高次元・高次曲率項を含む重力理論での特異点形成

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論点
＊4dim, 5dim, 6dim, … ダイナミクスはどう変化するか
＊Gauss-Bonnet項は，ダイナミクスにどう影響するか

1. GR 4dim vs 5dim
2. Field Equations (dual-null formulation)
2. 平面対称時空：Colliding Scalar Waves
3. 球対称時空：Wormhole-BH transition

一般相対論研究の面白さ

非線形性・複雑さ

ブラックホール，膨張宇宙，重力波

Einsteinも信じなかった事実

結果の美しさ

特異点定理　特異点はアインシュタイン方程式の解として必然として存在

宇宙検閲官仮説　特異点はBHホライズンの内側に隠されていて欲しい

フープ仮説　ホライズン形成は，物質分布がある程度コンパクトな場合

バーコフの定理　球対称，静的，真空時空はSchwarzchild

BH唯一性定理　定常ブラックホール時空はKerr

脱毛定理　BH形成により，M, Q, J の3つだけが物理情報として残る

一般相対論研究＋高次元研究の面白さ

動機　　膜宇宙論による新しいパラダイムの提示
LHCによる高次元空間の検証可能性

想定外のBH解の発見　　"Black Objects"

4-dim BHs　　　　　　　　Higher-dim BHs :

Schwarzschild → Tangherlini
--- unique & stable

Kerr → Myers-Perry
--- maybe unstable in higher J

black ring (Emparan-Reall)
black Saturn
di-rings, orthogonal di-rings, ...

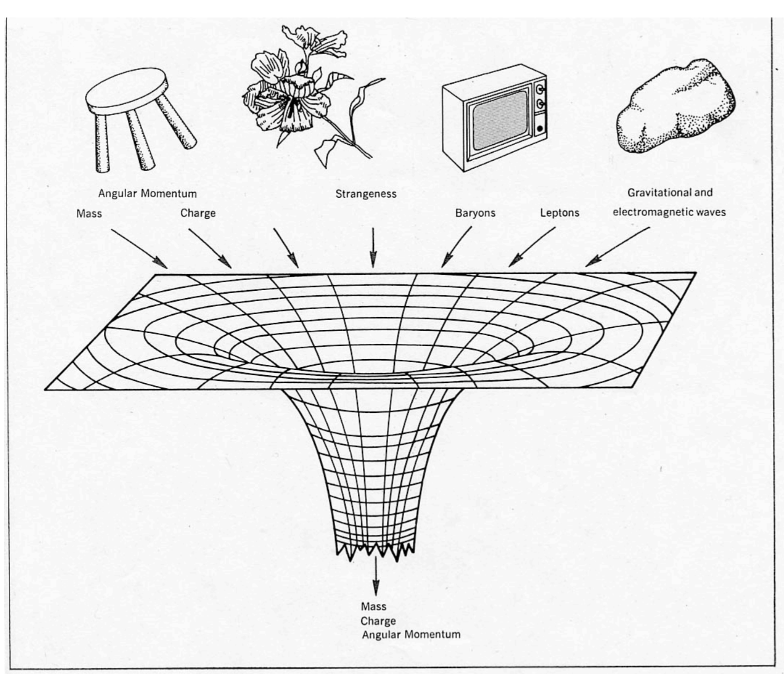
Higher-dim Black Holes have Rich Structures

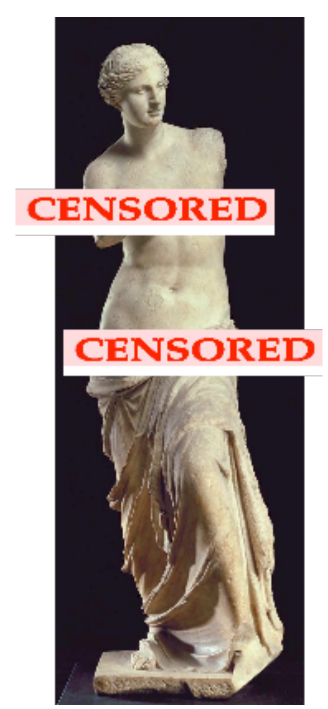
"Black Objects"

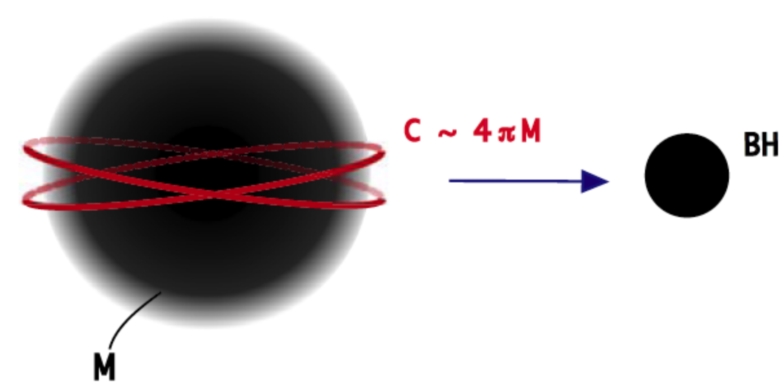
black hole
black string
black ring
black Saturn
di-rings, orthogonal di-rings ...

Uniqueness (only in spherical sym.)
Stability?
Formation Process?
Dynamical Features? ...

No Hair Conjecture?
Cosmic Censorship?
Hoop Conjecture?







Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **has minimum mass for static spherical BH solution**

  T Torii & H Maeda, PRD 71 (2005) 124002
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated in full gravity.)
- **new topic in numerical relativity.**

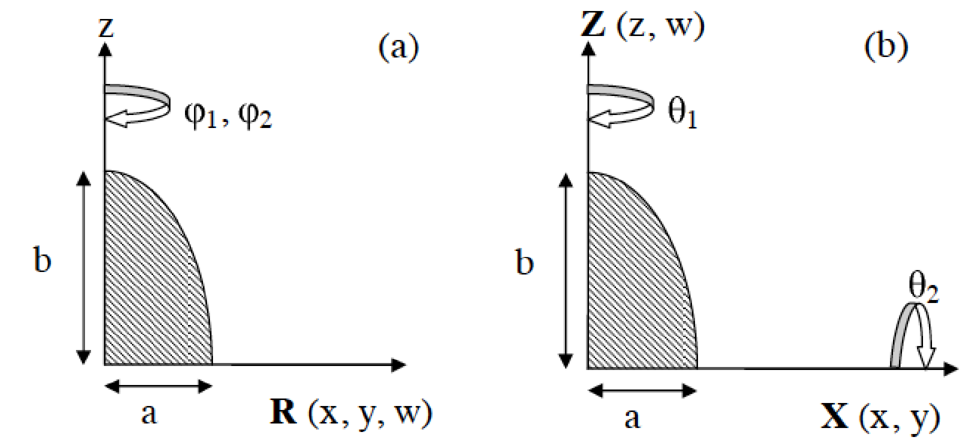
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496
- **much attentions in WH community**

  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

Dynamics in 5dim GR gravity?

2. Spheroidal matter collapse
Initial data analysis, Evolutions

Yamada & HS, CQG 27 (2010) 045012
Yamada & HS, PRD 83 (2011) 064006



3. Wormhole dynamics in GR
linear stability,
dynamical stability

Torii & HS, PRD 88 (2013) 064027
HS & Torii, in preparation

Dynamics in Gauss-Bonnet gravity?

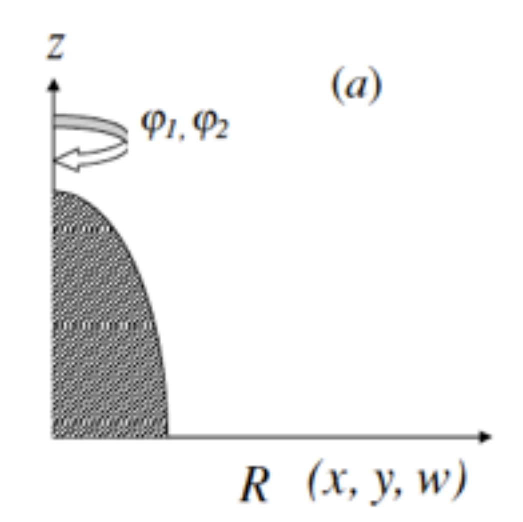
4. Wormhole dynamics in GB
5. Plane-wave collision in GB

HS & Torii, in preparation

2. Spheroidal matter collapse
A. Initial data construction

- time symmetric, asymptotically flat
- conformal flat
- non-rotating homogeneous dust

- solve the Hamiltonian constraint eq. 512^2 grids
- Apparent Horizon Search
- Define Hoop and check the Hoop Conjecture



$$ds^2 = \psi(R,z)^2 \left[dR^2 + R^2(d\varphi_1^2 + \sin^2\varphi_1 d\varphi_2^2) + dz^2\right]$$

$$R = \sqrt{x^2+y^2+z^2}, \quad \varphi_1 = \tan^{-1}\left(\frac{w}{\sqrt{x^2+y^2}}\right), \quad \varphi_2 = \tan^{-1}\left(\frac{y}{x}\right).$$

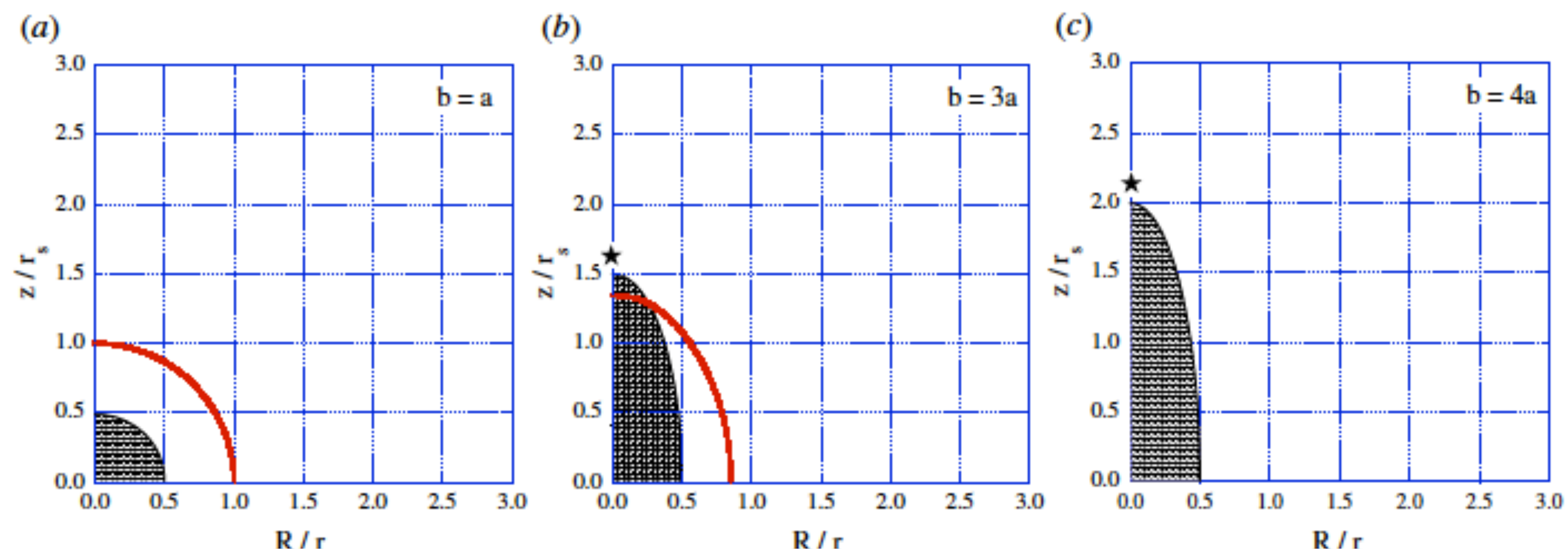
$$\frac{\partial^2\psi}{\partial R^2} + \frac{2}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial z^2} = -4\pi^2 G_5 \rho$$

