Gordon equations

$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}.$$

Scalar field variables

$$\begin{aligned} \pi_\pm &\equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi \\ p_\pm &\equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi \end{aligned}$$

Klein-Gordon eqs.

$$\begin{aligned} \Box\phi &= -\frac{e^f}{r}\left(2r\phi_{uv} + (n-2)r_u\phi_v + (n-2)r_v\phi_u\right) \\ &= -2e^f\phi_{uv} - e^f\Omega^2\left(\vartheta_- p_+ + \vartheta_+ p_-\right) \end{aligned}$$

Energy-momentum tensor

$$\begin{aligned} T_{++} &= \Omega^2(\pi_+^2 - p_+^2) \\ T_{--} &= \Omega^2(\pi_-^2 - p_-^2) \\ T_{+-} &= -e^{-f}\left(V_1(\psi) + V_2(\phi)\right) \\ T_{zz} &= e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}\left(V_1(\psi) - V_2(\phi)\right) \end{aligned}$$

evolution equations (1)

Equations for x^+ direction

$$
\begin{aligned}
\partial_+\Omega &= -\frac{1}{n-2}\vartheta_+\Omega^2 && (7)\\
\partial_+\vartheta_+ &= -\vartheta_+\nu_+ - \frac{1}{\Omega A}\kappa^2 T_{++} = -\vartheta_+\nu_+ - \frac{1}{A}\kappa^2\Omega(\pi_+^2 - p_+^2) && (8)\\
\partial_+\vartheta_- &= \frac{1}{A}\frac{e^{-f}}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}Z + \Lambda + \kappa^2(V_1+V_2)\right] - \frac{\tilde{\alpha}}{A}\Omega^3 e^{-f}\frac{(n-2)(n-5)}{2}\left[Z^2+W\right] && (9)\\
\partial_+ f &= \nu_+ && (10)\\
\partial_+\nu_+ &= \text{no evolution eq. exists}\\
\partial_+\nu_- &= \frac{\alpha_1}{A}Ze^{-f}\Omega^2\frac{(n-3)}{2}\left\{-\frac{\alpha_1}{A}2(n-3)+n-4\right\}\\
&\quad +\frac{1}{A}\Omega^2 e^{-f}\kappa^2(\pi_+\pi_- - p_+p_-) + \frac{1}{A}e^{-f}\left\{\frac{\alpha_1}{A}\frac{2(n-3)}{(n-2)} - 1\right\}\{\Lambda+\kappa^2(V_1+V_2)\}\\
&\quad -\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{\alpha_1}{A}\Omega^2(n-3)\left\{k^2+2WZ+2Z^2\right\} + \frac{\tilde{\alpha}}{A}\Omega^4 2(n-5)\left\{k^2+2WZ\right\}Z\right]\\
&\quad +\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{1}{2}\Omega^2\left\{(n-2)k^2+2WZ-4Z^2\right\} + \frac{1}{A}\frac{4}{n-2}Z\left\{\Lambda+\kappa^2(V_1+V_2)\right\}\right]\\
&\quad -\frac{\tilde{\alpha}}{A}e^{f}\Omega^2\frac{4}{(n-2)^2}\left\{\nu_+\vartheta_+(\partial_-\vartheta_-) + \nu_-\vartheta_-(\partial_+\vartheta_+) + (\partial_+\vartheta_+)(\partial_-\vartheta_-) + \nu_+\nu_-\vartheta_+\vartheta_- - (\partial_-\vartheta_+)^2\right\} && (11)\\
\partial_+\psi &= \Omega\pi_+ && (12)\\
\partial_+\phi &= \Omega p_+ && (13)\\
\partial_+\pi_+ &= \text{no evolution eq. exists}\\
\partial_+\pi_- &= \left(\frac{1}{n-2}-\frac{1}{2}\right)\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} && (14)\\
\partial_+p_+ &= \text{no evolution eq. exists}\\
\partial_+p_- &= \left(\frac{1}{n-2}-\frac{1}{2}\right)\Omega\vartheta_+p_- - \frac{1}{2}\Omega\vartheta_-p_+ - \frac{1}{2e^f\Omega}\frac{d V_2}{d\phi} && (15)
\end{aligned}
$$

evolution equations (2)

Equations for x^- direction

$$\partial_- \Omega = -\frac{1}{n-2}\vartheta_- \Omega^2 \tag{16}$$

$$\partial_- \vartheta_+ = (9) \tag{17}$$

$$\partial_- \vartheta_- = -\vartheta_- \nu_- - \frac{1}{\Omega A}\kappa^2 T_{--} = -\vartheta_- \nu_- - \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2) \tag{18}$$

$$\partial_- f = \nu_- \tag{19}$$

$$\partial_- \nu_+ = (11) \tag{20}$$

$$\partial_- \nu_- = \text{no evolution eq. exists}$$

$$\partial_- \psi = \Omega \pi_- \tag{21}$$

$$\partial_- \phi = \Omega p_- \tag{22}$$

$$\partial_- \pi_+ = -\frac{1}{2}\Omega\vartheta_+ \pi_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_- \pi_+ - \frac{1}{2e^f \Omega}\frac{dV_1}{d\psi} \tag{23}$$

$$\partial_- \pi_- = \text{no evolution eq. exists}$$

$$\partial_- p_+ = -\frac{1}{2}\Omega\vartheta_+ p_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_- p_+ - \frac{1}{2e^f \Omega}\frac{dV_2}{d\phi} \tag{24}$$

$$\partial_- p_- = \text{no evolution eq. exists}$$

This constitutes the first-order dual-null form, suitable for numerical coding.

GR 5d: small amplitude waves

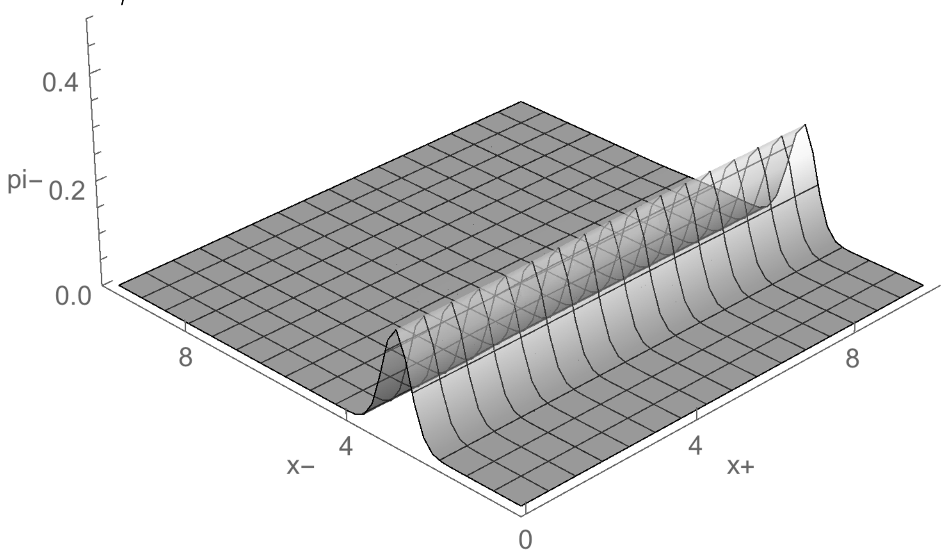
flat background, normal scalar field

Initial data:

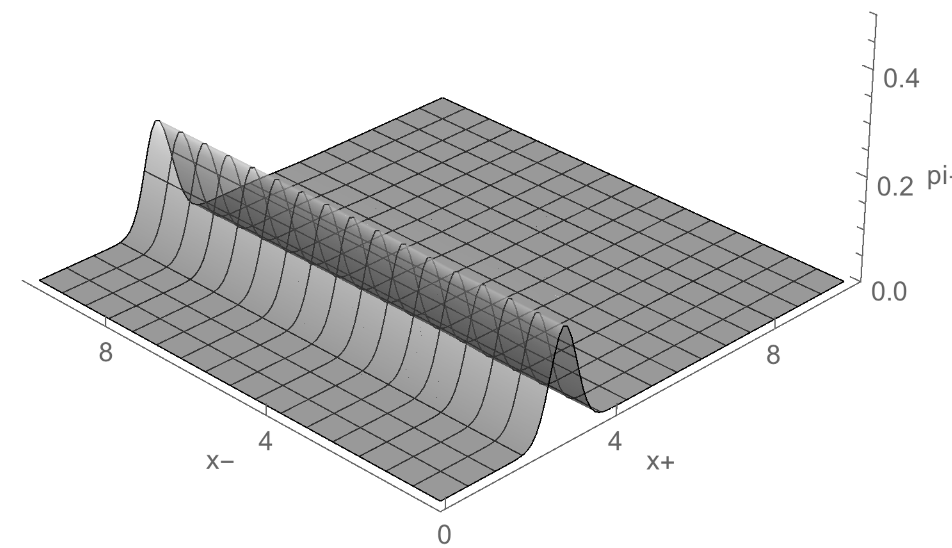
$\psi = 0,\ \pi_+ = a\exp(-b(z-c)^2)$ on $x_- = 0$ surface, where $z = x^+/\sqrt{2}$

$\psi = 0,\ \pi_- = a\exp(-b(z-c)^2)$ on $x_+ = 0$ surface, where $z = x^-/\sqrt{2}$

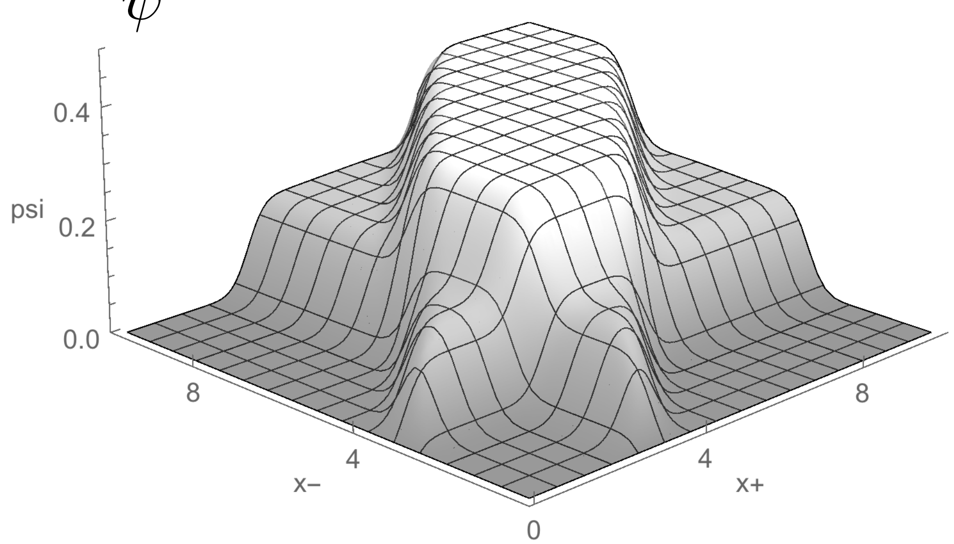
$\partial_- \psi$



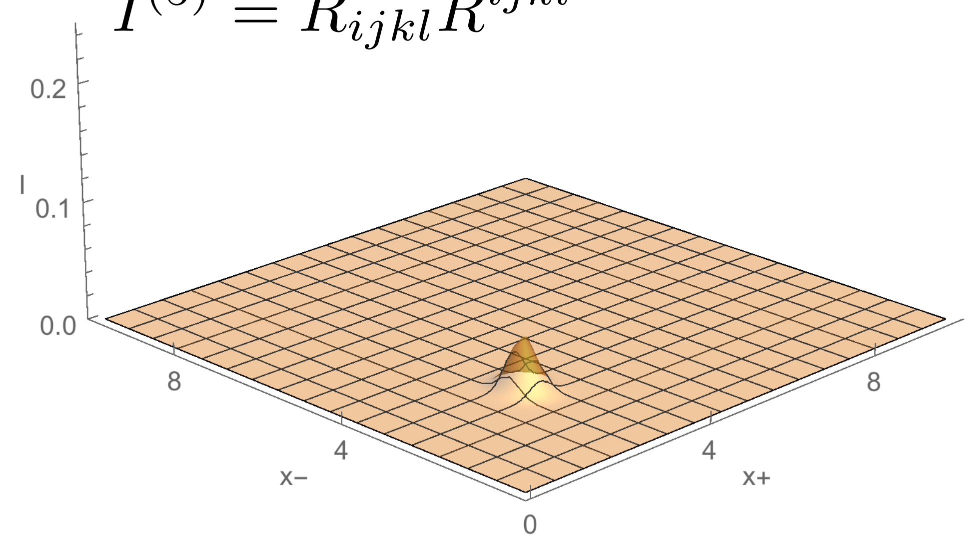
$\partial_+ \psi$



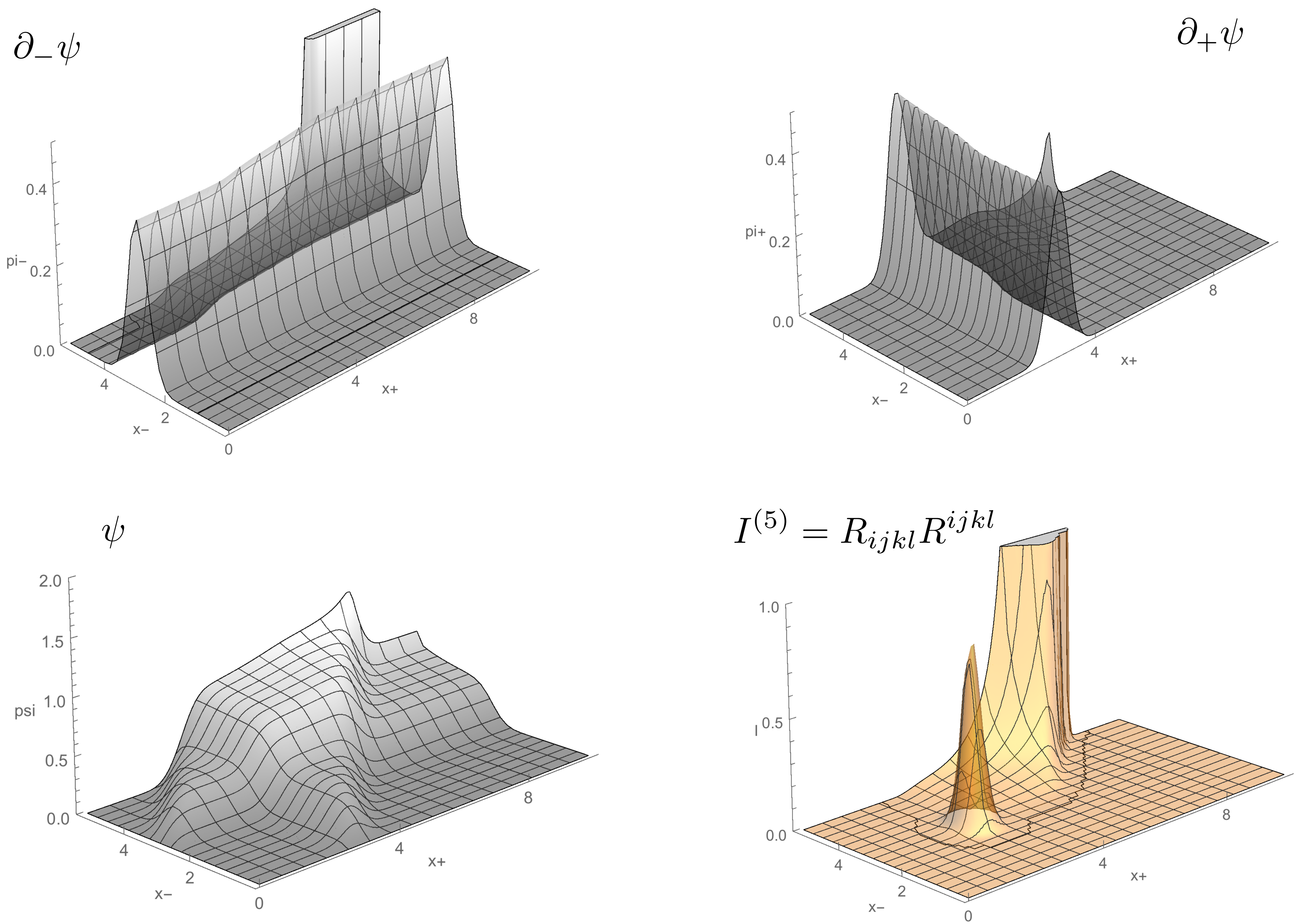
ψ

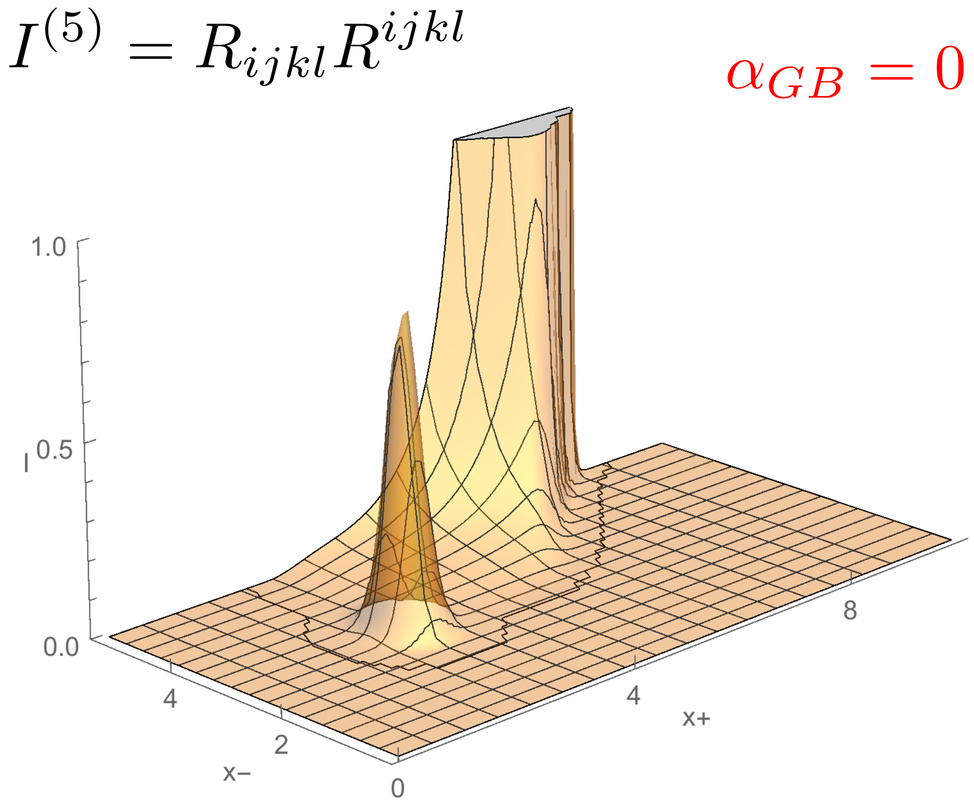


$I^{(5)} = R_{ijkl}R^{ijkl}$

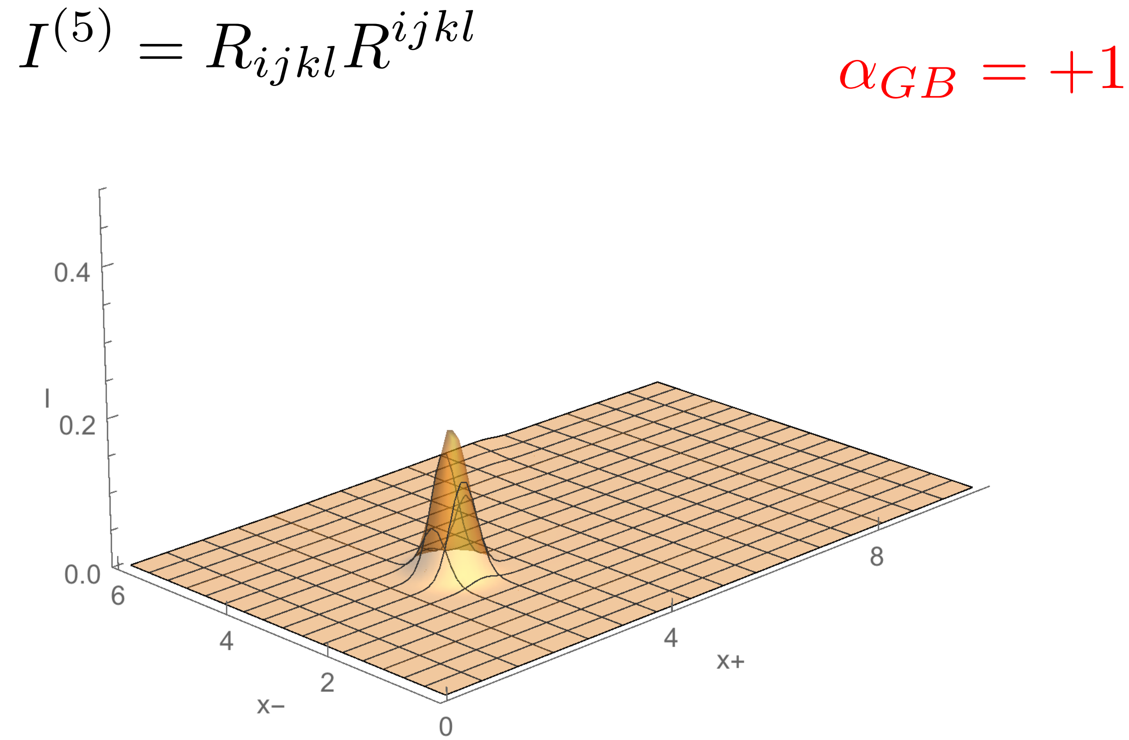


GR 5d: large amplitude waves

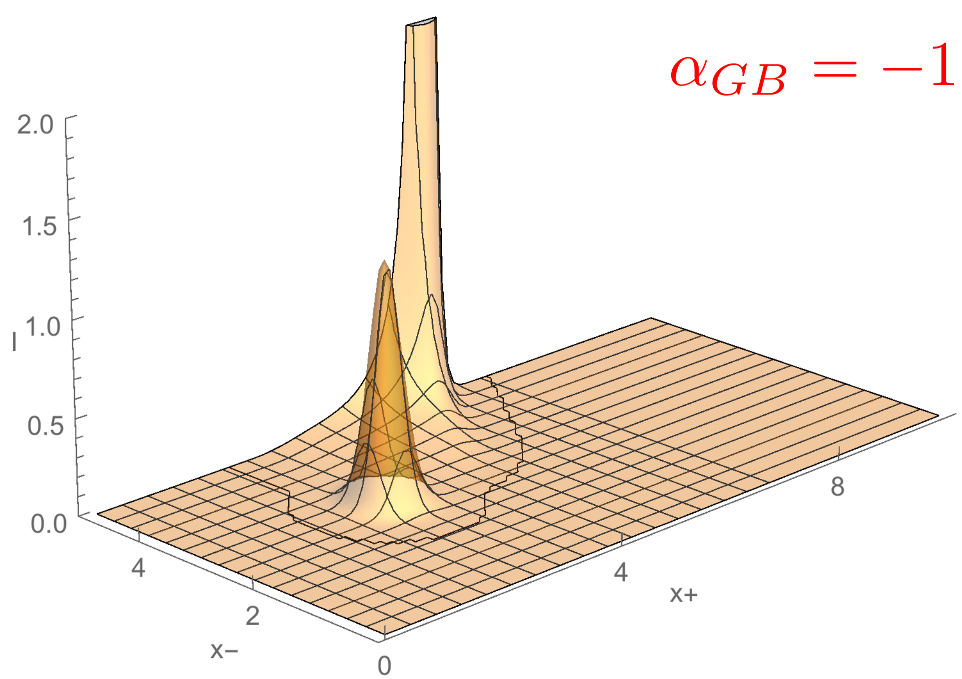




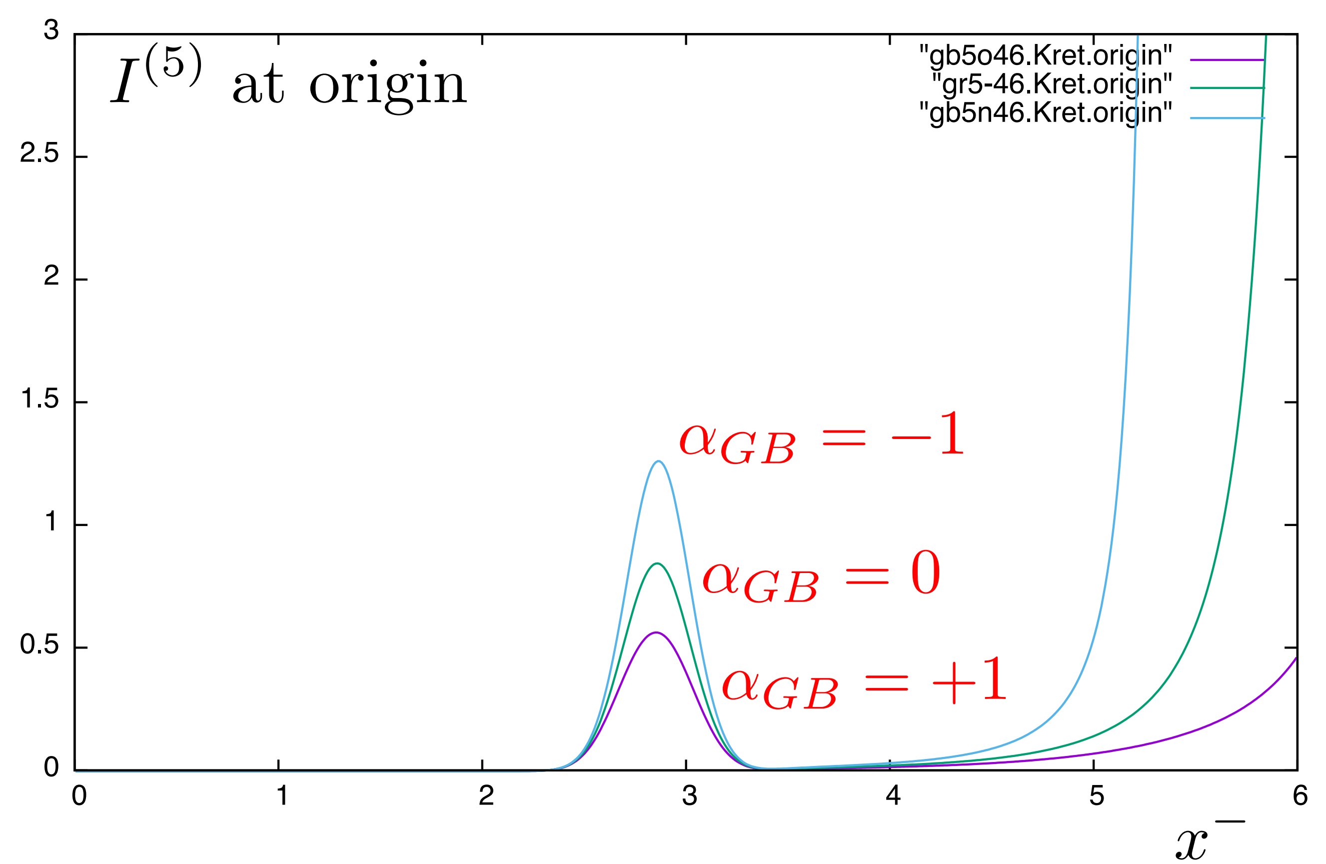
GR 5d



GaussBonnet 5d

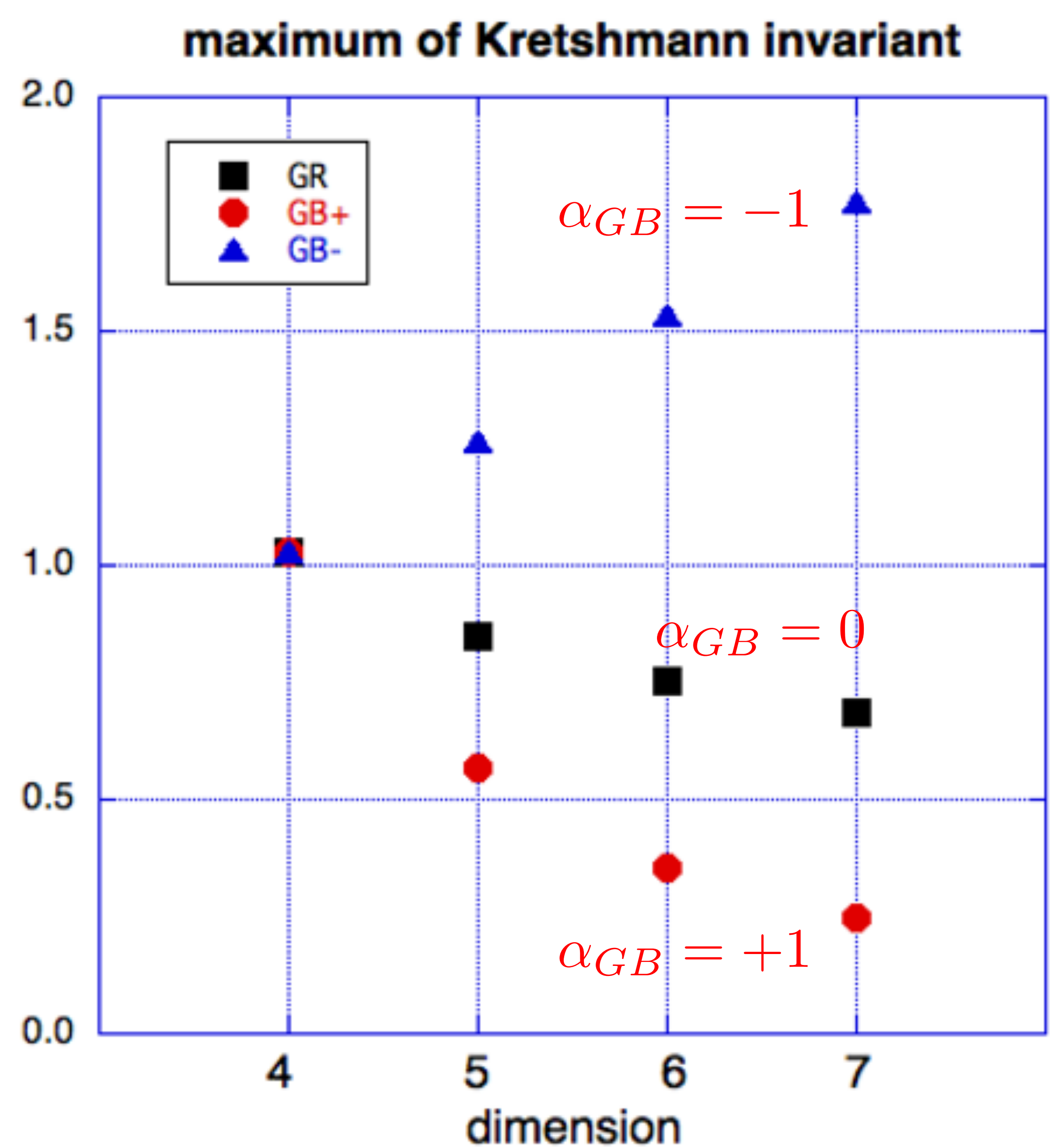


GaussBonnet 5d (negative α)



Colliding Scalar Waves massless scalar waveの衝突による特異点形成

$\max\ (R_{ijkl}R^{ijkl})$



5,6,7次元 Gauss-Bonnet

＊4dim, 5dim, 6dim,…　高次元化