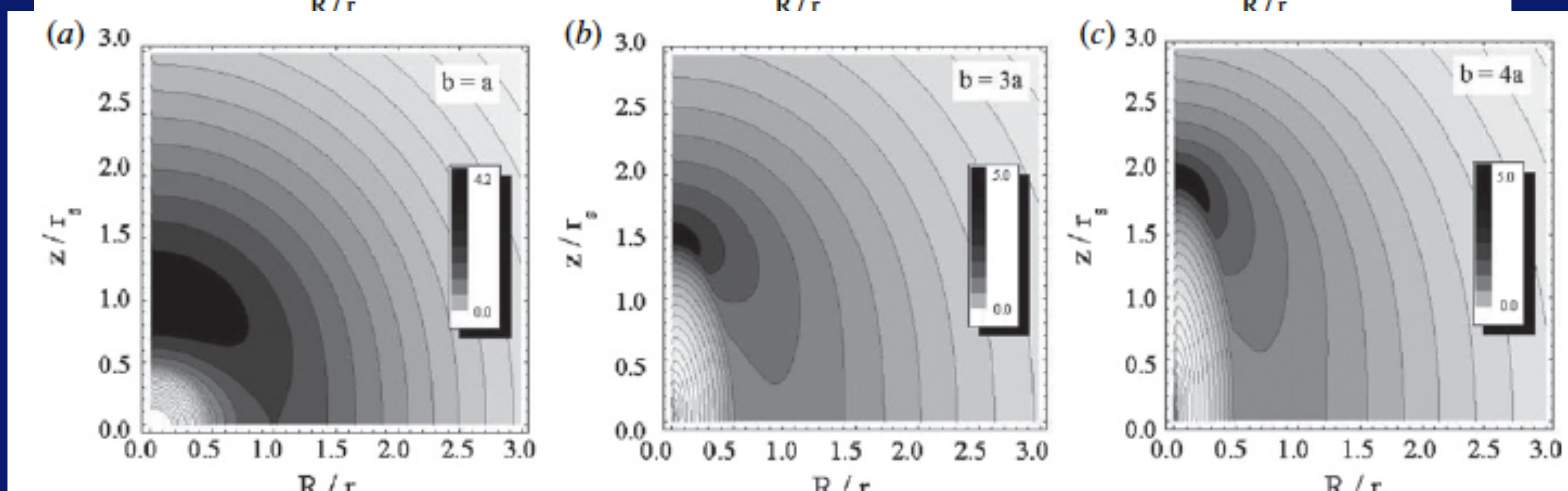
2. Spheroidal matter collapse
B. Initial data sequence

cf. (3-dim.) Nakamura-Shapiro-Teukolsky (1988)

4+1
initial data

Class. Quantum Grav. **27** (2010) 045012 Y Yamada and H Shinkai





Contour Plot of the Kretchmann invariant, $R_{abcd}R^{abcd}$

2. Spheroidal matter collapse
C. Evolution method

- ADM 2+1 Double Axisym Cartoon
- 130^2 x 2^2 grids

- lapse function: Maximal slicing condition
- shift vectors: Minimum distortion condition
- asymptotically flat

- Collisionless Particles (5000)
- the same total mass
- no rotation

- Apparent Horizon Search

2. Spheroidal matter collapse
C. Evolution examples (4D, ST1991)

VOLUME 66, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1991

Formation of Naked Singularities: The Violation of Cosmic Censorship

Stuart L. Shapiro and Saul A. Teukolsky

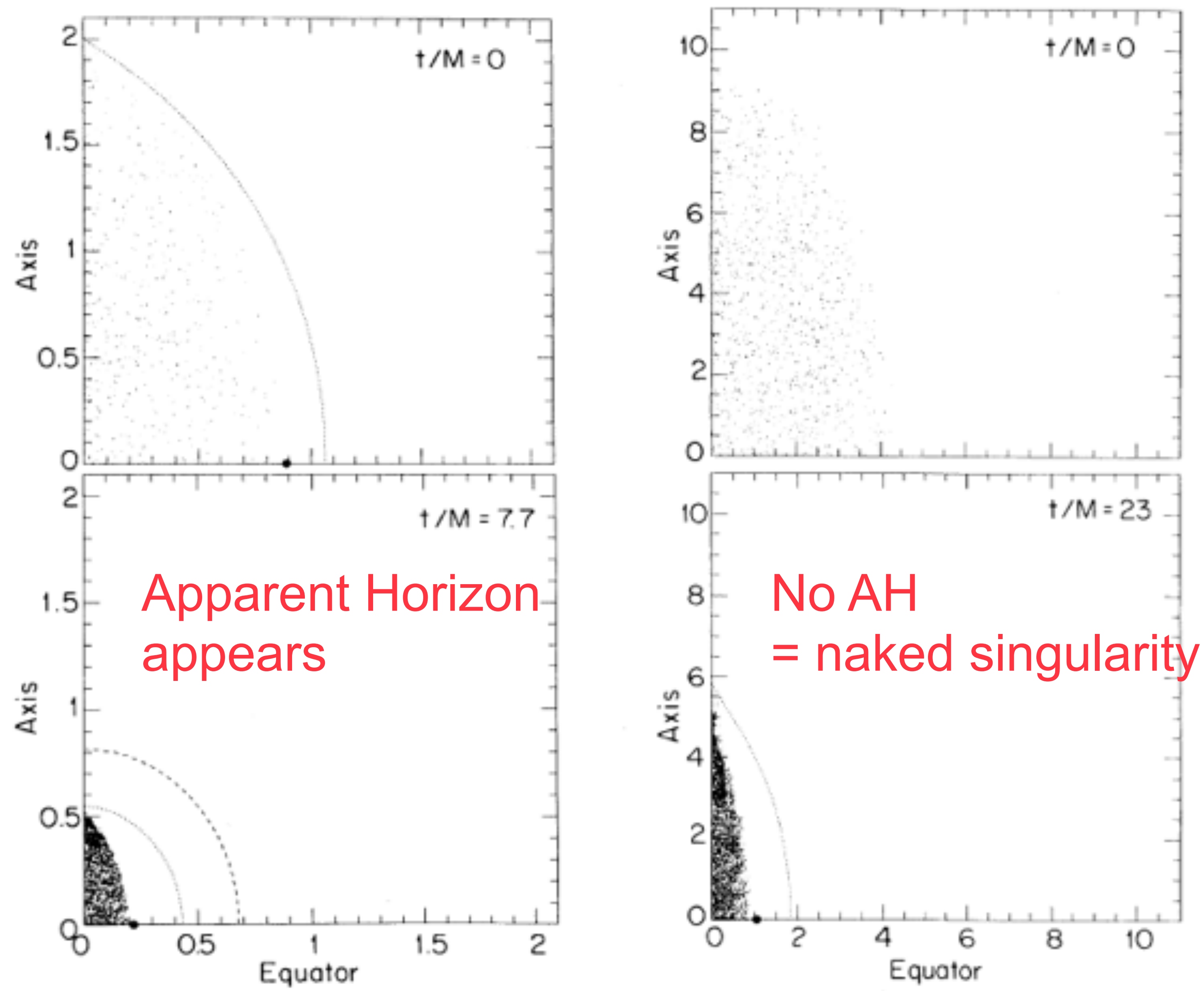


FIG. 1. Snapshots of the particle positions at initial and late times for prolate collapse. The positions (in units of M) are projected onto a meridional plane. Initially the semimajor axis of the spheroid is $2M$ and the eccentricity is 0.9. The collapse proceeds nonhomologously and terminates with the formation of a spindle singularity on the axis. However, an apparent horizon (dashed line) forms to cover the singularity. At $t/M = 7.7$ its area is $\mathcal{A}/16\pi M^2 = 0.98$, close to the asymptotic theoretical limit of 1. Its polar and equatorial circumferences at that time are $\mathcal{C}_{\rm pole}^{\rm AH}/4\pi M = 1.03$ and $\mathcal{C}_{\rm eq}^{\rm AH}/4\pi M = 0.91$. At later times these circumferences become equal and approach the expected theoretical value 1. The minimum exterior polar circumference is shown by a dotted line when it does not coincide with the matter surface. Likewise, the minimum equatorial circumference, which is a circle, is indicated by a solid dot. Here $\mathcal{C}_{\rm eq}^{\rm min}/4\pi M = 0.59$ and $\mathcal{C}_{\rm pole}^{\rm min}/4\pi M = 0.99$. The formation of a black hole is thus consistent with the hoop conjecture.

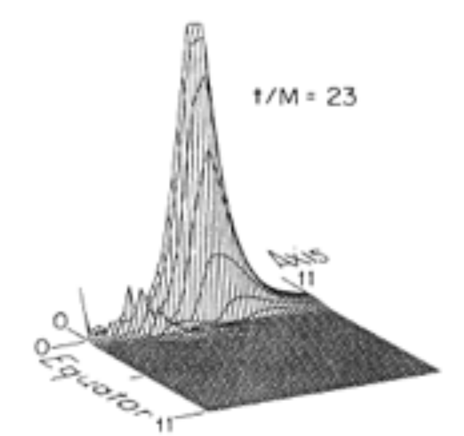


FIG. 4. Profile of I in a meridional plane for the collapse shown in Fig. 2. For the case of 32 angular zones shown here, the peak value of I is $24/M^4$ and occurs on the axis just outside the matter.

2. Spheroidal matter collapse
C. Evolution examples (5D, ours)

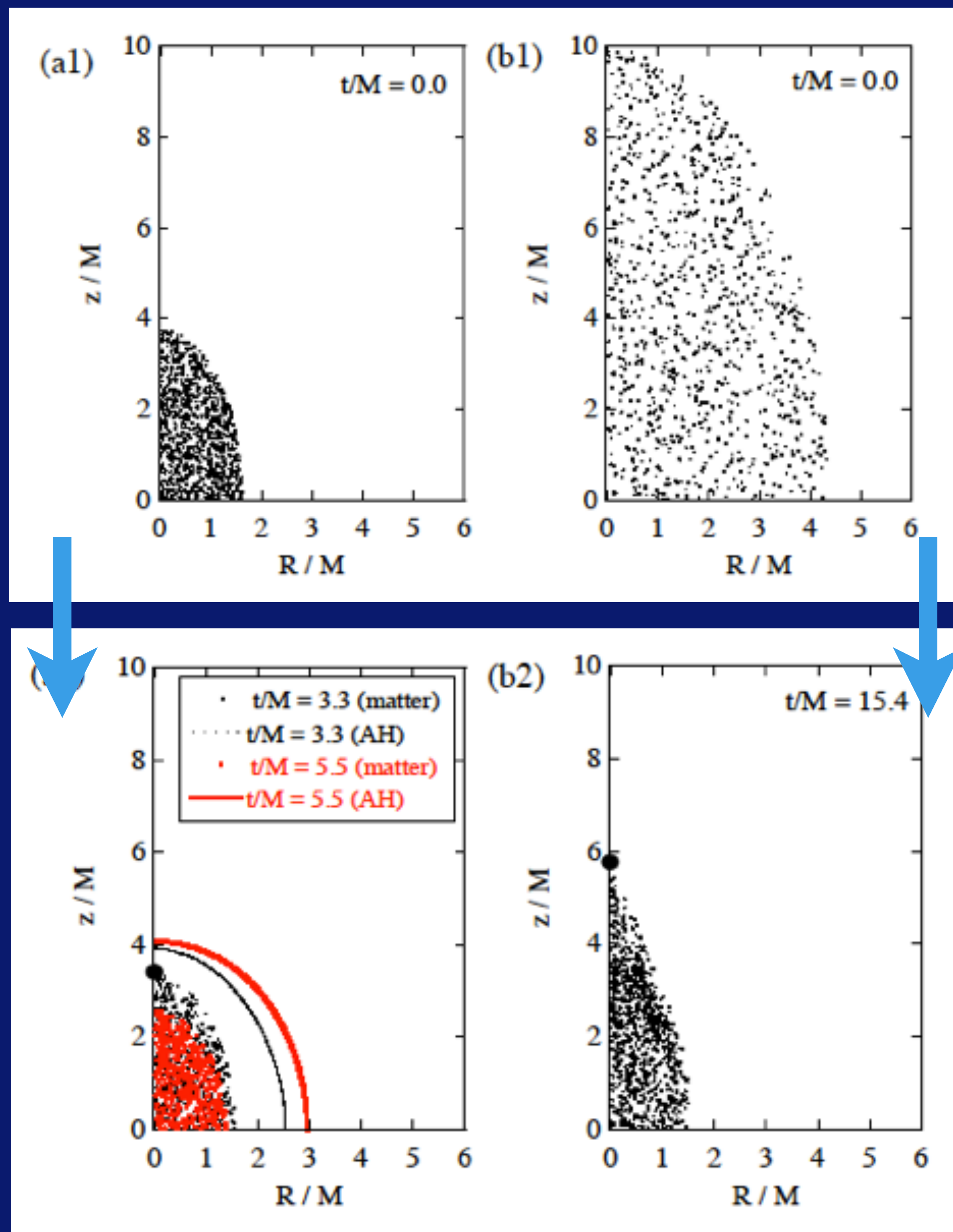


FIG. 2: Snapshots of 5D axisymmetric evolution with the initial matter distribution of $b/M = 4$ [Fig.(a1) and (a2); model 5DSβ in Table I] and 10 [Fig.(b1) and (b2); model 5DSδ]. We see the apparent horizon (AH) is formed at the coordinate time $t/M = 3.3$ for the former model and the area of AH increases, while AH is not observed for the latter model up to the time $t/M = 15.4$ when our code stops due to the large curvature. The big circle indicates the location of the maximum Kretschmann invariant $\mathcal{I}_{\max}$ at the final time at each evolution. Number of particles are reduced to 1/10 for figures.