＊Gauss-Bonnet項（正αの項）

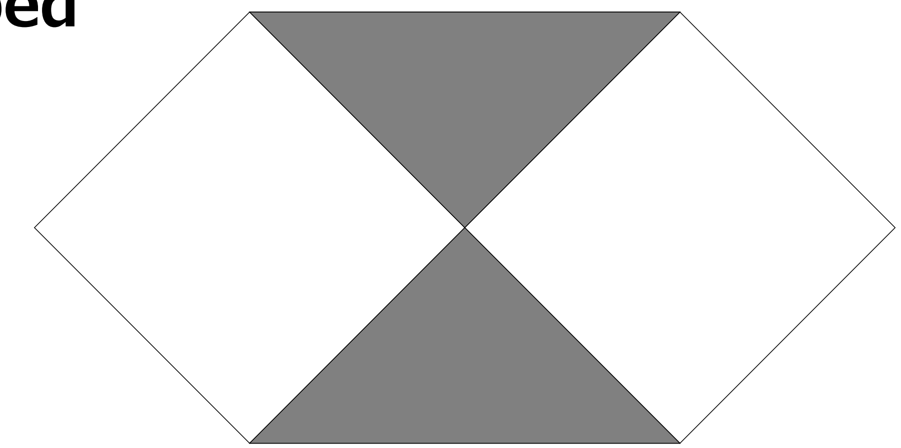
は，どちらも特異点形成条件を緩くさせる

BH & WH are interconvertible?

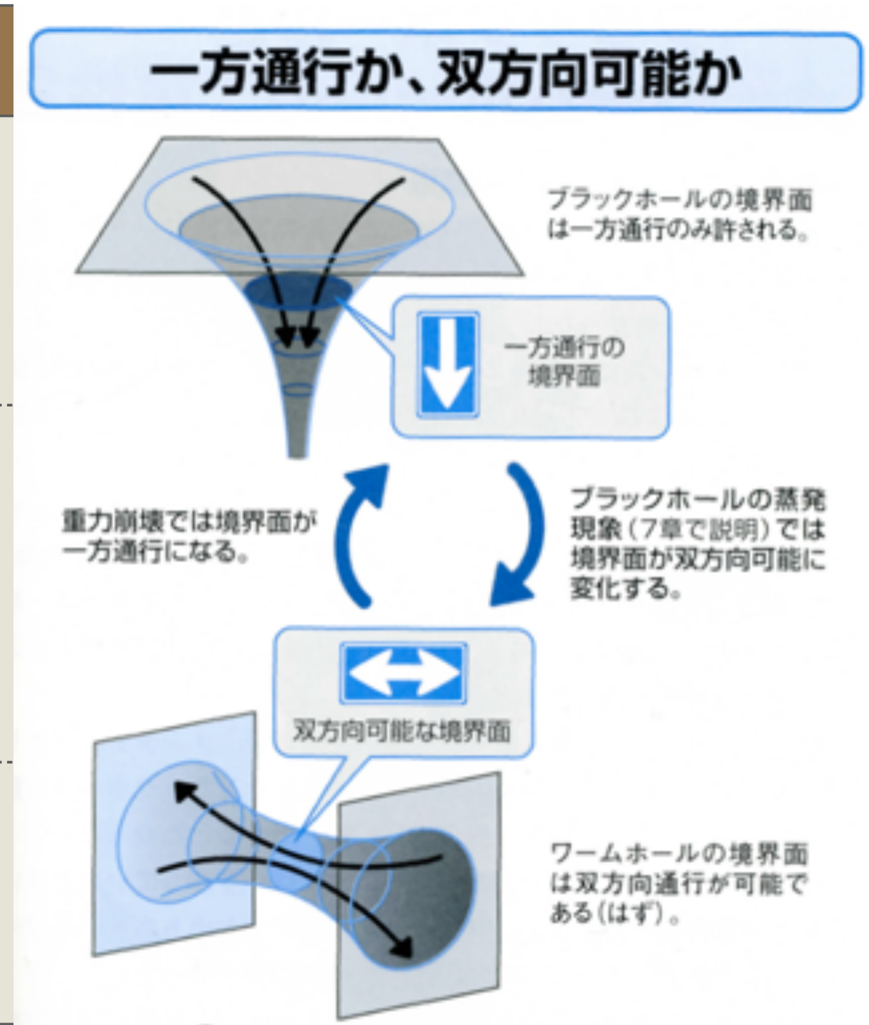
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)

Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.

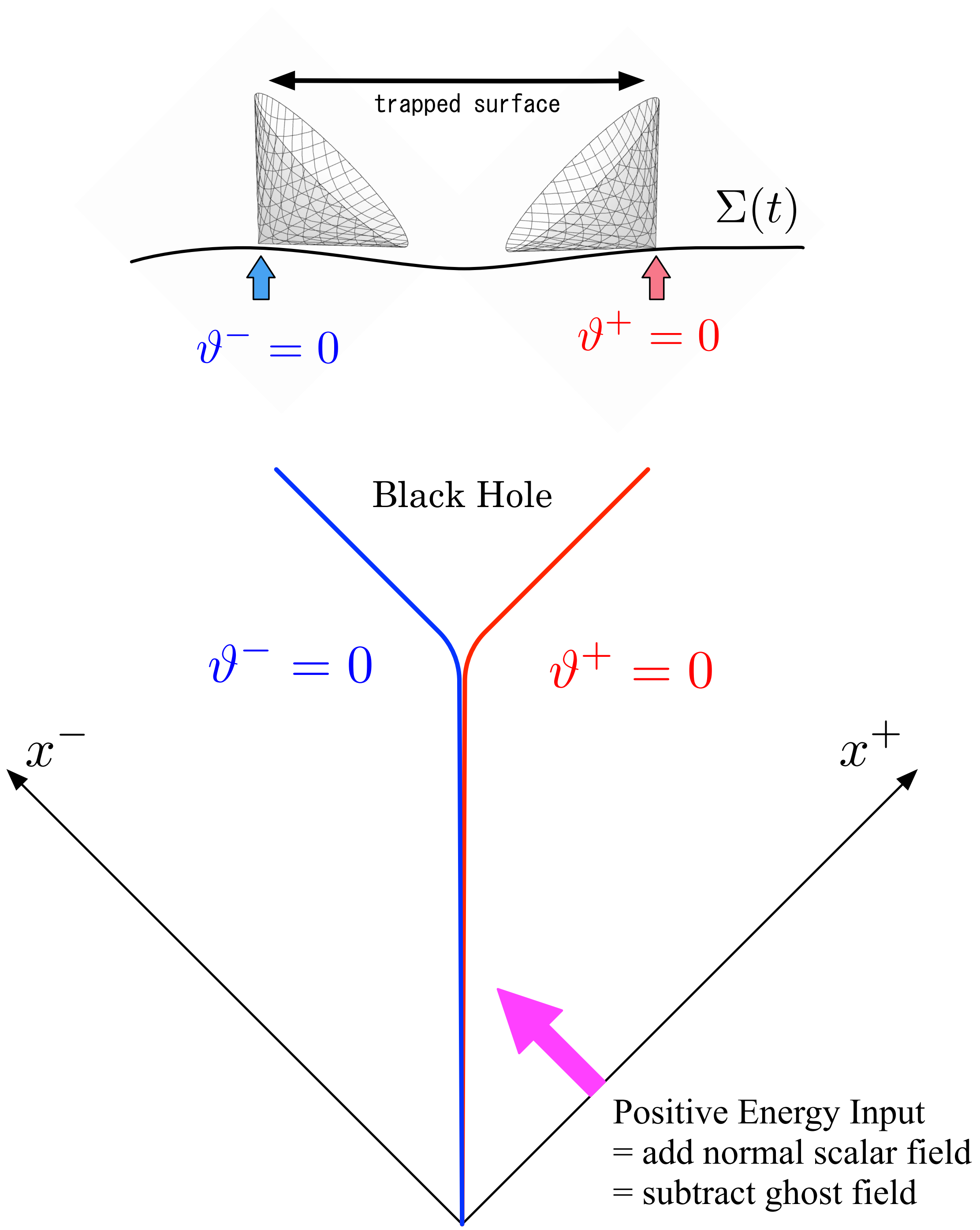


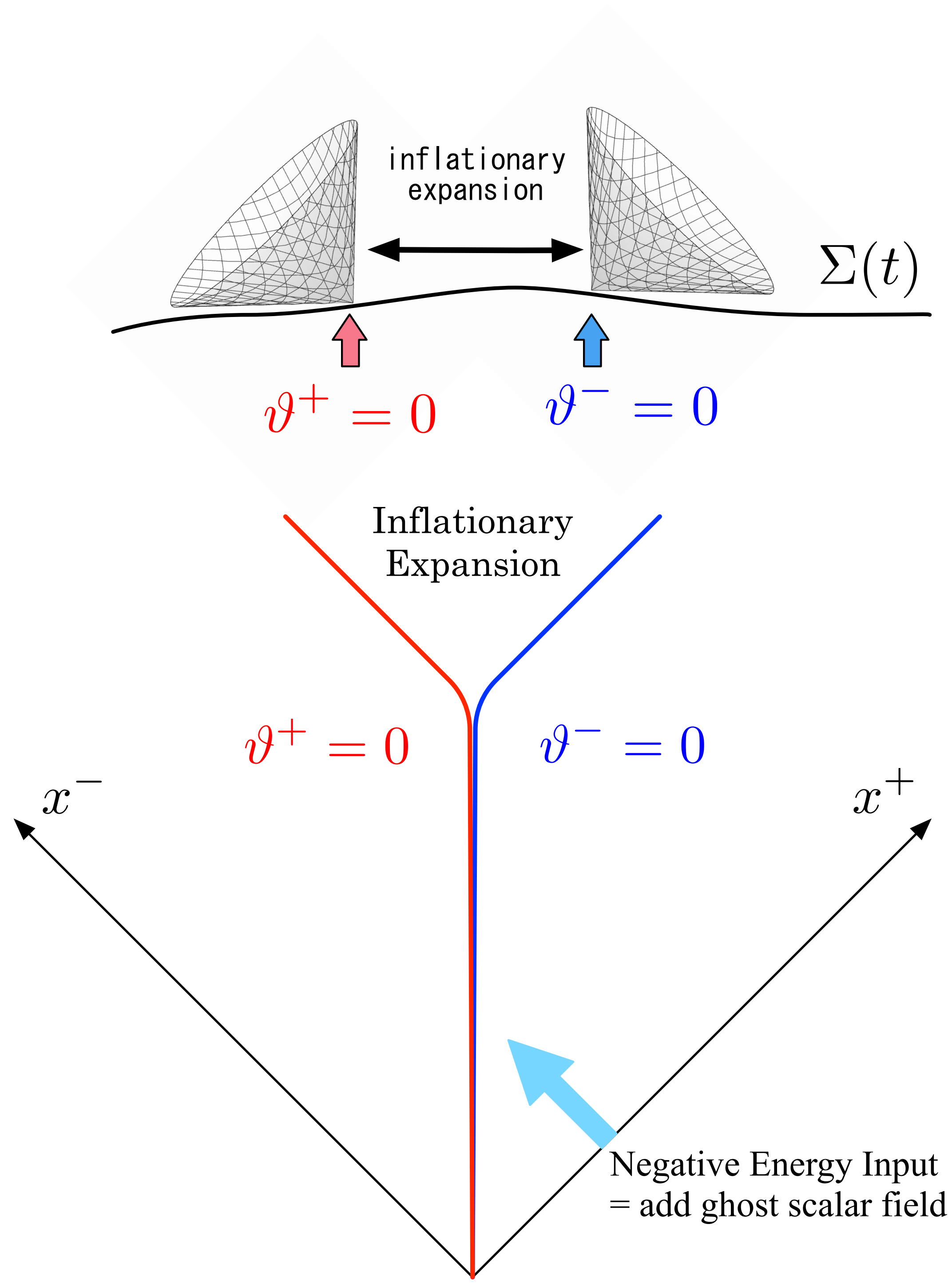
	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH ⇨ 1-way traversable	Temporal (timelike) outer THs ⇨ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density “exotic” matter
Appear-ance	occur naturally	Unlikely to occur naturally. but constructible??