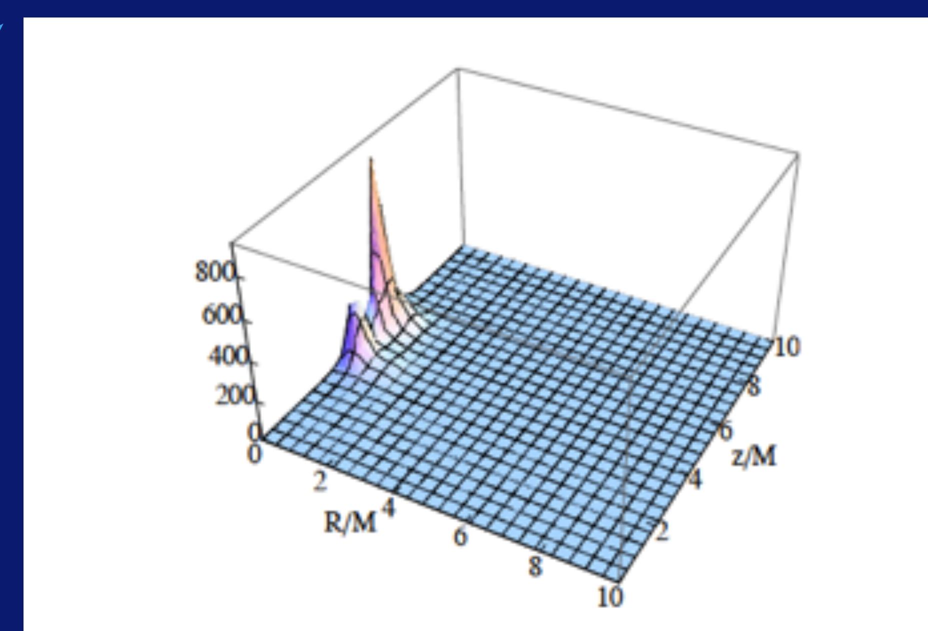
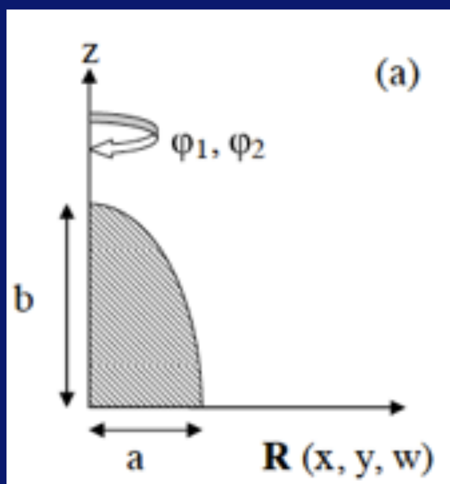
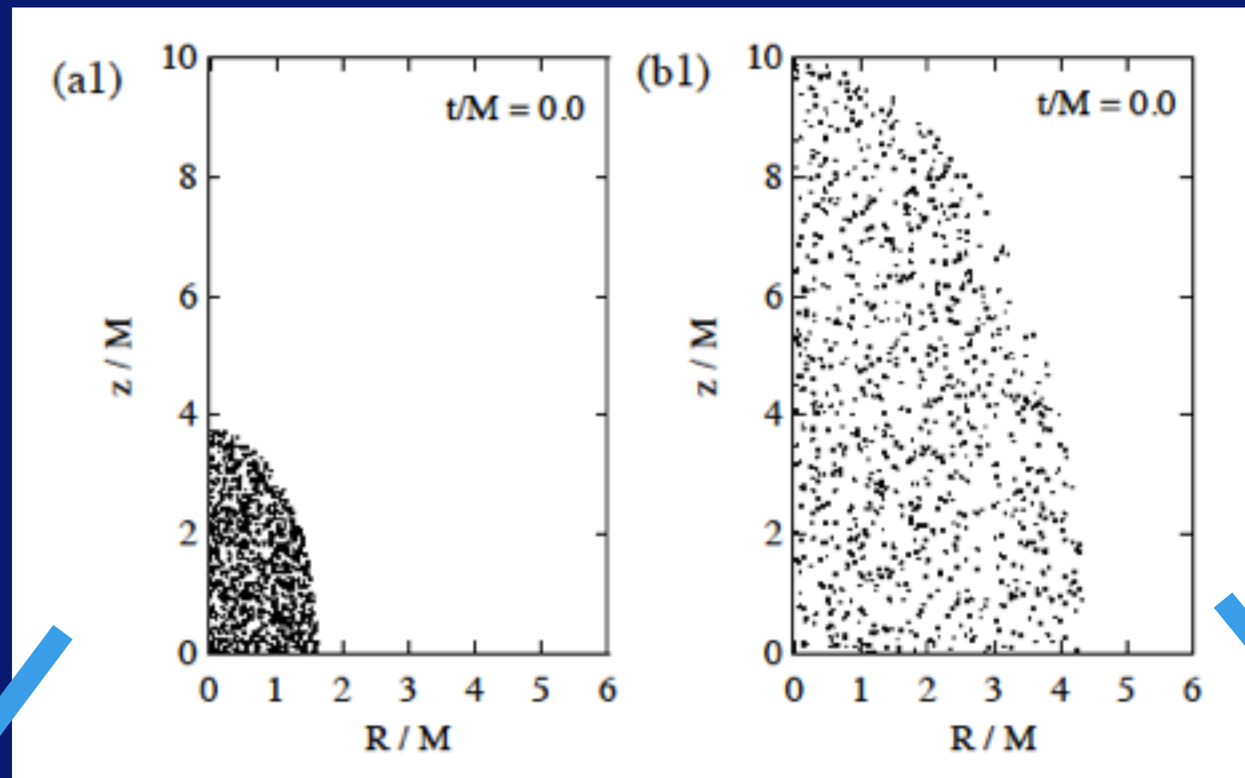


FIG. 3: Kretschmann invariant $\mathcal{I}$ for model 5DSδ at $t/M = 15.4$. The maximum is $O(1000)$, and its location is on z-axis, just outside of the matter.

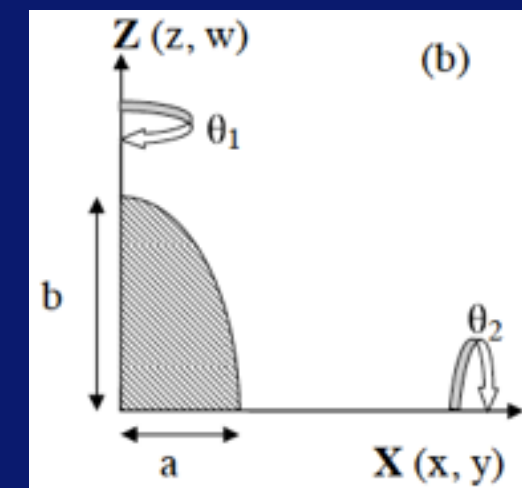
2. Spheroidal matter collapse
C. Evolution examples (5D, ours)