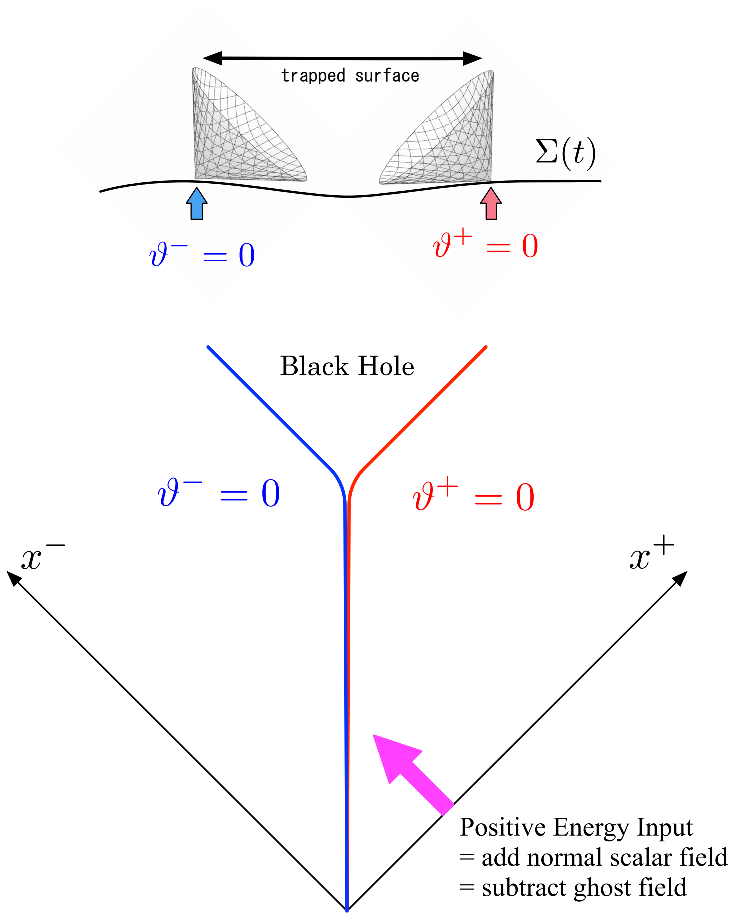
4d GR $\alpha_{\mathrm{GB}} = 0$ HS-Hayward PRD 66(2002) 044005

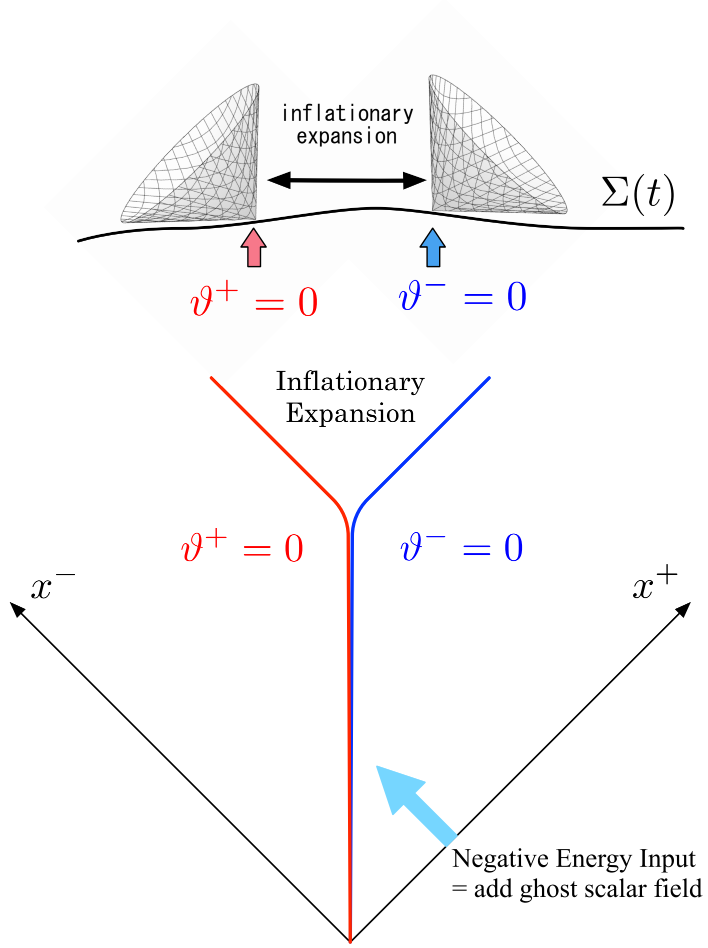
Wormhole evolutionのおさらい

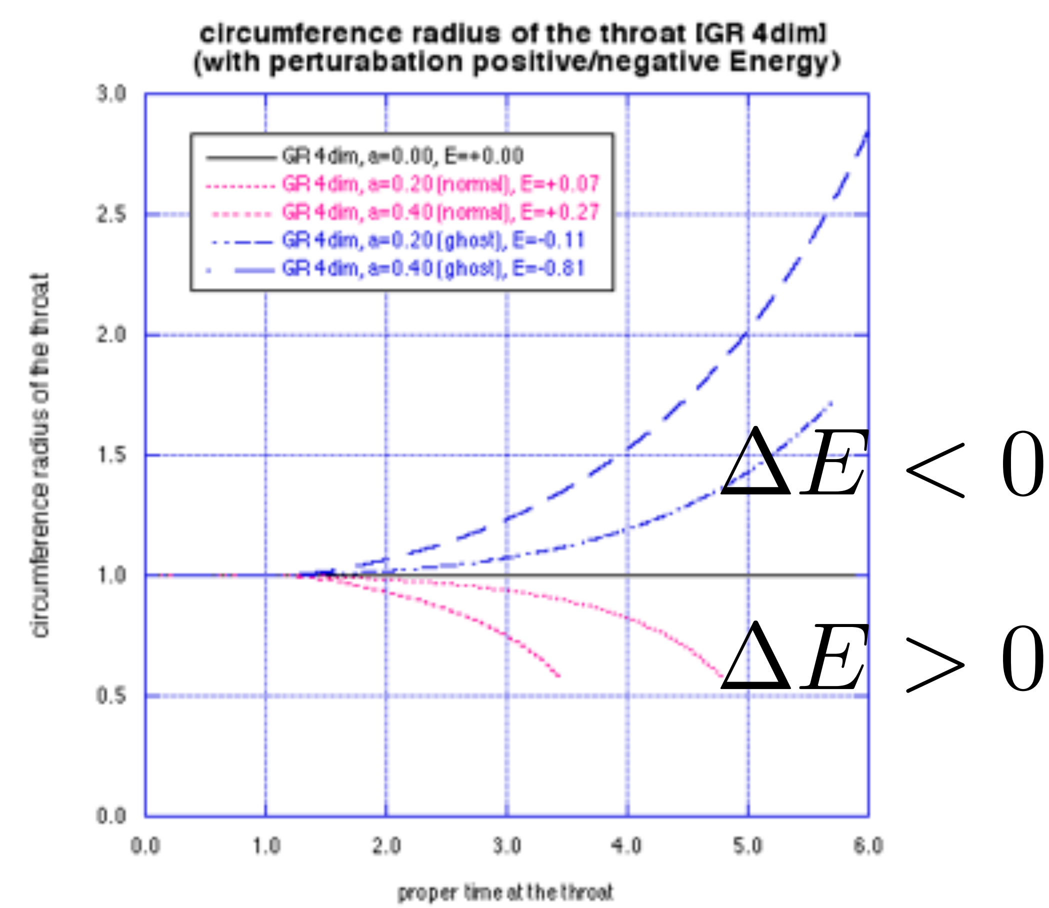




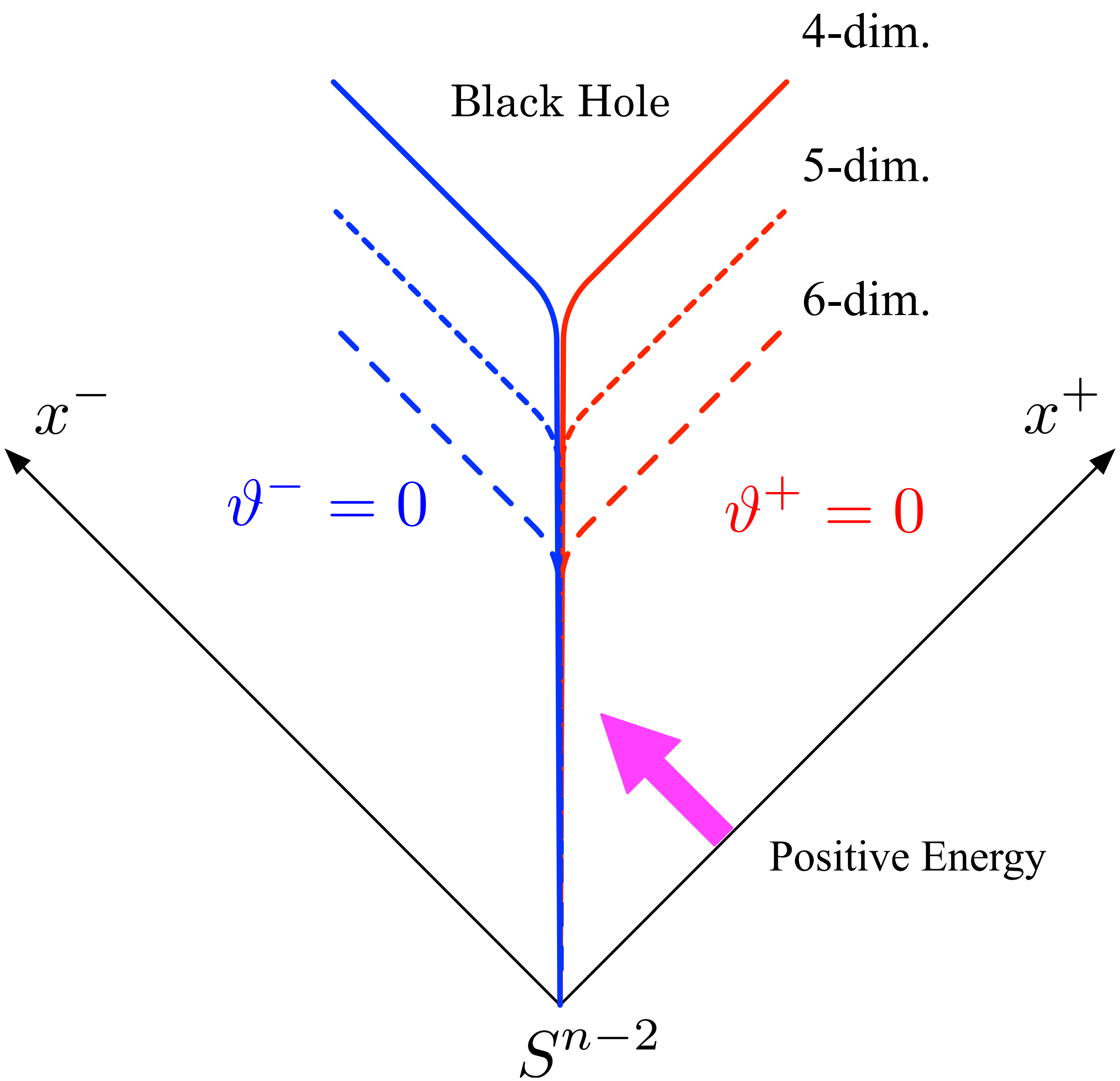
Wormhole evolutionのおさらい







Wormhole evolution in n-dim のおさらい



PHYSICAL REVIEW D **88,** 064027 (2013)

TABLE I. The negative eigenvalues ω^2.

n	ω^2
4	−1.39705243371511
5	−2.98495893027790
6	−4.68662054299460
7	−6.46258414126318
8	−8.28975936306259
9	−10.1535530451867
10	−12.0442650147438
11	−13.9552091676647
20	−31.5751101285105
50	−91.3457759137153
100	−191.283017729717

$$f(t,r) = f_0(r) + \varepsilon f_1(r) e^{i\omega t}, \tag{3.1}$$

$$\delta(t,r) = \delta_0(r) + \varepsilon \delta_1(r) e^{i\omega t}, \tag{3.2}$$

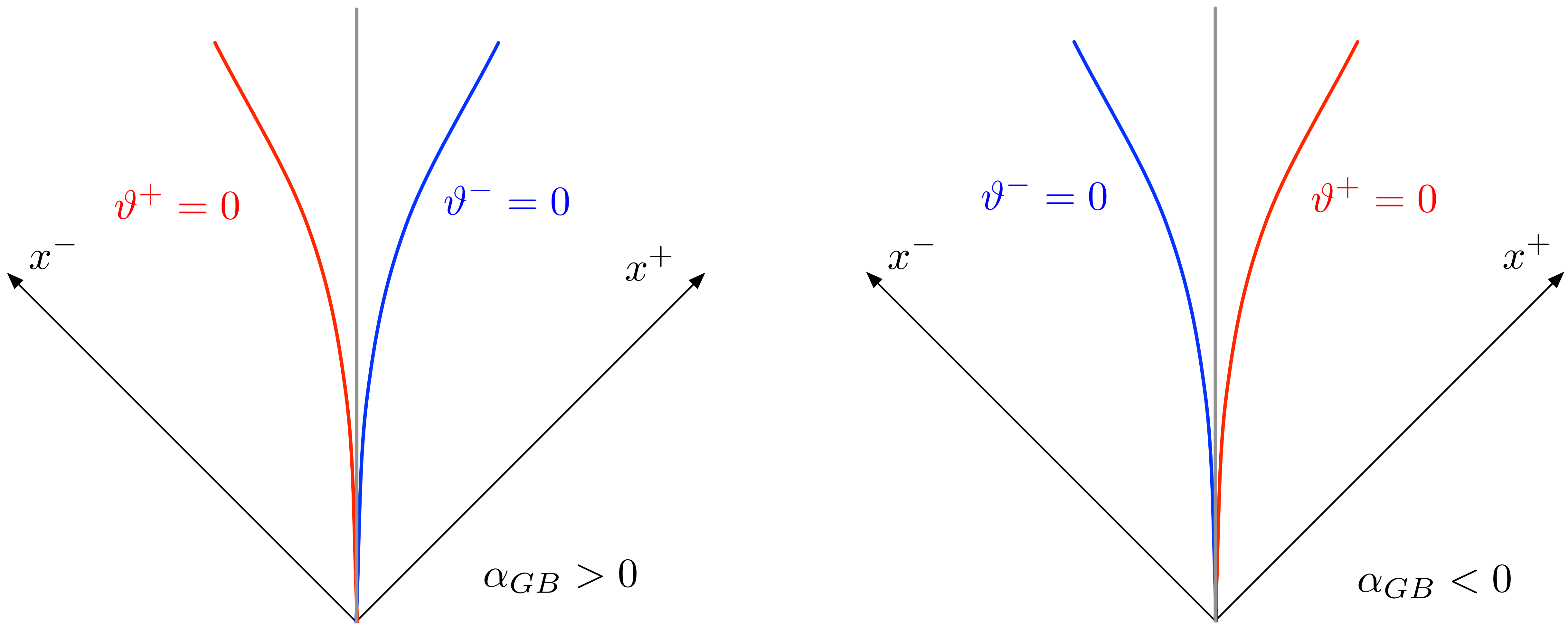
$$R(t,r) = R_0(r) + \varepsilon R_1(r) e^{i\omega t}, \tag{3.3}$$

$$\phi(t,r) = \phi_0(r) + \varepsilon \phi_1(r) e^{i\omega t}. \tag{3.4}$$

次元が大きいほど，不安定モードを拾う．

（線形解析）

5d GR vs Gauss-Bonnet WH : instability appears



不安定化する.