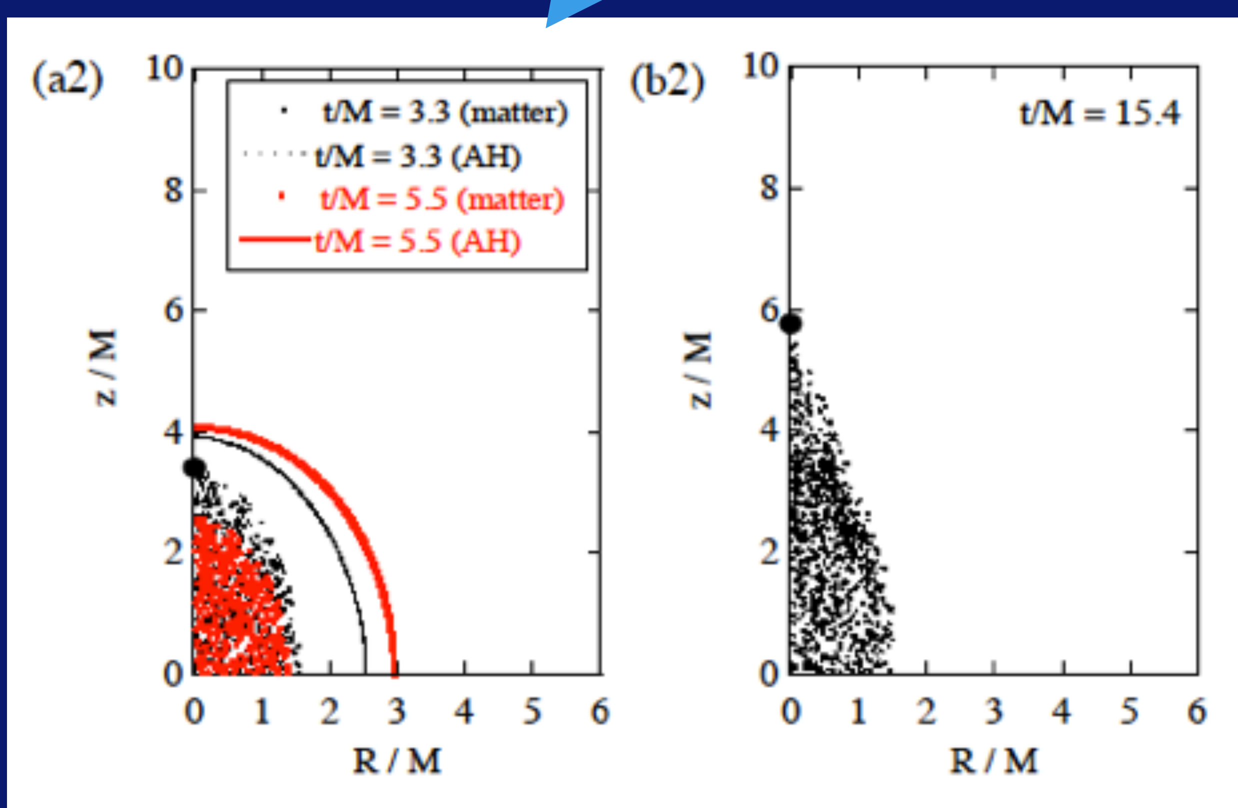
$SO(3)$

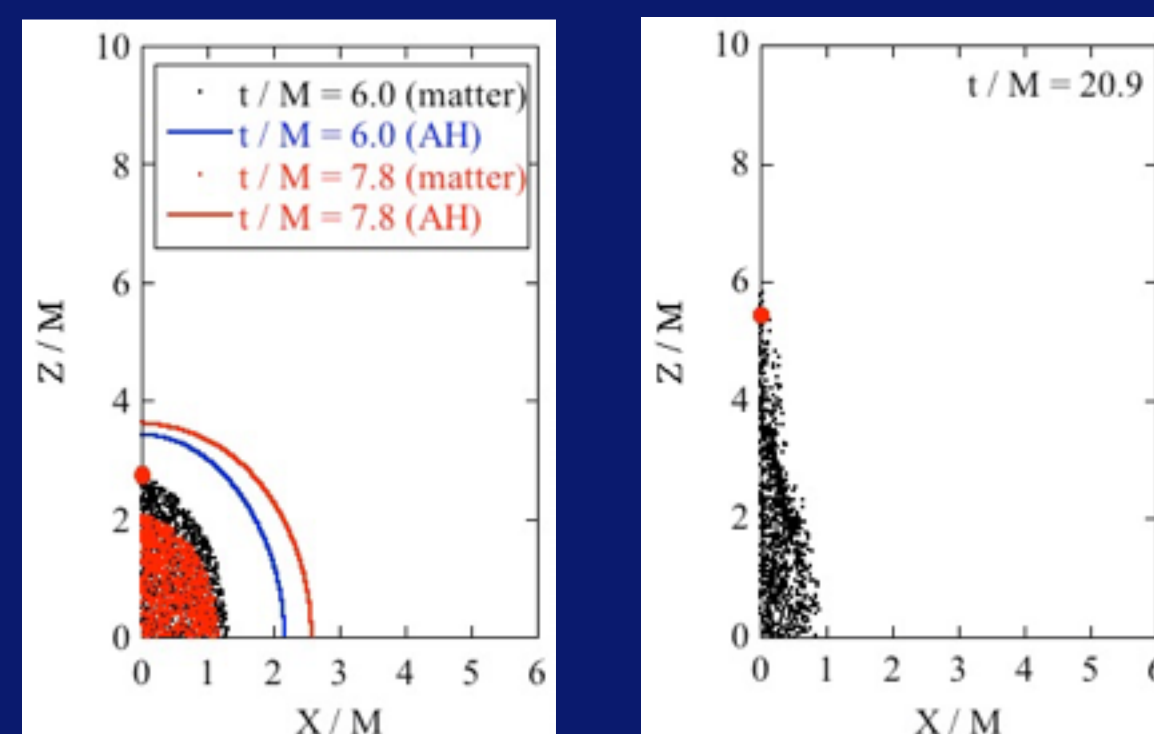




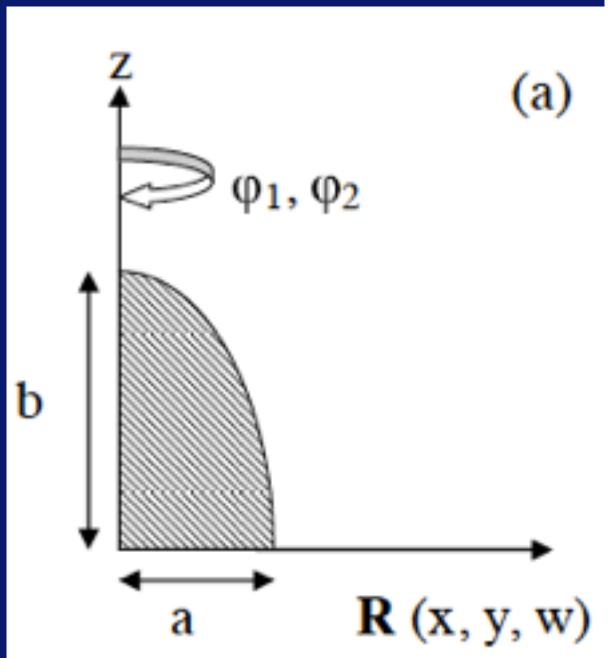
$U(1) \times U(1)$

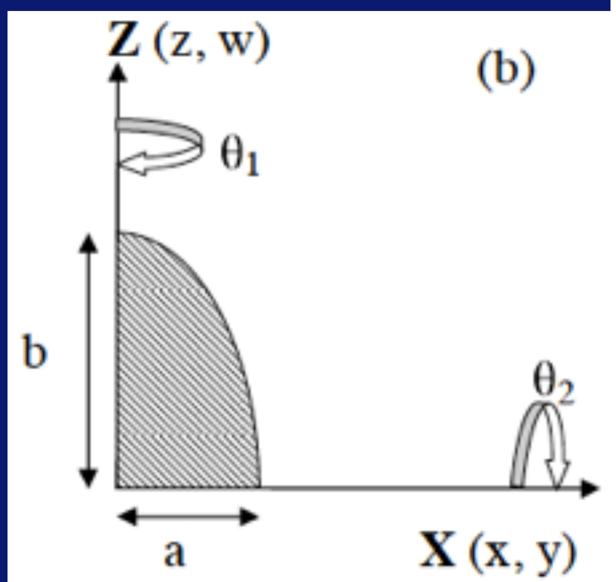






2. Spheroidal matter collapse
D. Comparisons 4D vs. 5D





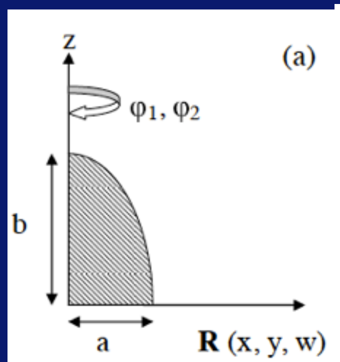
$b/M\ (t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα AH-formed $e_{\rm AH}=0.90$ $e_{\rm f}=0.92$	4Dβ no $e_{\rm f}=0.89$	4Dγ no $e_{\rm f}=0.92$	4Dδ no $e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.82$	5DSβ AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.84$	5DSγ no $e_{\rm f}=0.88$	5DSδ no $e_{\rm f}=0.96$

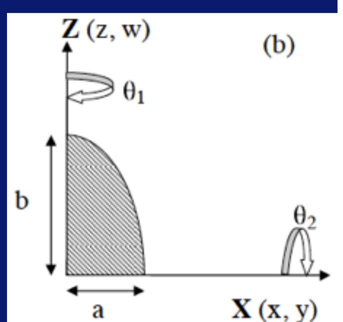
towards spindle

towards spherical

towards spherical towards spindle

2. Spheroidal matter collapse
D. Comparisons 4D vs. 5D

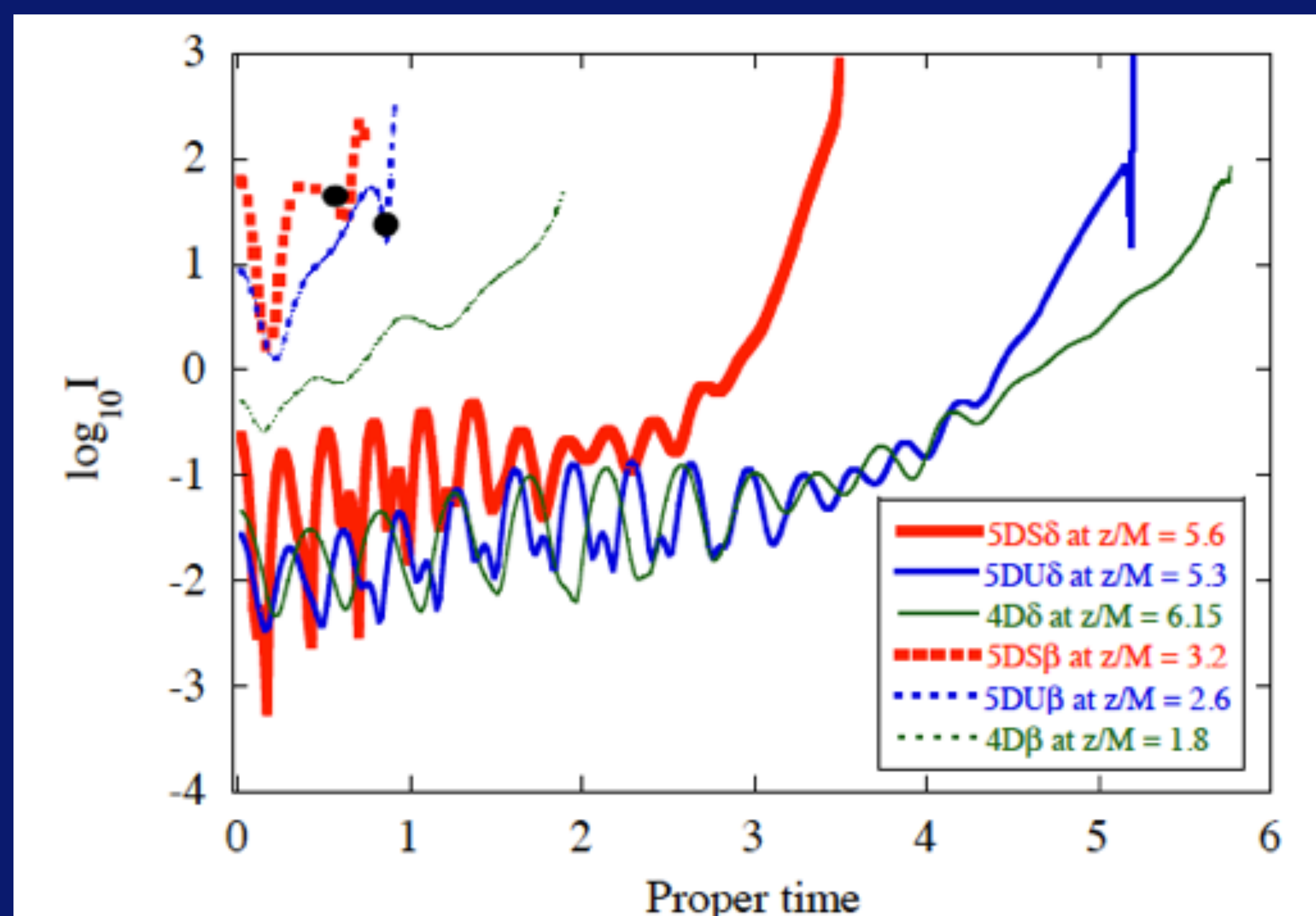




b/M $(t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα AH-formed $e_{\rm AH}=0.90$ $e_{\rm f}=0.92$	4Dβ no $e_{\rm f}=0.89$	4Dγ no $e_{\rm f}=0.92$	4Dδ no $e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.82$	5DSβ AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.84$	5DSγ no $e_{\rm f}=0.88$	5DSδ no $e_{\rm f}=0.96$
5D double axisym. U(1)×U(1)	5DUα AH-formed $e_{\rm AH}=0.86$ $e_{\rm f}=0.79$	5DUβ AH-formed $e_{\rm AH}=0.87$ $e_{\rm f}=0.81$	5DUγ AH-formed $e_{\rm AH}=0.92$ $e_{\rm f}=0.90$	5DUδ no $e_{\rm f}=0.98$

5D collapses
- -- proceed rapidly.
- -- towards spherically.
- -- AH forms in wider ranges.

$$I = R_{abcd}R^{abcd}$$
at $I(t_{\rm end})$



2. Spheroidal matter collapse
C. Evolution examples

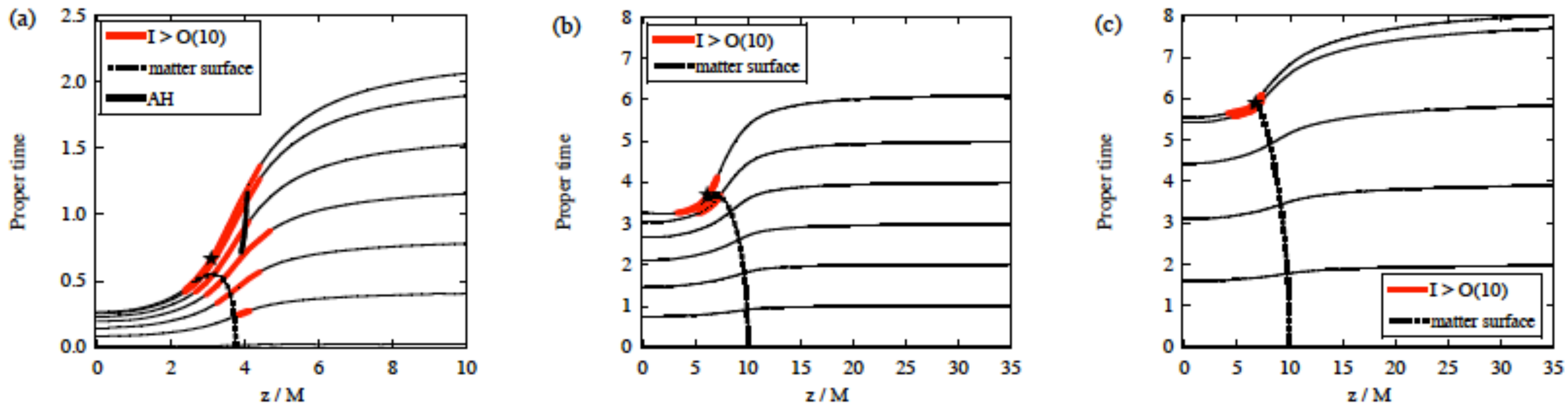
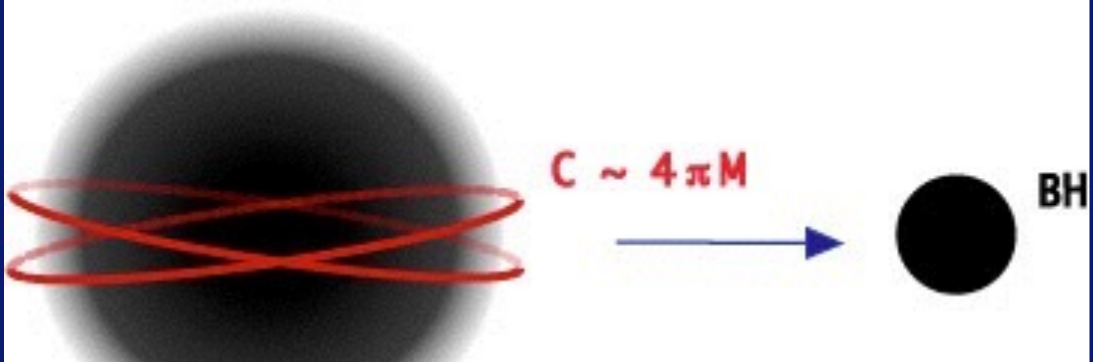


FIG. 4: The snapshots of the hypersurfaces on the z-axis in the proper-time versus coordinate diagram; (a) model 5DSβ, (b) model 5DSδ, and (c) model 4Dδ. The upper most hypersurface is the final data in numerical evolution. We also mark the matter surface and the location of AH if exist. The ranges with $\mathcal{I} \geq 10$ are marked with bold lines and peak value of $\mathcal{I}$ express by asterisks.