coupling の正負によって，最終的な時空が異なる.

coupling 正 （通常のGaussBonnet）$\Longrightarrow$ BHを形成しにくい

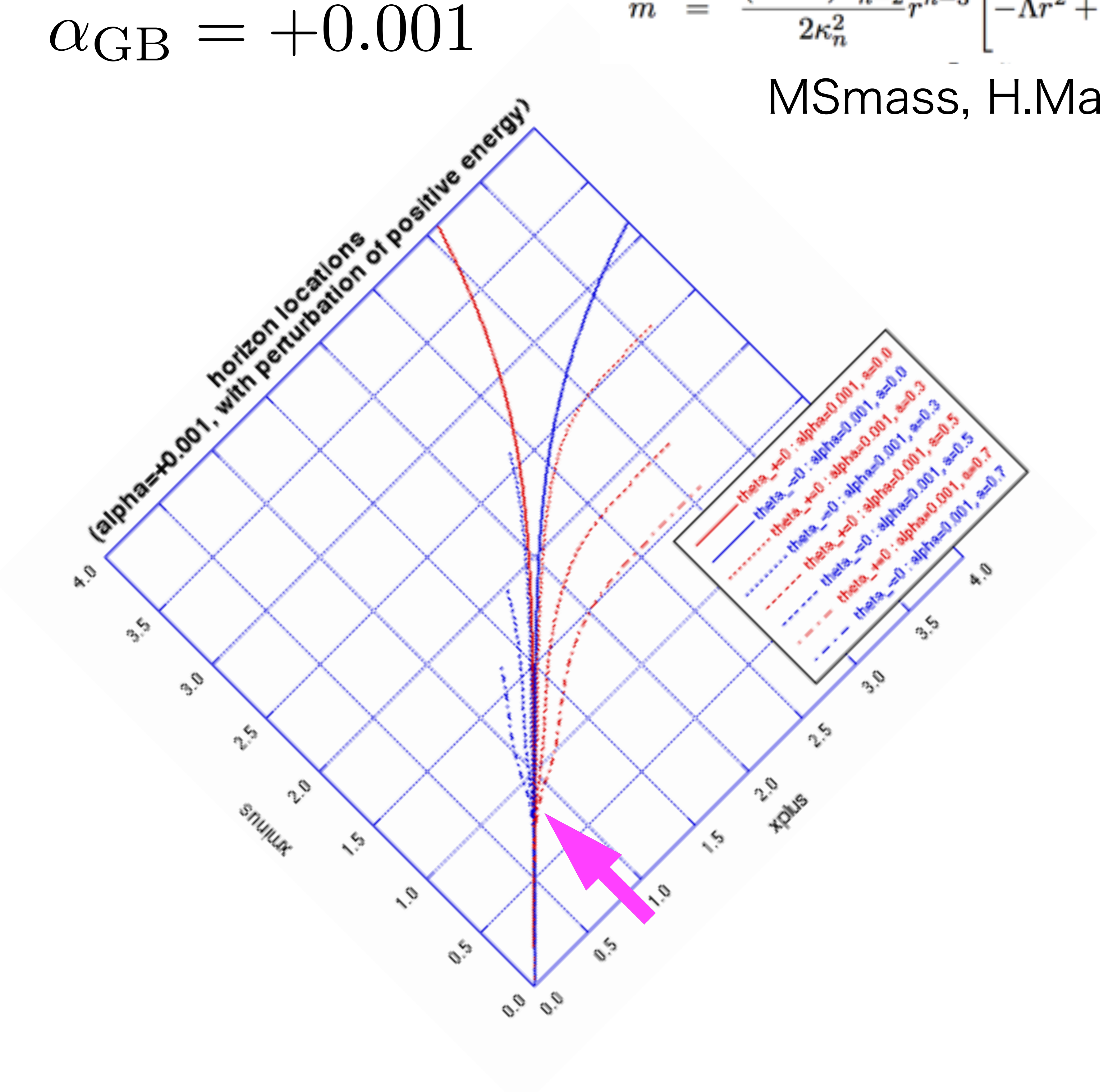
特異点回避の性質

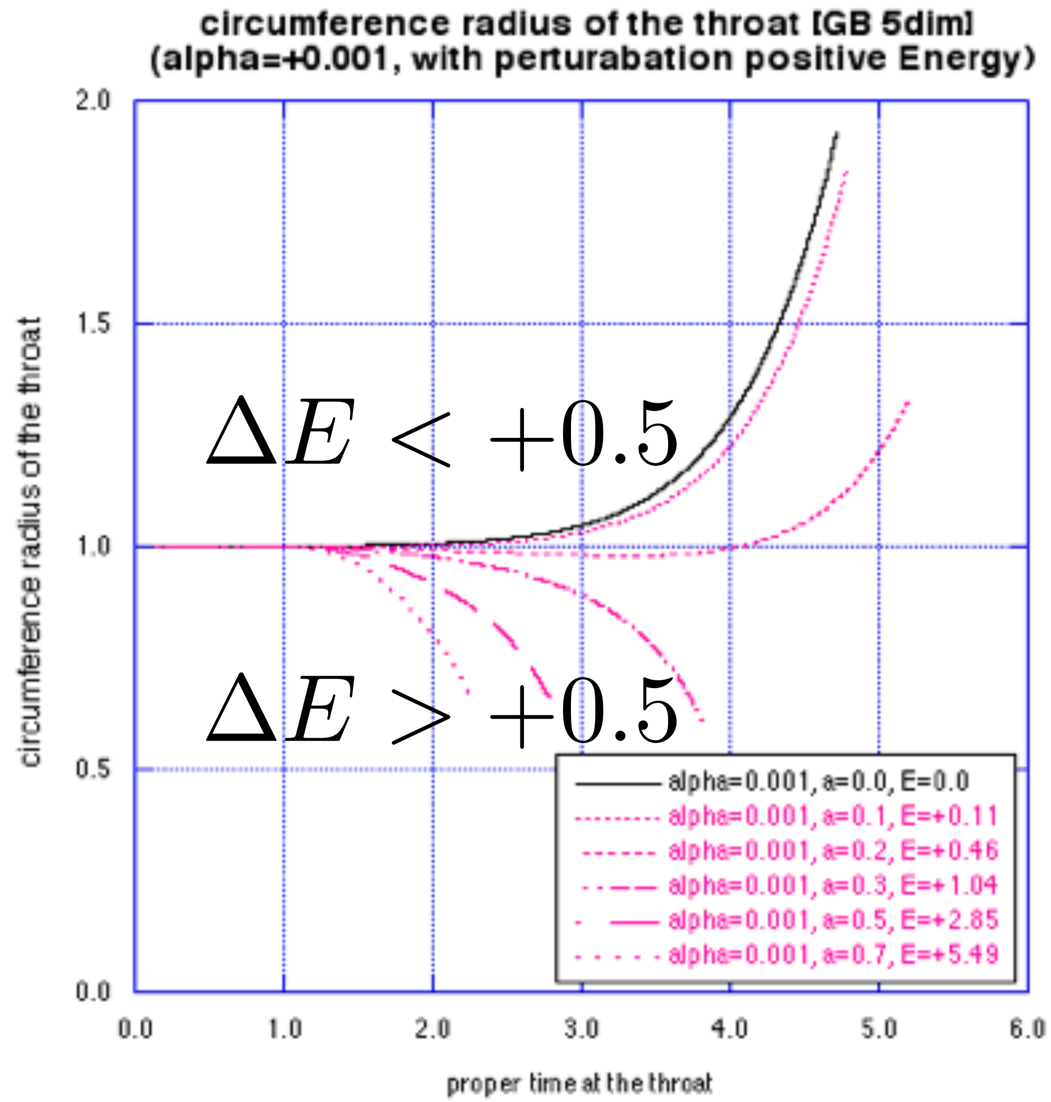
5d Gauss-Bonnet WH : positive energy injection (1)

$\alpha_{\rm GB} = +0.001$

$$m = \frac{(n-2)V^k_{n-2}}{2\kappa_n^2} r^{n-3}\left[-\tilde{\Lambda}r^2 + \left(k + \frac{2}{(n-2)^2}r^2 e^f \theta_+\theta_-\right) + \tilde{\alpha}r^{-2}\left(k + \frac{2}{(n-2)^2}r^2 e^f \theta_+\theta_-\right)^2\right]$$

MSmass, H.Maeda-Nozawa, PRD77 (2008) 063031





coupling 正 （通常のGaussBonnet） ⇒ BHを形成しにくい

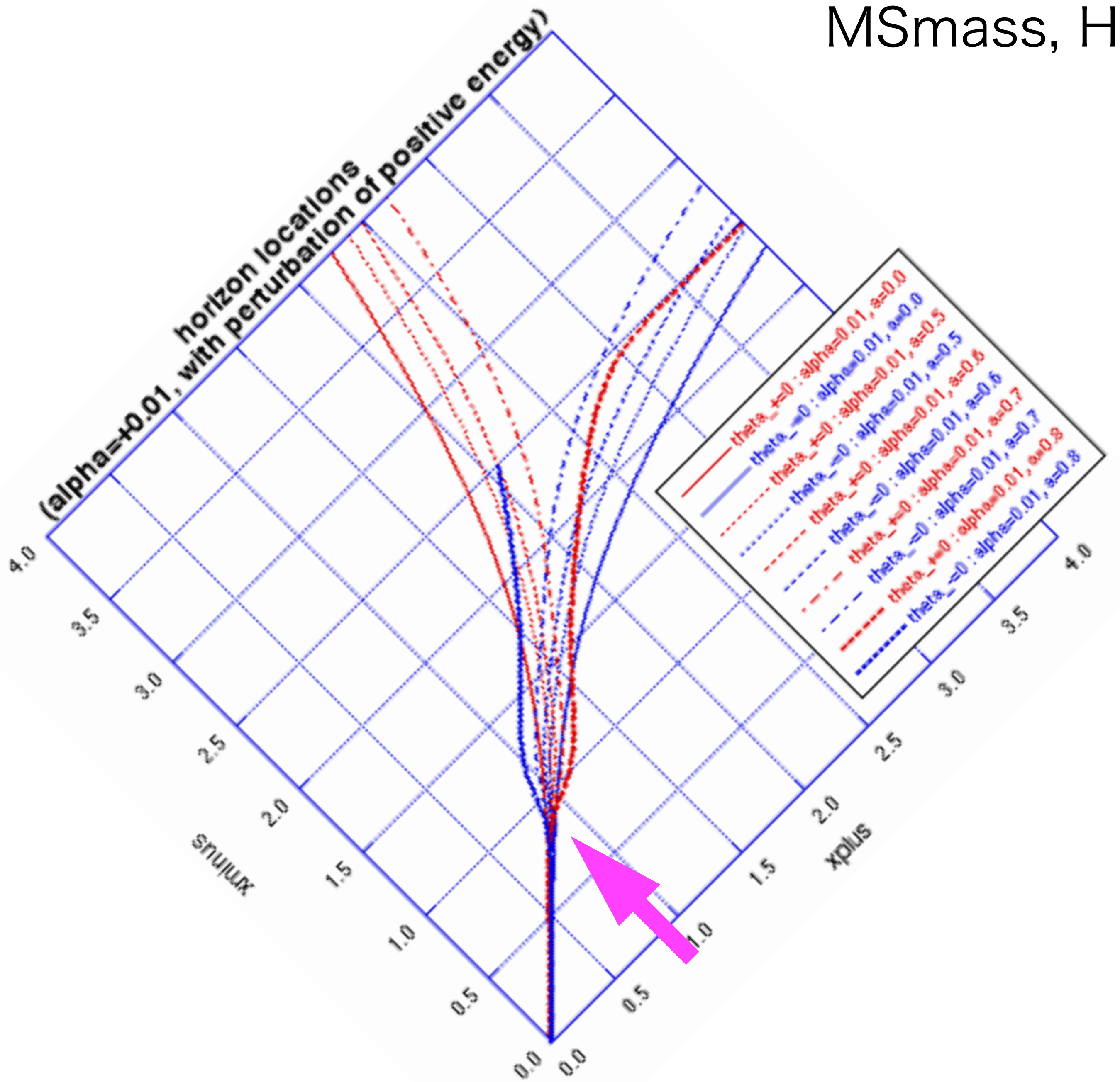
ある程度以上の正エネルギーを追加 ⇒ BH形成 に転じる

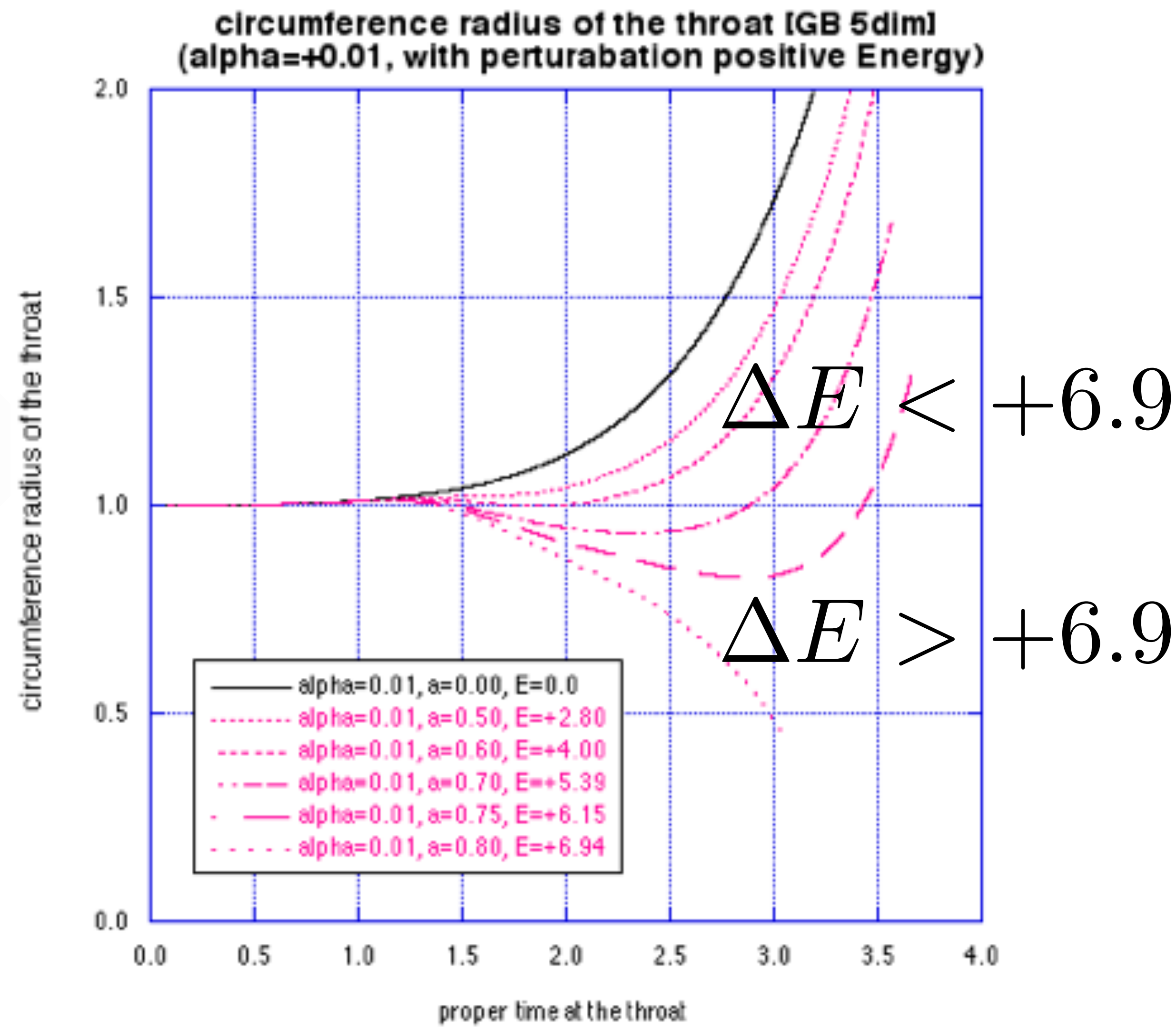
5d Gauss-Bonnet WH : positive energy injection (2)

$\alpha_{\rm GB} = +0.01$

$$m = \frac{(n-2)V^k_{n-2}}{2\kappa_n^2} r^{n-3} \left[-\tilde{\Lambda} r^2 + \left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+ \theta_- \right) + \tilde{\alpha} r^{-2} \left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+ \theta_- \right)^2 \right]$$

MSmass, H.Maeda-Nozawa, PRD77 (2008) 063031





alpha 大きければ，閾値大きい

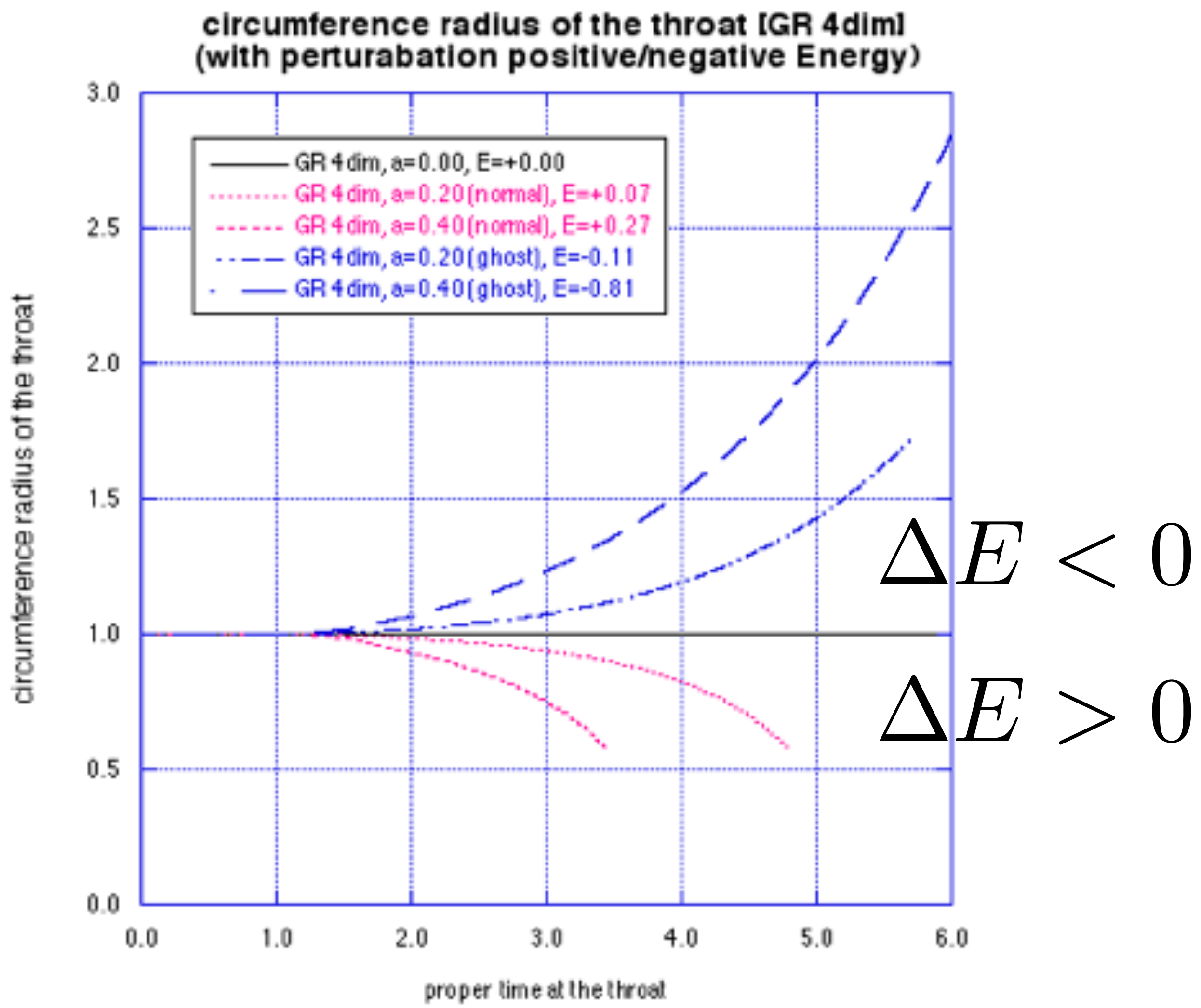
coupling 正　（通常のGaussBonnet）$\Rightarrow$ BHを形成しにくい

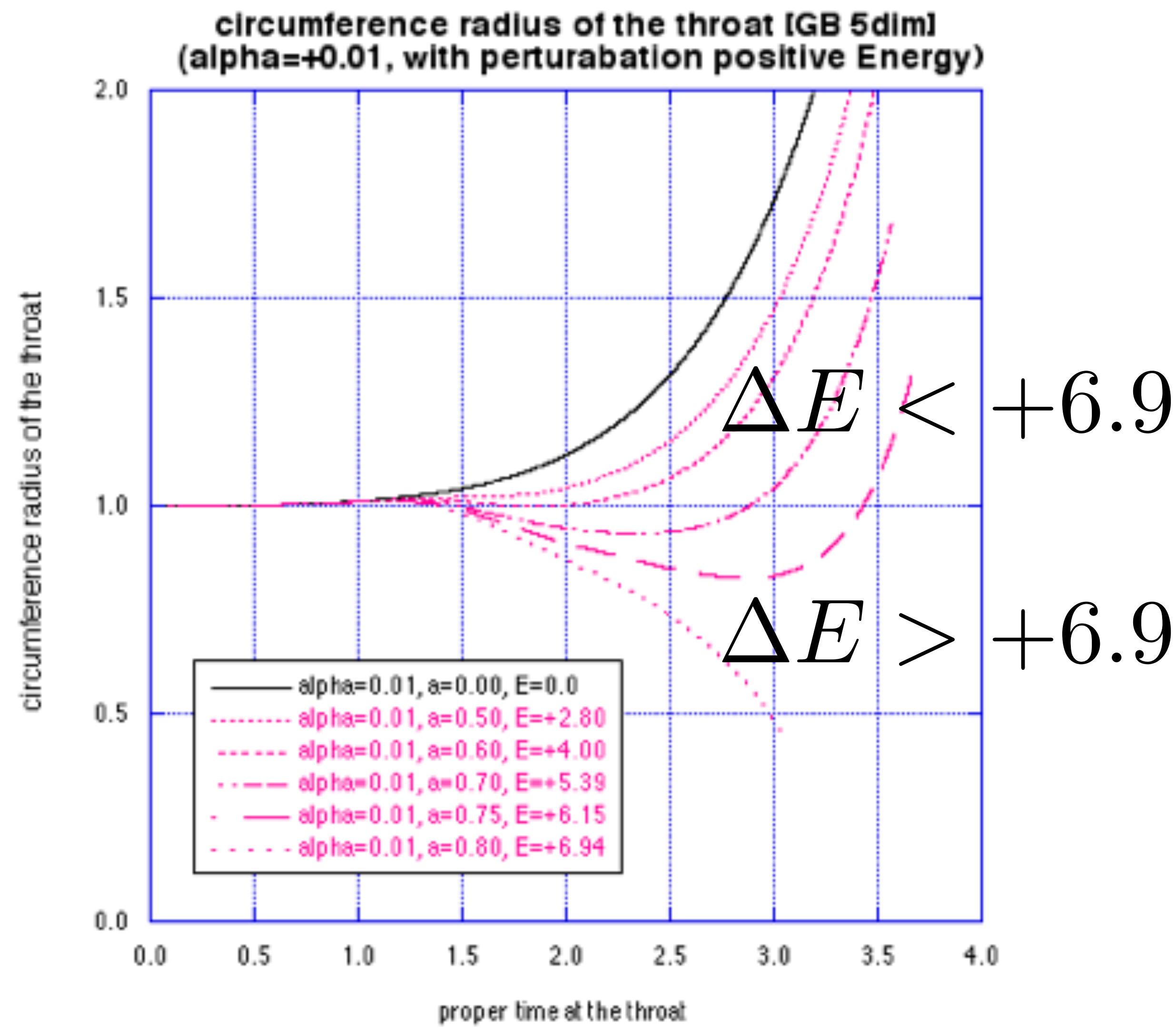
ある程度以上の正エネルギーを追加　$\Rightarrow$ BH形成 に転じる

5d Gauss-Bonnet WH : positive energy injection (3)