2'. Hoop Conjecture

A. Hyper-Hoop conjecture ?

Hoop Conjecture Thorne (1972)



$$C \leq 4\pi M$$

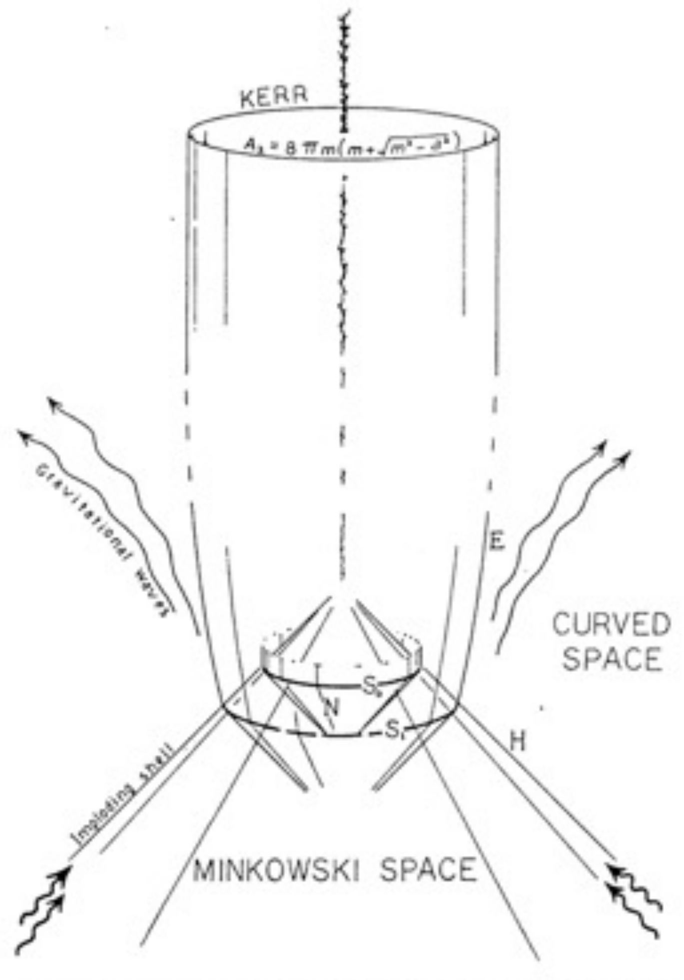


Figure 3. An attempt to derive a contradiction with the "establishment" viewpoint. A shell of photons collapses into a flat space-time. The assumption that a K black hole results implies the existence of a certain inequality on the initial geom

Penrose (1969)

$$A \leq 16\pi M^2$$

Hyper-Hoop Conjecture

Ida-Nakao (2002)

$$V_{D-3} \leq G_D M$$

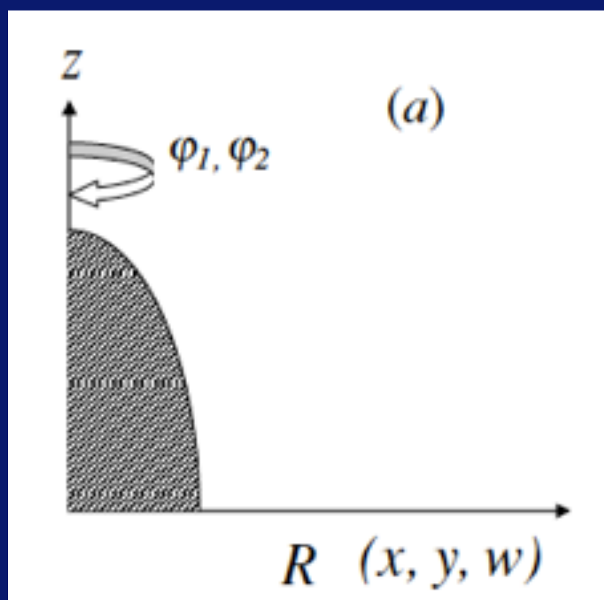
In 5-D, if mass gets compacted in some area, ...

2'. Hoop Conjecture

B. Spheroidal Cases

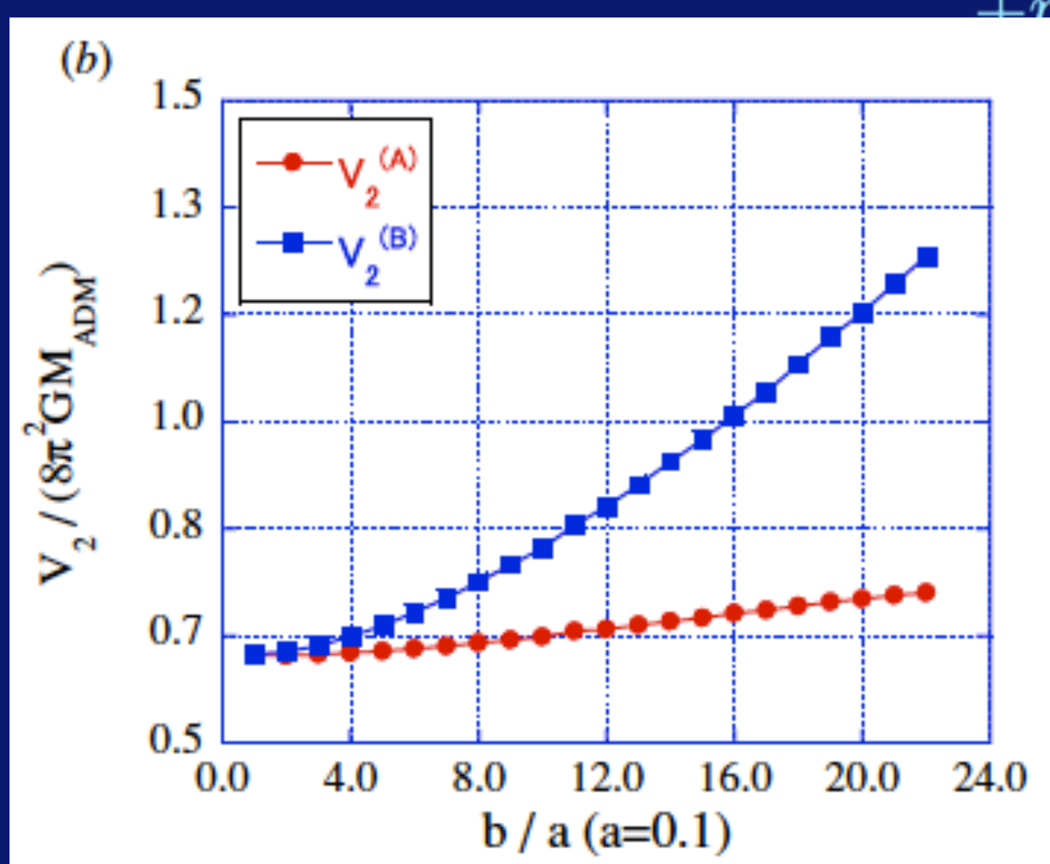
$$V_2 \le \frac{\pi}{2} 16\pi G_5 M$$

Define Hyper-Hoop as the surface $\delta V_2 = 0$



$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r}_h^{\,2} + r_h^2}\, r_h \sin\theta \, d\theta$$

$$\ddot{r}_h - \frac{3\dot{r}_h^{\,2}}{r_h} - 2r_h + \frac{r_h^2 + \dot{r}_h^{\,2}}{r_h}\left[\frac{\dot{r}_h}{r_h}\cot\theta - \frac{2}{\psi}(\dot{r}_h \sin\theta + r_h\cos\theta)\frac{\partial\psi}{\partial z} - \frac{2}{\psi}(r_h\sin\theta - \dot{r}_h\cos\theta)\frac{\partial\psi}{\partial R}\right] = 0$$



$$\int_0^{\pi/2} \psi^2 \sqrt{\dot{r}_h^{\,2} + r_h^2}\, r_h \cos\theta \, d\theta$$

$$\frac{\dot{r}_h^{\,2}}{r_h} - 2r_h - \frac{r_h^2 + \dot{r}_h^{\,2}}{r_h}\left[\frac{\dot{r}_h}{r_h}\tan\theta + \frac{2}{\psi}(r_h\sin\theta \ldots \dot{r}_h\cos\theta)\frac{\partial\psi}{\partial R} + \frac{2}{\psi}(r_h\cos\theta + \dot{r}_h\sin\theta)\frac{\partial\psi}{\partial z}\right] = 0$$

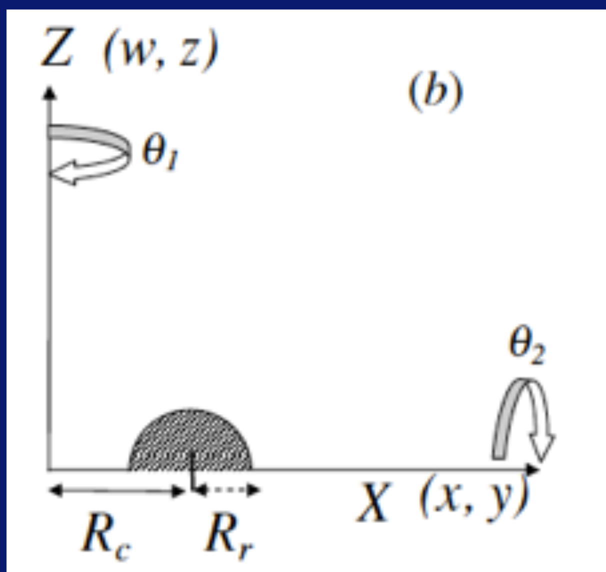
Hyper-Hoop $V_2^{(A)}$ does work for spheroidal horizons.

2'. Hoop Conjecture
c. Toroidal Cases

$$V_2 \le \frac{\pi}{2} 16\pi G_5 M$$

$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \cos\phi \, d\phi$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h + \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\cot\phi - \frac{2}{\psi}(\dot{r_h}\sin\phi + r_h\cos\phi)\frac{\partial\psi}{\partial X} - \frac{2}{\psi}(r_h\sin\phi - \dot{r_h}\cos\phi)\frac{\partial\psi}{\partial Z}\right] = 0$$

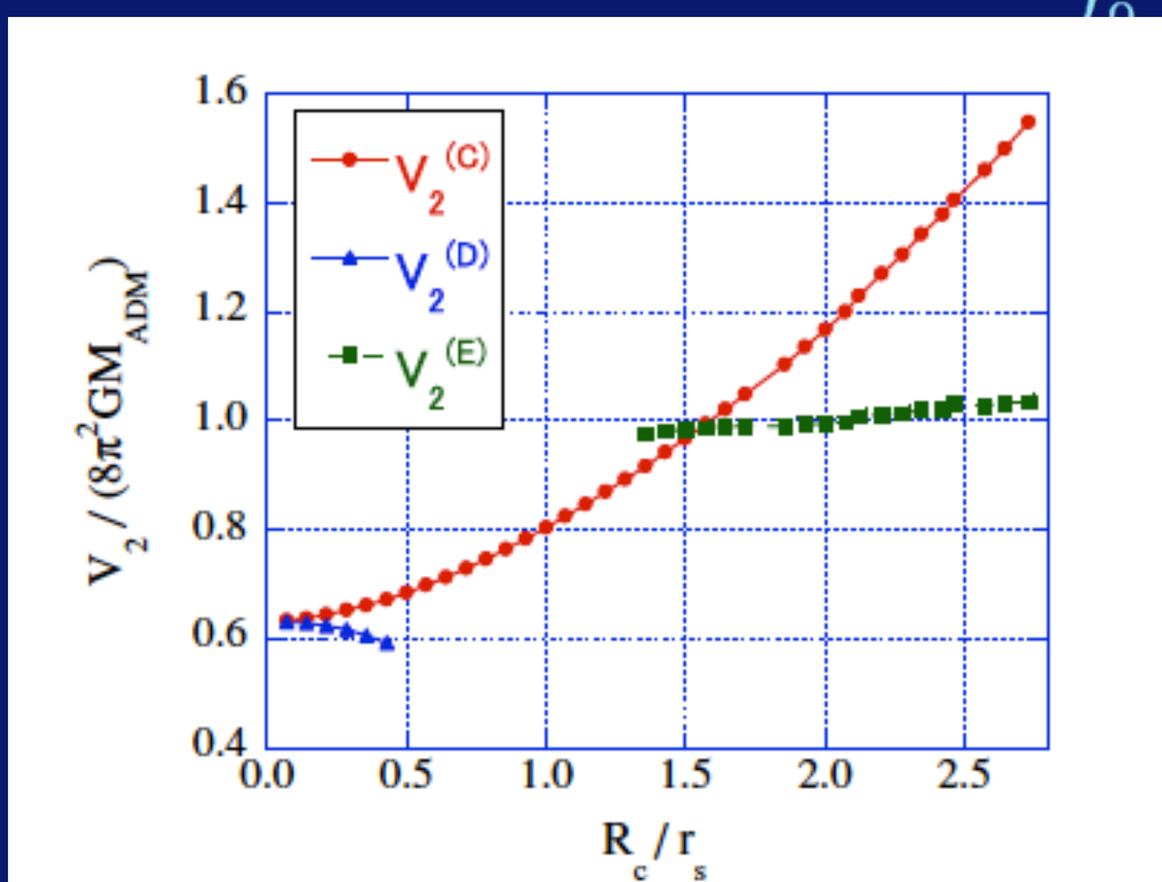


$$V_2^{(D)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \sin\phi \, d\phi$$

$$- 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\tan\phi + \frac{2}{\psi}(r_h\sin\phi \ldots s\phi)\frac{\partial\psi}{\partial X} + \frac{2}{\psi}(r_h\cos\phi + \dot{r_h}\sin\phi)\frac{\partial\psi}{\partial Z}\right] = 0$$

$$\psi^2 \sqrt{\dot{r_h}^2 + r_h^2}(r_h\cos\xi + R_c)\, d\xi$$

$$- 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{-R_c + \dot{r_h}\sin\xi}{R_c + r_h\cos\xi} + \frac{2}{\psi}(\dot{r_h}\sin\xi + r_h\cos\xi)\frac{\partial\psi}{\partial X} + \frac{2}{\psi}(r_h\sin\xi - \dot{r_h}\cos\xi)\frac{\partial\psi}{\partial Z}\right] = 0$$



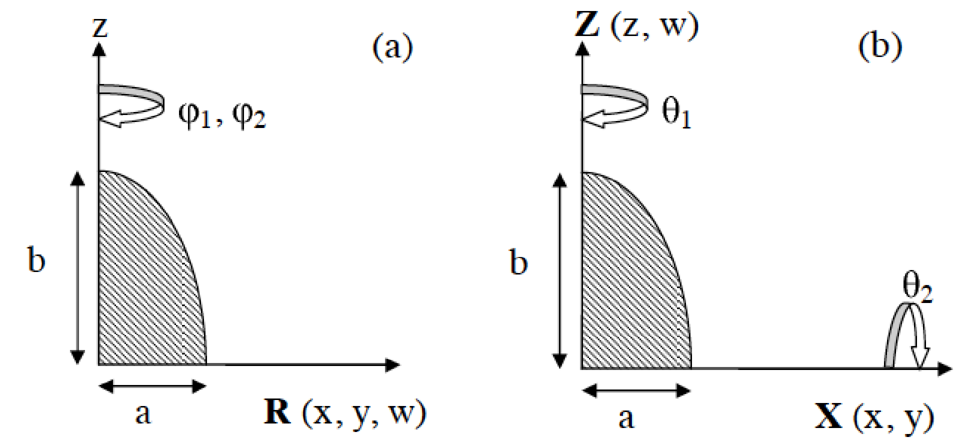
Hyper-Hoop does not work for ring horizons.