$\alpha_{\rm GB} = 0$ **GR**

$\alpha_{\rm GB} = +0.01$ **GB**





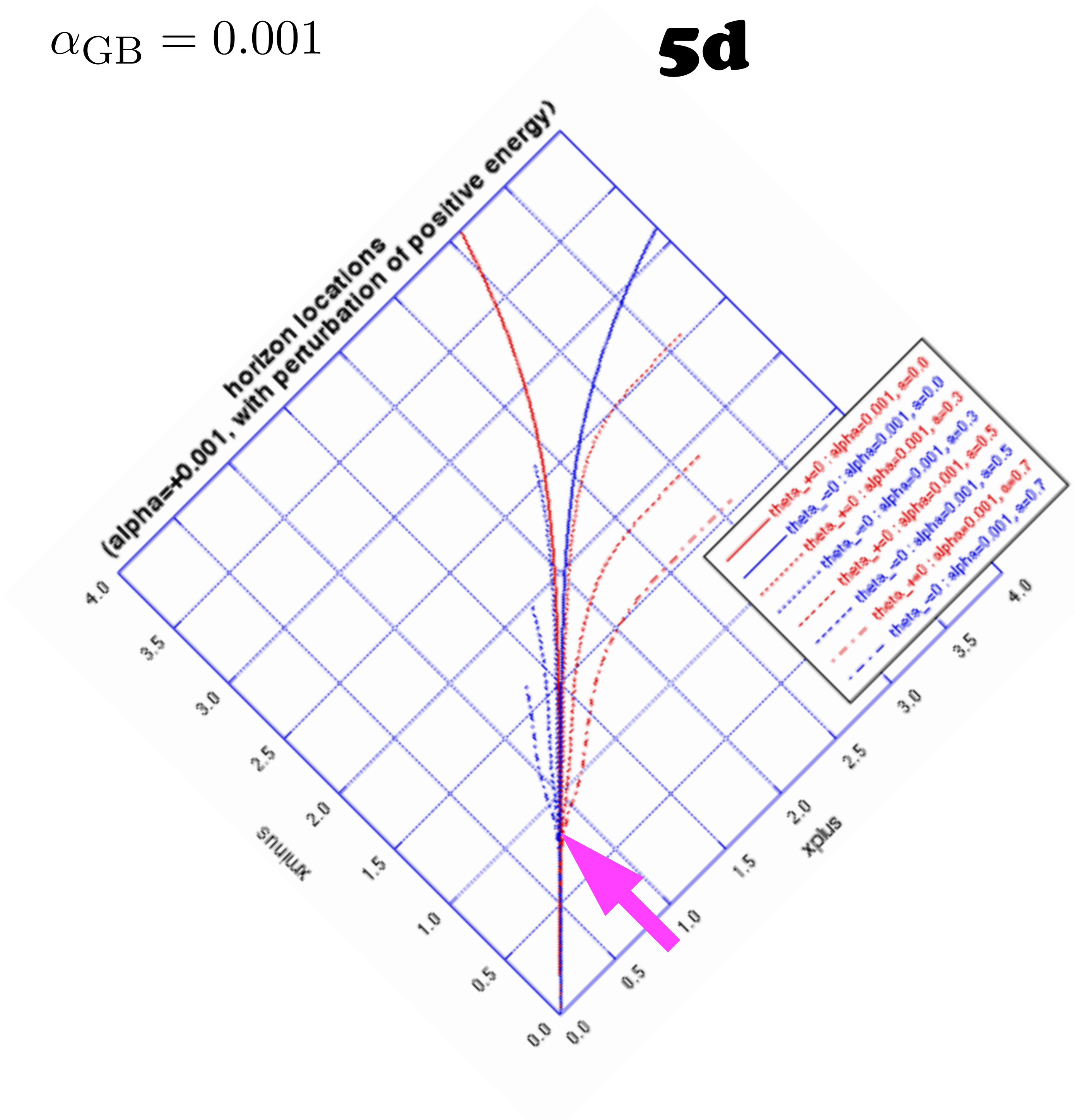
alpha 大きければ，閾値大きい

coupling 正 （通常のGaussBonnet）⇒ BHを形成しにくい

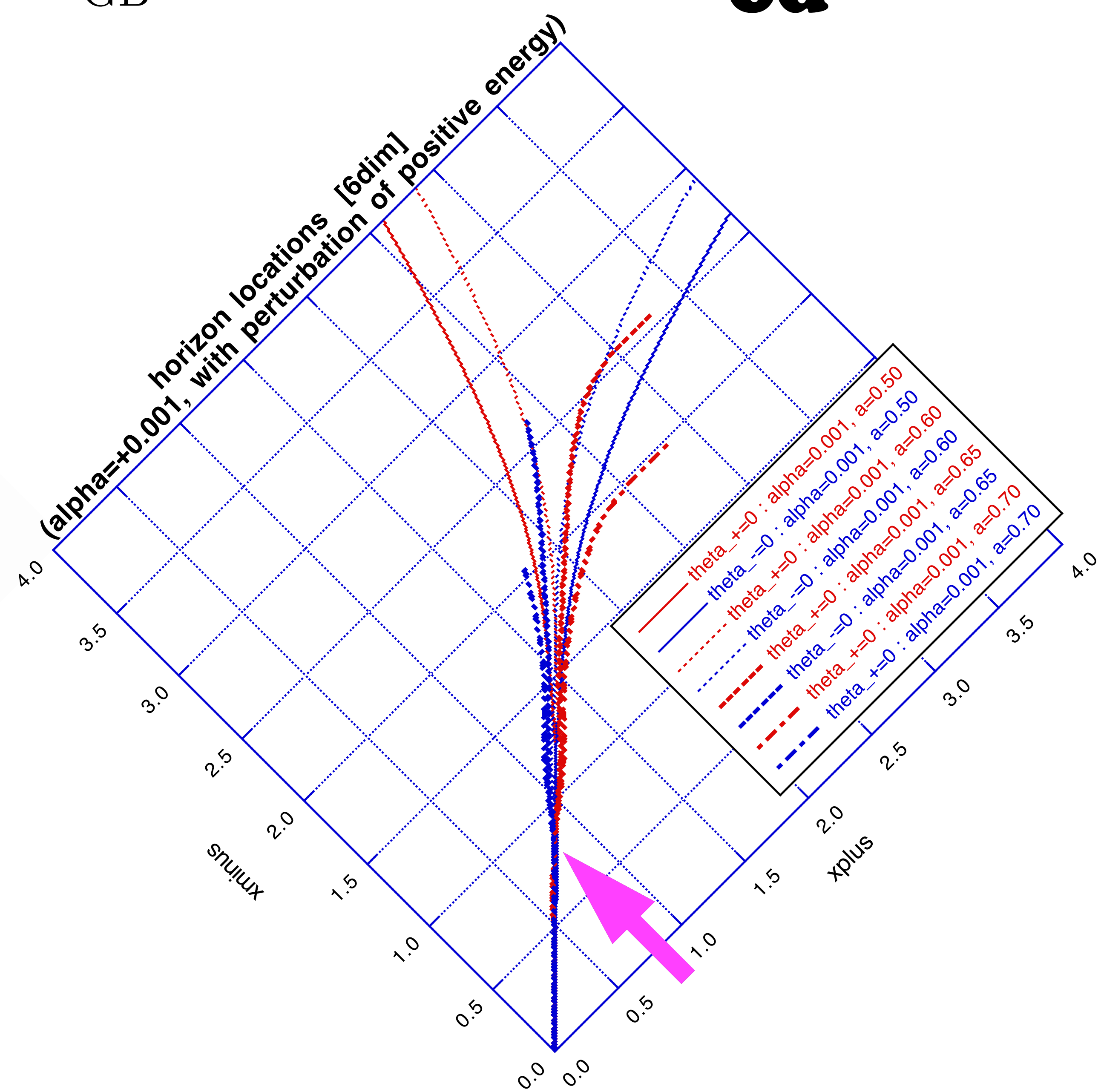
ある程度以上の正エネルギーを追加 ⇒ BH形成 に転じる

5d, 6d Gauss-Bonnet WH

$\alpha_{\rm GB} = 0.001$ **5d**



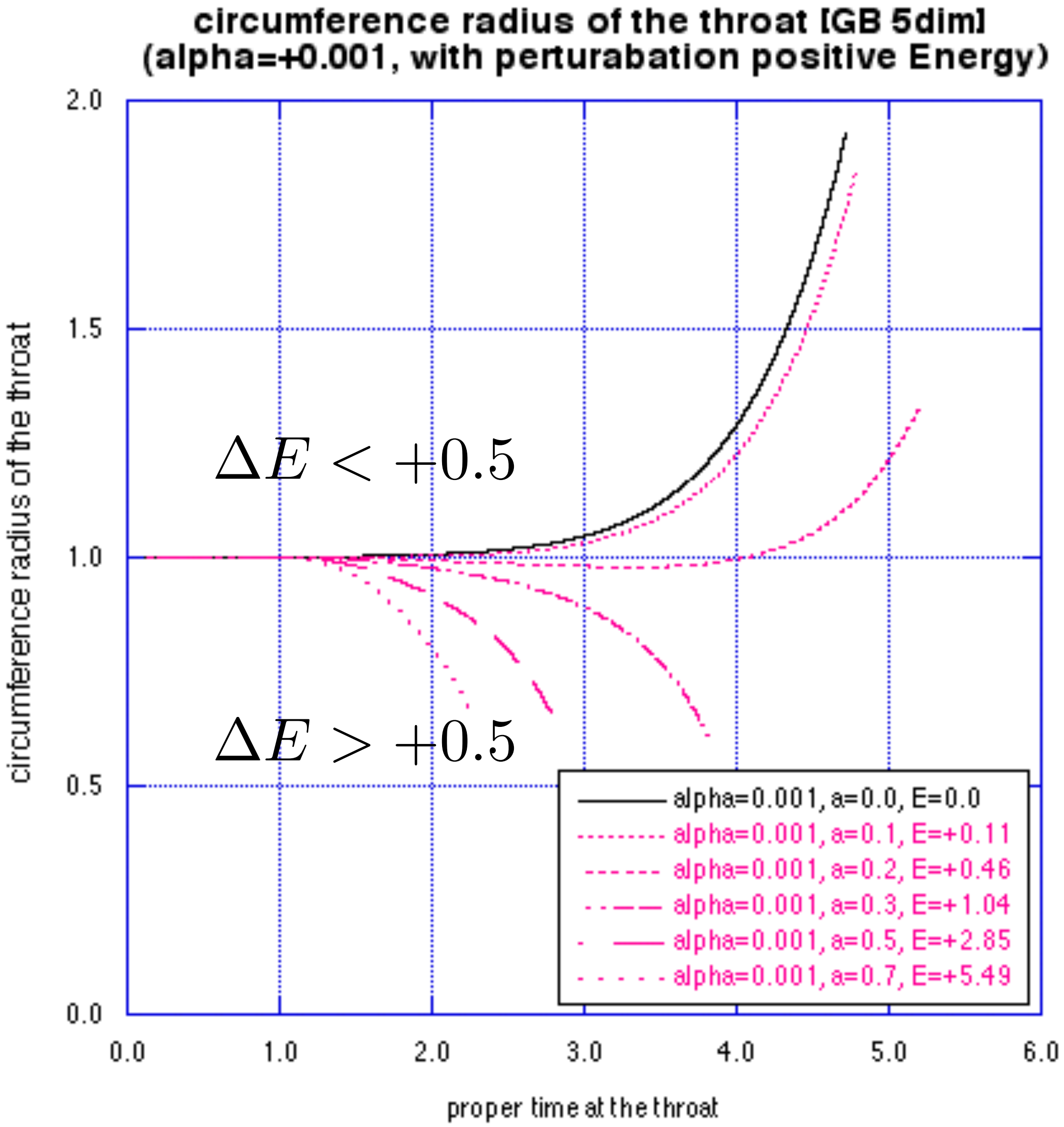
$\alpha_{\rm GB} = 0.001$ **6d**



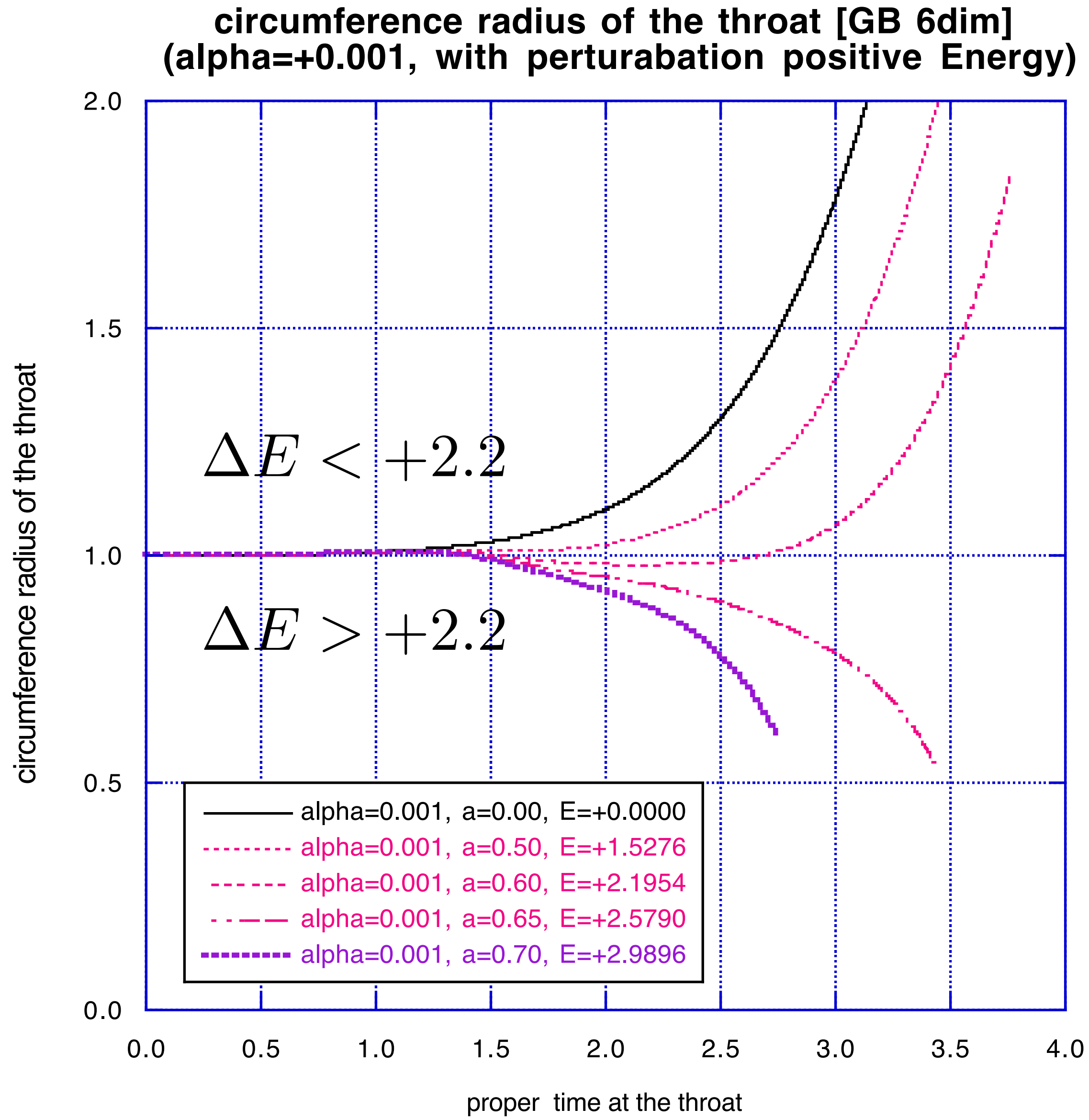
need more positive energy for transition to BH in 6dim

5d, 6d Gauss-Bonnet WH

$\alpha_{\rm GB} = 0.001$ **5d**



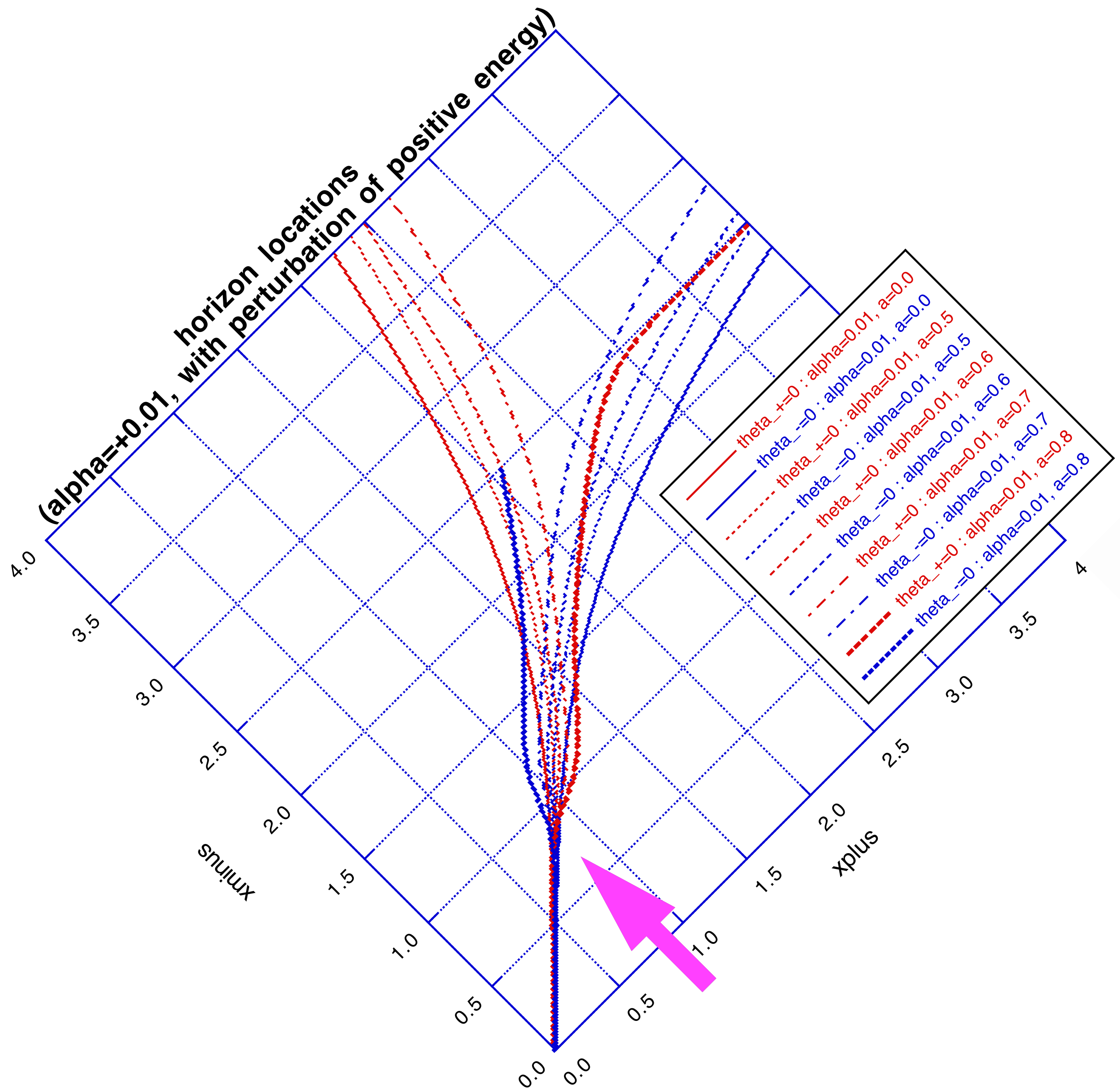
$\alpha_{\rm GB} = 0.001$ **6d**



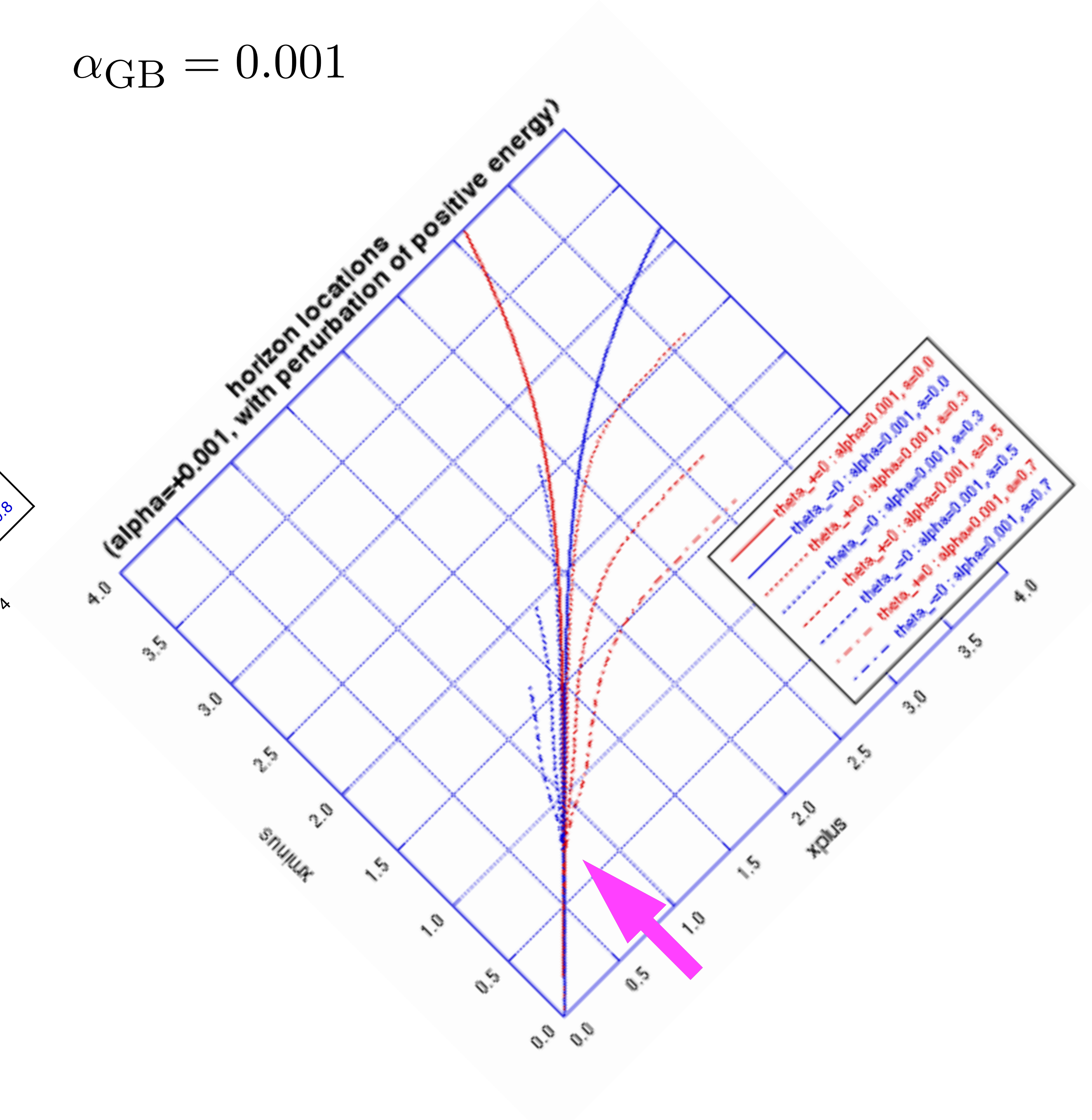
need more positive energy for transition to BH in 6dim

5d Gauss-Bonnet WH : trapped surface

$\alpha_{\mathrm{GB}} = 0.01$



$\alpha_{\mathrm{GB}} = 0.001$

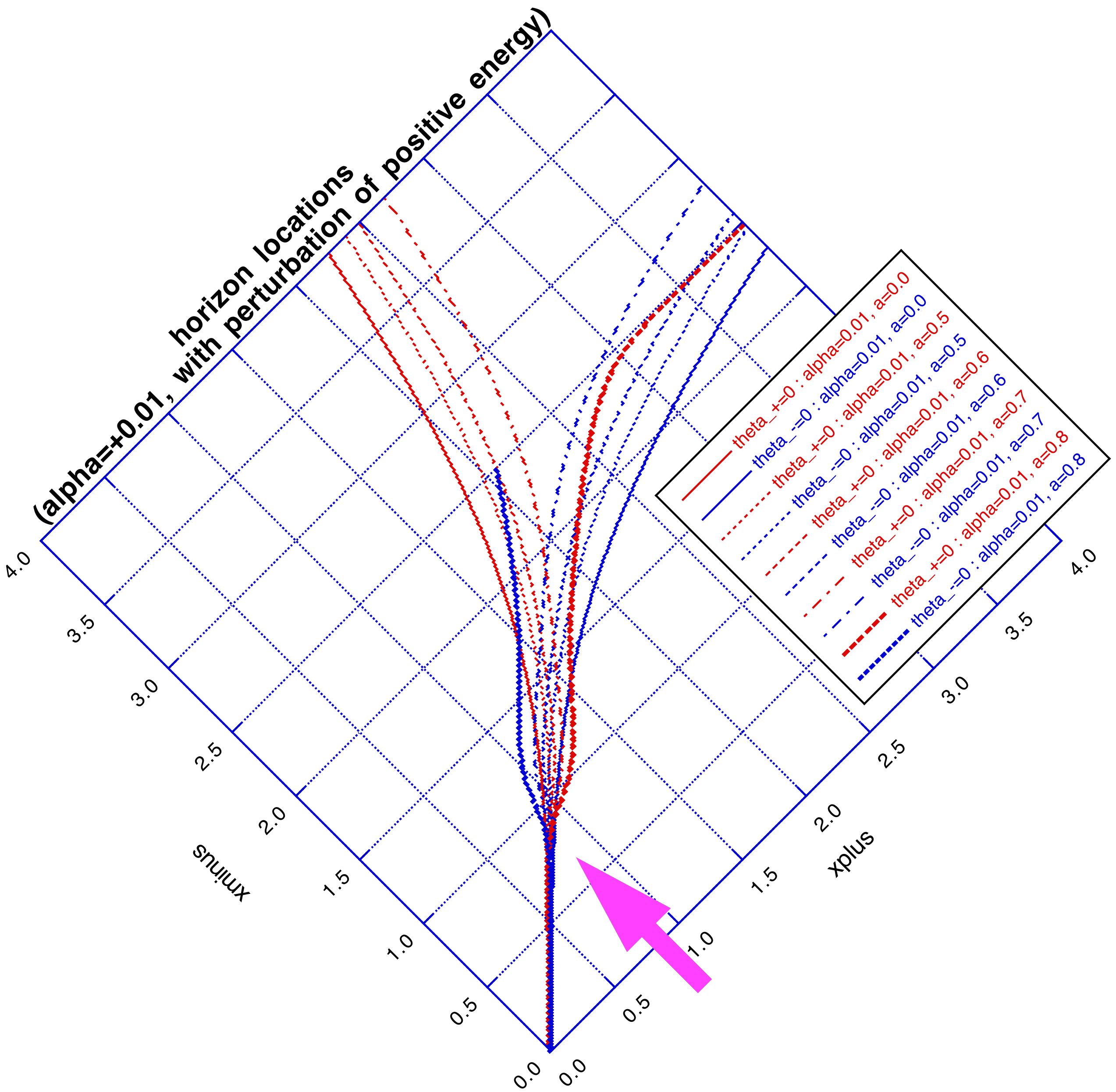


existence of trapped surface

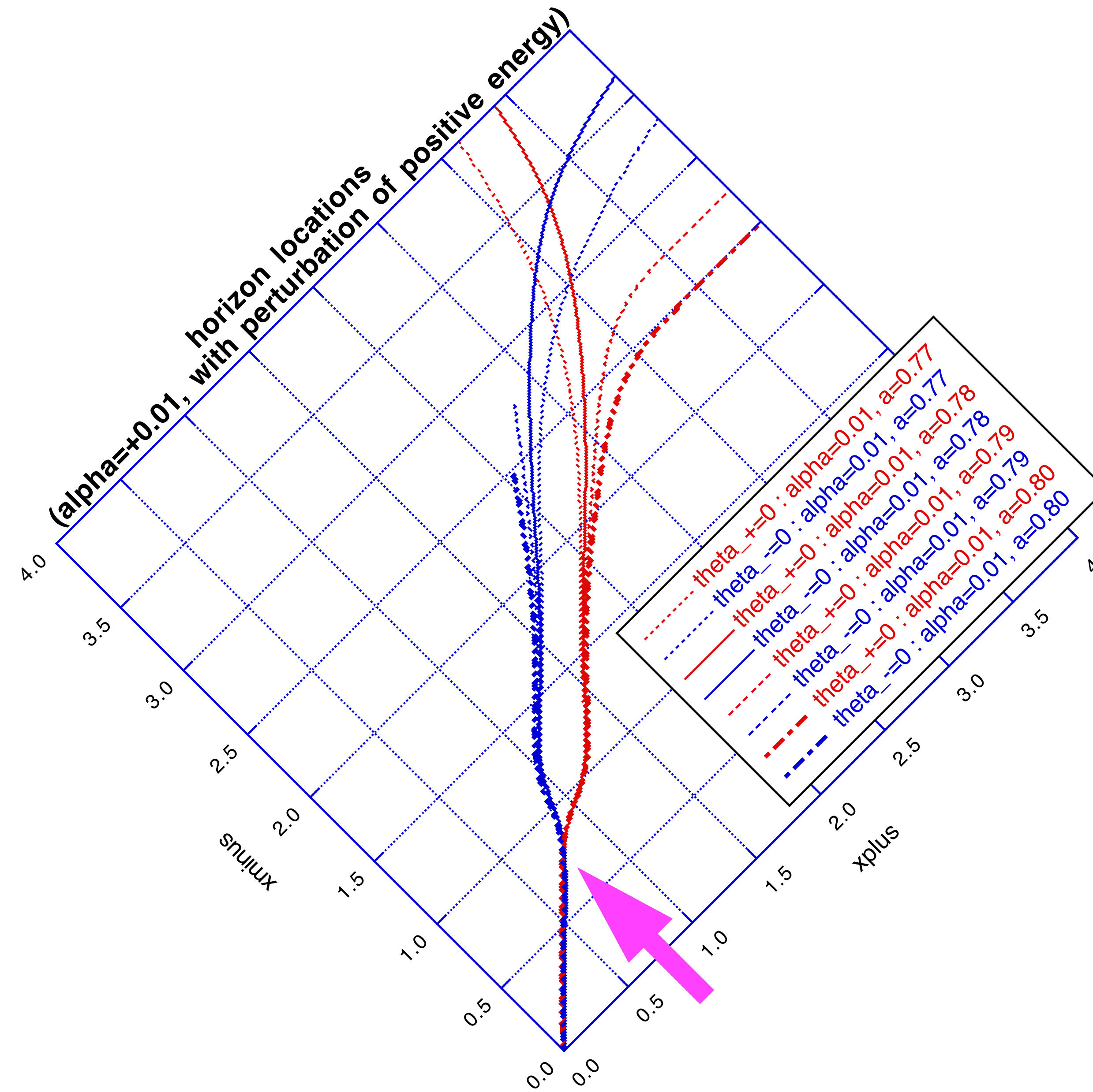
—> not necessary to form BH

5d Gauss-Bonnet WH : trapped surface

$\alpha_{\rm GB} = 0.01$



critical behavior



existence of trapped surface

—> not necessary to form a BH

Summary

✅ 平面スカラー波の衝突

✅ 球対称ワームホールのBHへの転移現象

4dim, 5dim, 6dim, ⋯

高次元になるほど，同じ初期条件でもBHは形成しにくい

Yamada-HS (2011) [naked singularity形成]とconsistent

高次元になるほど，不安定性は拡大する

Torii-HS (2013) [WH不安定性]とconsistent

$$F \sim \frac{1}{r^{n-2}}$$

Gauss-Bonnet重力の特色

正のcouplingでは，同じ初期条件でも特異点形成は遅くなる.

正のcouplingでは，同じ初期条件でもBHは形成しにくい.

エネルギー底上げ・特異点回避の傾向がある.

高次元になるほど，不安定性は拡大する.

trapped surfaceの存在は，必ずしもBH形成を意味しない.

（面積定理が成立しない解系列があることに対応か）