Dynamics in 5dim GR gravity?

Yamada & HS, CQG 27 (2010) 045012
Yamada & HS, PRD 83 (2011) 064006

2. Spheroidal matter collapse
Initial data analysis, Evolutions

（回転なしのスピンドル形状の重力崩壊）



＊5D は，4Dよりもはやく重力崩壊をおこす　（局所的に強い重力）
＊5Dでの崩壊は，4Dよりも球状になりやすい　（重力が多くの自由度持つ）
＊Apparent Horizonは5Dの方が形成しやすい.　（球状進化の結果）

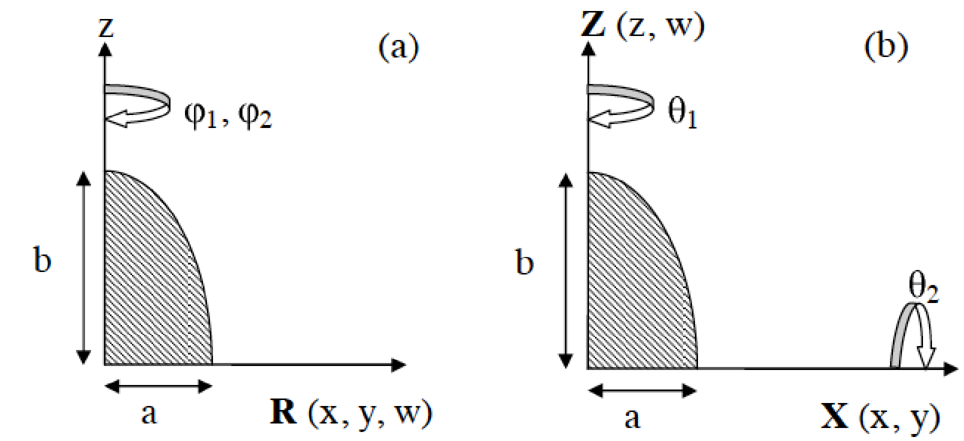
＊極端なスピンドルでは裸の特異点が出現する

＊Hyper-Hoop conjectureはスピンドル形状に対して成立.
リング形状に対して不成立.

Dynamics in 5dim GR gravity?

2. Spheroidal matter collapse
Initial data analysis, Evolutions

Yamada & HS, CQG 27 (2010) 045012
Yamada & HS, PRD 83 (2011) 064006



3. Wormhole dynamics in GR
linear stability,
dynamical stability

Torii & HS, PRD 88 (2013) 064027
HS & Torii, in preparation

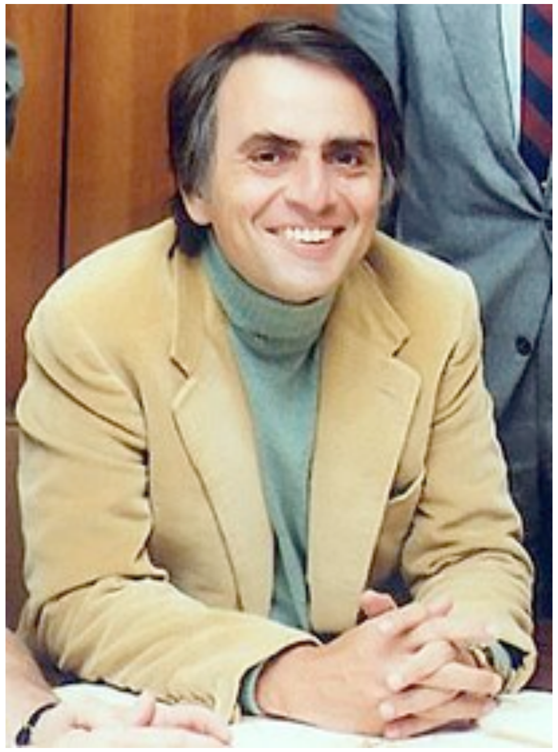
Dynamics in Gauss-Bonnet gravity?

4. Wormhole dynamics in GB
5. Plane-wave collision in GB

HS & Torii, in preparation

Why Wormhole?

They make great science fiction -- short cuts between otherwise distant regions.
Morris & Thorne 1988, Sagan "Contact" etc



US movie 1997

Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity

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(Received 16 March 1987; accepted for publication 17 July 1987)

Rapid interstellar travel by means of spacetime wormholes is described in a way that is useful for teaching elementary general relativity. The description touches base with Carl Sagan's novel *Contact*, which, unlike most science fiction novels, treats such travel in a manner that accords with the best 1986 knowledge of the laws of physics. Many objections are given against the use of black holes or Schwarzschild wormholes for rapid interstellar travel. A new class of solutions of the Einstein field equations is presented, which describe wormholes that, in principle, could be traversed by human beings. It is essential in these solutions that the wormhole possess a throat at which there is no horizon; and this property, together with the Einstein field equations, places an extreme constraint on the material that generates the wormhole's spacetime curvature: In the wormhole's throat that material must possess a radial tension τ_0 with the enormous magnitude $\tau_0 \sim$ (pressure at the center of the most massive of neutron stars) $\times$ (20 km)^2/(circumference of throat)^2. Moreover, this tension must exceed the material's density of mass-energy, $\rho_0 c^2$. No known material has this $\tau_0 > \rho_0 c^2$ property, and such material would violate all the "energy conditions" that underlie some deeply cherished theorems in general relativity. However, it is not possible today to rule out firmly the existence of such material; and quantum field theory gives tantalizing hints that such material might, in fact, be possible.

Box 1. Excerpts from *Contact* by Carl Sagan.[19]

After traveling through some sort of "tunnel" that took them in less than an hour from Earth to an orbit around the star Vega, five of the characters in the novel speculate on the nature of the tunnel:

"You see," Eda explained softly, "if the tunnels are black holes there are real contradictions implied. There is an interior tunnel in the exact Kerr solution of the Einstein Field Equations, but it's unstable. The slightest perturbation would seal it off and convert the tunnel into a physical singularity through which nothing can pass. I have tried to imagine a superior civilization that would control the internal structure of a collapsing star to keep the interior tunnel stable. This is very difficult. The civilization would have to monitor and stabilize the tunnel forever. It would be especially difficult with something as large as the dodecahedron falling through."

"Even if Abonnema can discover how to keep the tunnel open, there are many other problems," Vaygay said. "Too many. Black holes collect problems faster than they collect matter. There are the tidal forces. We should have been torn apart in the black hole's gravitational field. We should have been stretched like people in the paintings of El Greco or the sculptures of . . . Giacometti. Then other problems: As measured from Earth it takes an infinite amount of time for us to pass through a black hole, and we could never, never return to Earth. Maybe this is what happened. Maybe we will never go home. Then, there should be an inferno of radiation near the singularity. This is a quantum mechanical instability. . . "

"And finally," Eda continued, "a Kerr-type tunnel can lead to grotesque causality violations. With a modest change of trajectory inside the tunnel, one could emerge from the other end as early in the history of the universe as you might like—a picosecond after the big bang, for example. That would be a very disorderly universe.

"Look, fellas," she said, "I'm no expert in General Relativity. But didn't we see black holes? Didn't we fall into them? Didn't we emerge out of them? Isn't a gram of observation worth a ton of theory?"

"I know, I know," Vaygay said in mild agony. "It has to be something else. Our understanding of physics can't be so far off. Can it?"

He addressed this last question, a little plaintively, to Eda, who only replied, "A naturally occurring black hole can't be a tunnel; they have impassible singularities at their centers."

pages 347,348

Eda was, considering the circumstances, very relaxed. She soon understood why. While she and Vaygay had been undergoing lengthy interrogations, he had been calculating.

"I think the tunnels are Einstein–Rosen bridges," he said. "General relativity admits a class of solutions, called wormholes, similar to black holes, but with no evolutionary connection—they cannot be generated, as black holes can, by the gravitational collapse of a star. But the usual sort of wormhole, once made, expands and contracts before anything can cross through; it exerts disastrous tidal forces, and it also requires—at least as seen by an observer left behind—an infinite amount of time to get through."

Ellie did not see how this represented much progress, and asked him to clarify. The key problem was holding the wormhole open. Eda had found a class of solutions to his field equations that suggested a new macroscopic field, a kind of tension that could be used to prevent a wormhole from contracting fully. Such a wormhole would pose none of the other problems of black holes; it would have much smaller tidal stresses, two-way access, quick transit times as measured by an exterior observer, and no devastating interior radiation field.

"I don't know whether the tunnel is stable against small perturbations," he said. "If not, they would have to build a very elaborate feedback system to monitor and correct the instabilities."

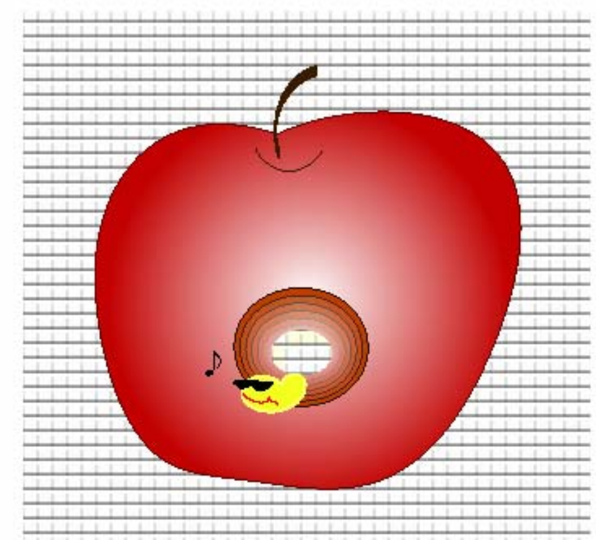
page 406

Morris-Thorne's "Traversable" wormhole

M.S. Morris and K.S. Thorne, Am. J. Phys. 56 (1988) 395
M.S. Morris, K.S. Thorne, and U. Yurtsever, PRL 61 (1988) 3182
H.G. Ellis, J. Math. Phys. 14 (1973) 104
(G. Clément, Am. J. Phys. 57 (1989) 967)

Desired properties of traversable WHs

1. Spherically symmetric and Static $\Rightarrow$ M. Visser, PRD 39(89) 3182 & NPB 328 (89) 203
2. Einstein gravity
3. Asymptotically flat
4. No horizon for travel through
5. Tidal gravitational forces should be small for traveler
6. Traveler should cross it in a finite and reasonably small proper time
7. Must have a physically reasonable stress-energy tensor
   $\Rightarrow$ Weak Energy Condition is violated at the WH throat.
   $\Rightarrow$ (Null EC is also violated in general cases.)
8. Should be perturbatively stable
9. Should be possible to assemble



"Ellis (Morris-Thorne) wormhole"

Why Wormhole?

They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.

They are very similar to black holes --both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole = Hypersurface foliated by marginally trapped surfaces

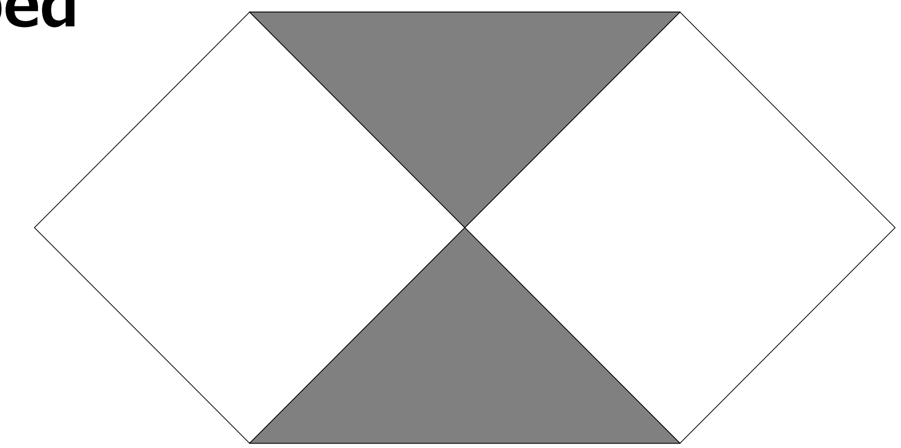
BH and WH are interconvertible? New duality?

BH & WH are interconvertible?

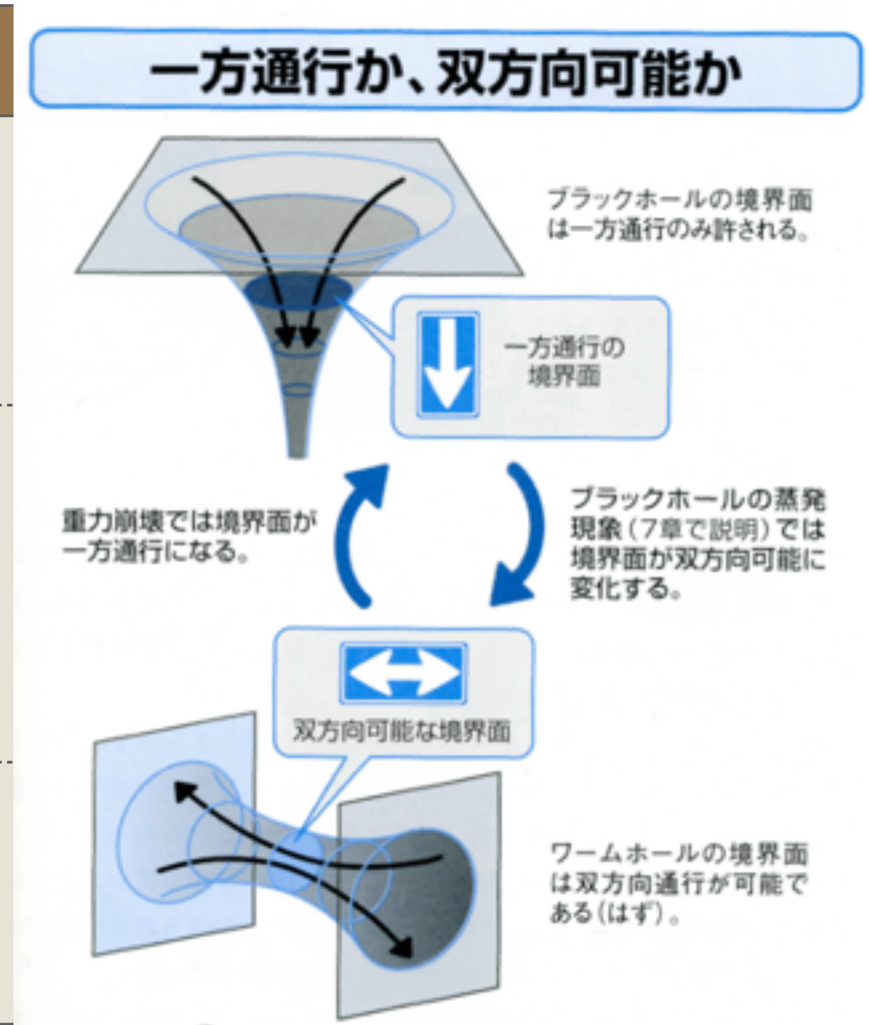
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)

Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.



	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH ⇨ 1-way traversable	Temporal (timelike) outer THs ⇨ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density “exotic” matter
Appear-ance	occur naturally	Unlikely to occur naturally. but constructible??



Part I Wormhole dynamics in 4-dim GR

PHYSICAL REVIEW D **66**, 044005 (2002)

Fate of the first traversible wormhole: Black-hole collapse or inflationary expansion

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(Received 10 May 2002; published 16 August 2002)

Fate of Morris-Thorne (Ellis) wormhole?

- "Dynamical wormhole" defined by local trapping horizon
- spherically symmetric, both normal/ghost KG field
- apply dual-null formulation in order to seek horizons
- Numerical simulation

ghost/normal Klein-Gordon fields

$$T_{\mu\nu} = T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) = \underbrace{\left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right]}_{\text{normal}} + \underbrace{\left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right]}_{\text{ghost}}$$

$$\Box\psi = \frac{dV_1(\psi)}{d\psi}, \qquad \Box\phi = \frac{dV_2(\phi)}{d\phi}.$$

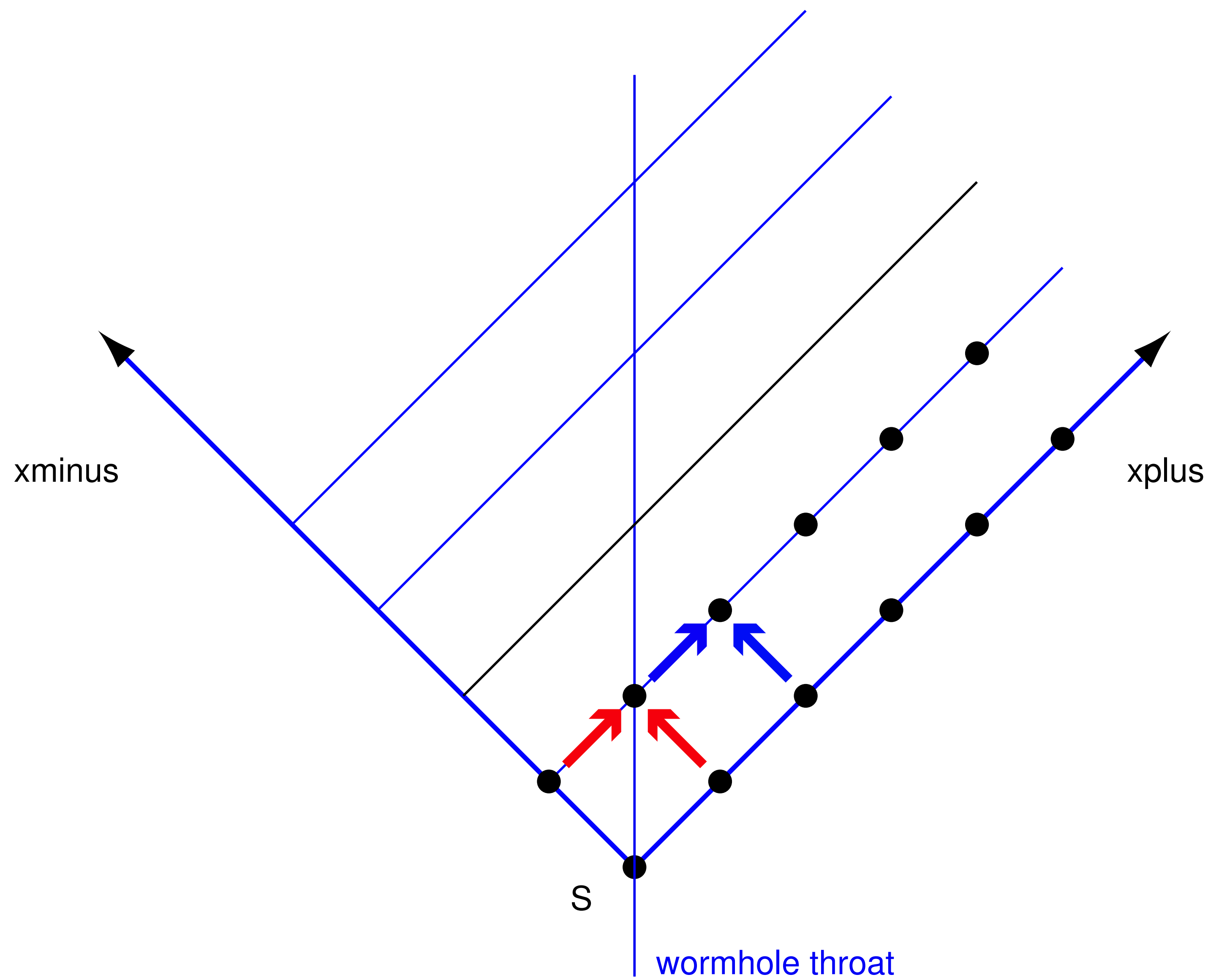
(Hereafter, we set $V_1(\psi) = 0, V_2(\phi) = 0$)

Initial data on $x^+ = 0$, $x^- = 0$ slices and on S

Generally, we have to set :

$$
\begin{aligned}
(\Omega, f, \vartheta_\pm, \phi, \psi) &\quad \text{on } S\text{: } x^+ = x^- = 0 \\
(\nu_\pm, \wp_\pm, \pi_\pm) &\quad \text{on } \Sigma_\pm\text{: } x^\mp = 0,\ x^\pm \geq 0
\end{aligned}
$$

Grid Structure for Numerical Evolution



dual-null formulation, spherically symmetric spacetime (4D)

- The spherically symmetric line-element:

$$ds^2 = -2e^{-f}dx^+dx^- + r^2dS^2, \quad \text{where} \quad r = r(x^+, x^-), f = f(x^+, x^-), \cdots$$

- To obtain a system accurate near $\Im^{\pm}$, we introduce the conformal factor $\boxed{\Omega = 1/r}$. We also define first-order variables, the conformally rescaled momenta

expansions	$\vartheta_\pm = 2\partial_\pm r = -2\Omega^{-2}\partial_\pm\Omega$	$(\theta_\pm = 2r^{-1}\partial_\pm r)$	(1)
inaffinities	$\nu_\pm = \partial_\pm f$		(2)
momenta of ϕ	$\wp_\pm = r\partial_\pm\phi = \Omega^{-1}\partial_\pm\phi$		(3)
momenta of ψ	$\pi_\pm = r\partial_\pm\psi = \Omega^{-1}\partial_\pm\psi$		(4)

The set of equations (remember the identity: $\partial_+\partial_- = \partial_-\partial_+$):

$$\partial_\pm\vartheta_\pm = -\nu_\pm\vartheta_\pm - 2\Omega\pi_\pm^2 + 2\Omega\wp_\pm^2, \tag{5}$$

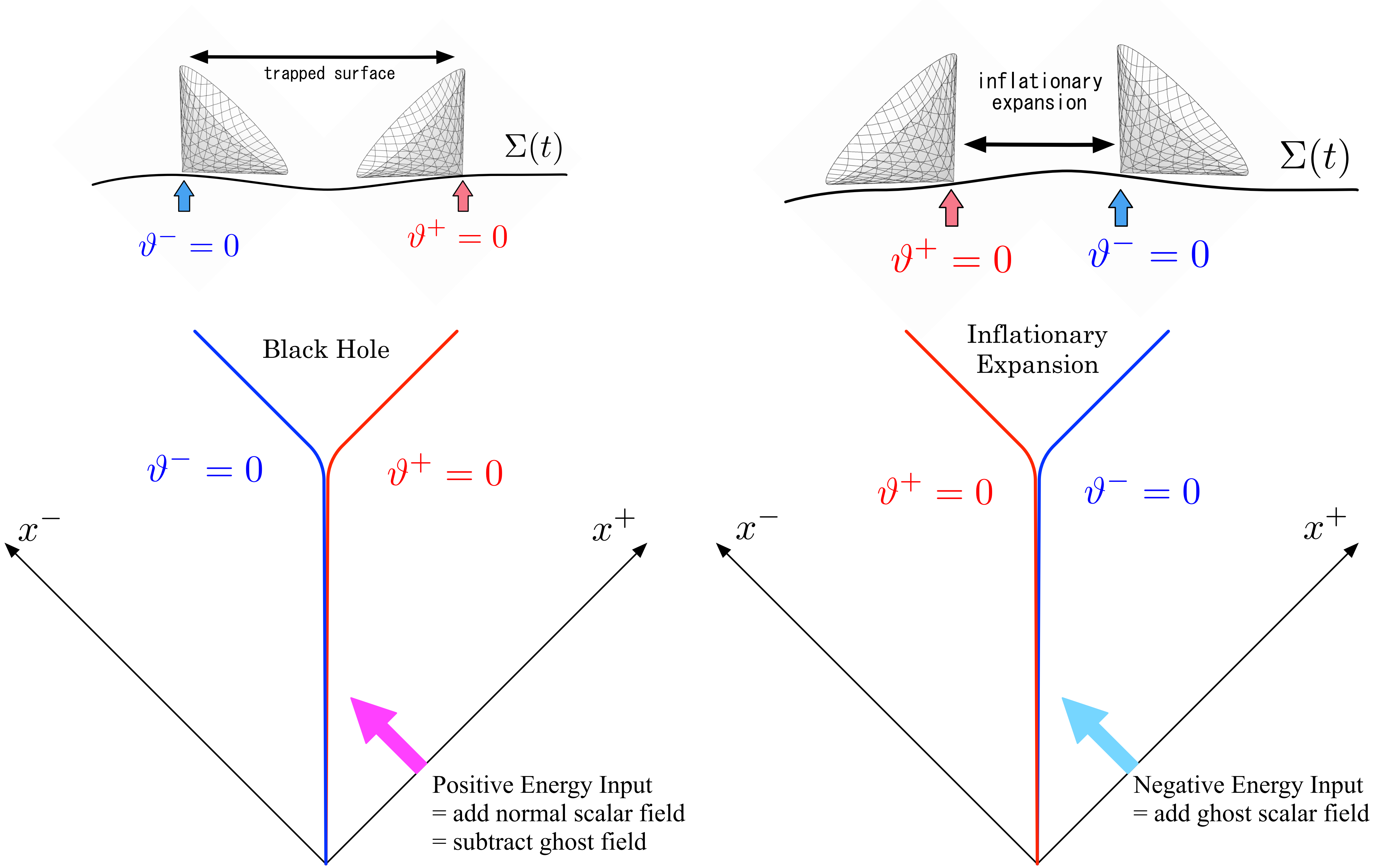
$$\partial_\pm\vartheta_\mp = -\Omega(\vartheta_+\vartheta_-/2 + e^{-f}), \tag{6}$$

$$\partial_\pm\nu_\mp = -\Omega^2(\vartheta_+\vartheta_-/2 + e^{-f} - 2\pi_+\pi_- + 2\wp_+\wp_-), \tag{7}$$

$$\partial_\pm\wp_\mp = -\Omega\vartheta_\mp\wp_\pm/2, \tag{8}$$

$$\partial_\pm\pi_\mp = -\Omega\vartheta_\mp\pi_\pm/2. \tag{9}$$

Wormhole evolutionの結果



Ghost pulse input -- Bifurcation of the horizons（4d)

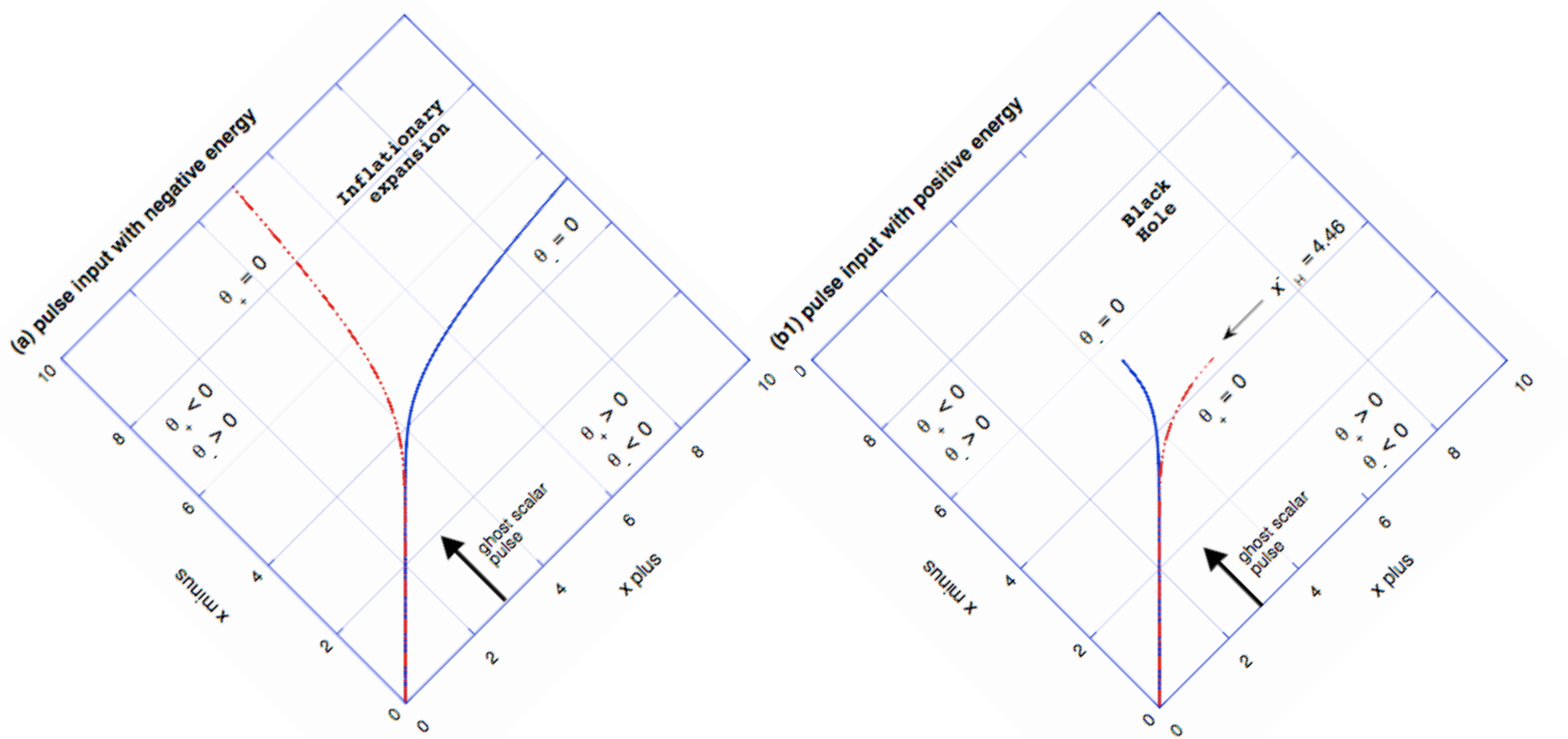
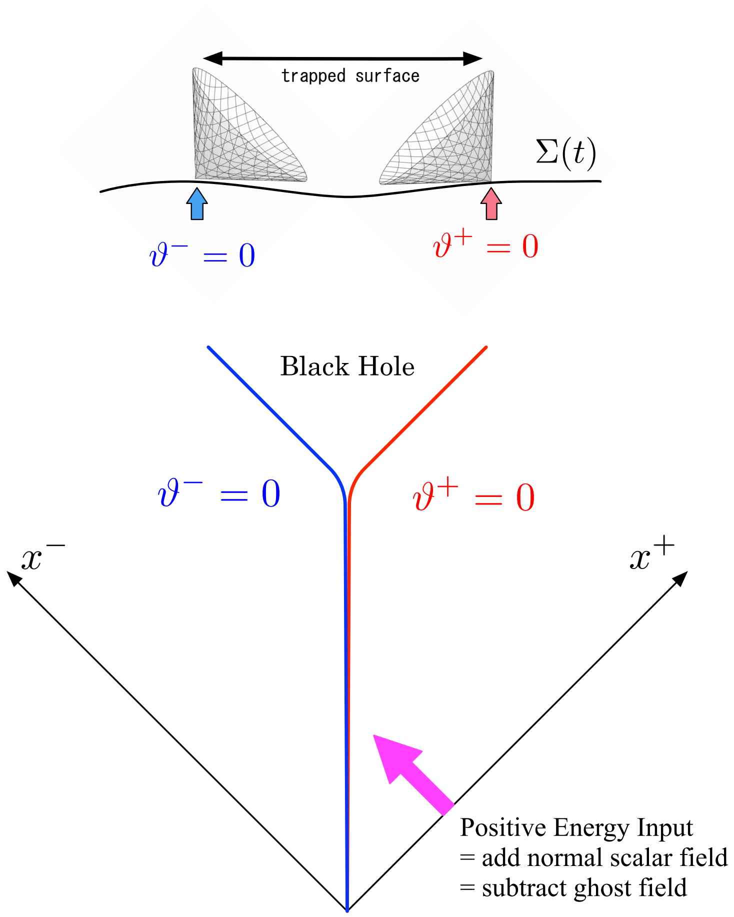
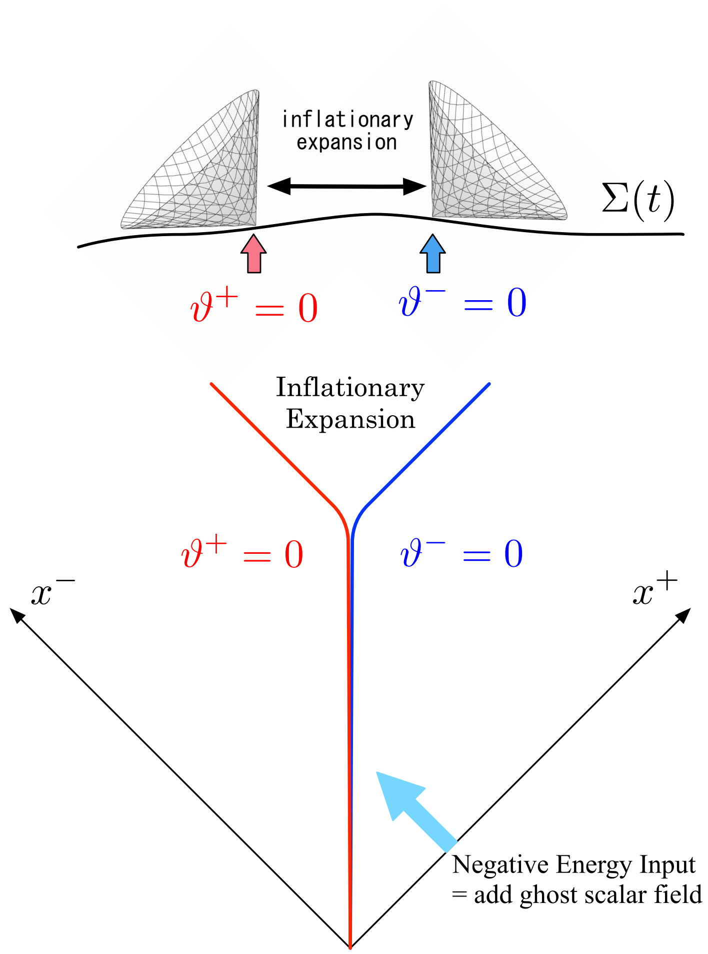
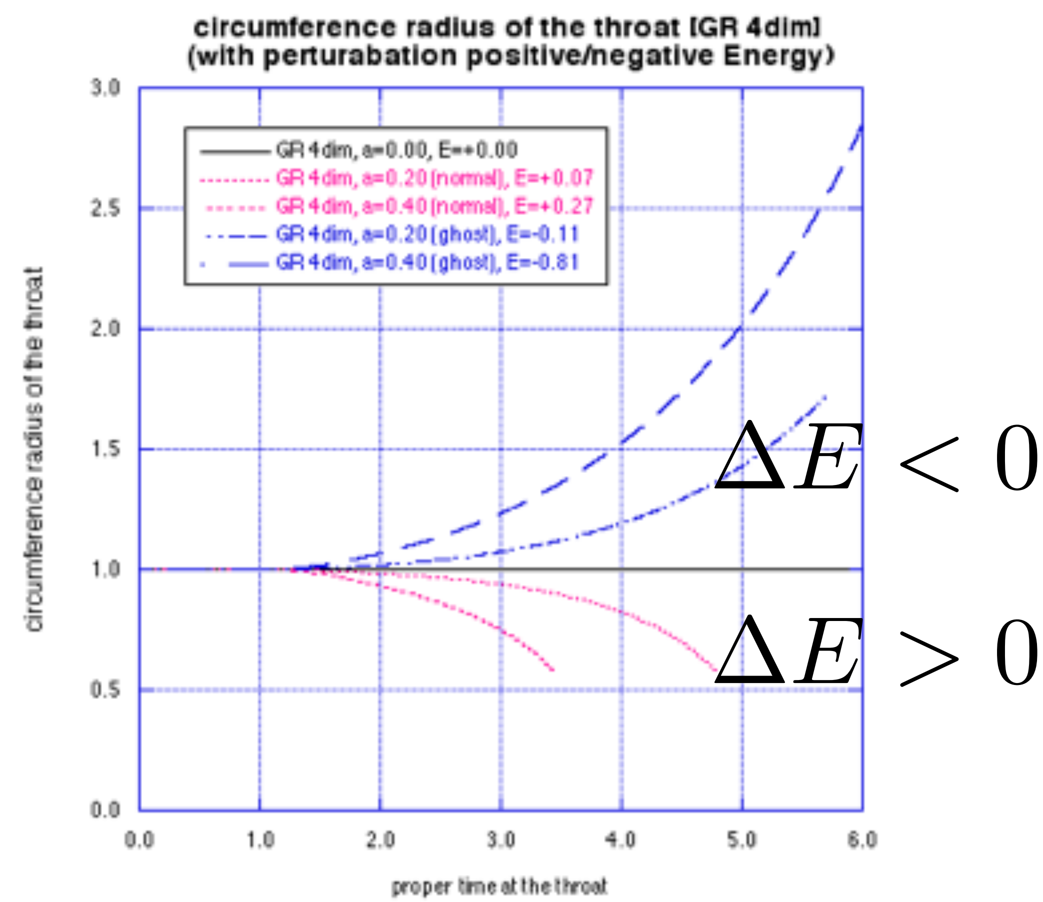


Figure 3: Horizon locations, $\vartheta_\pm = 0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

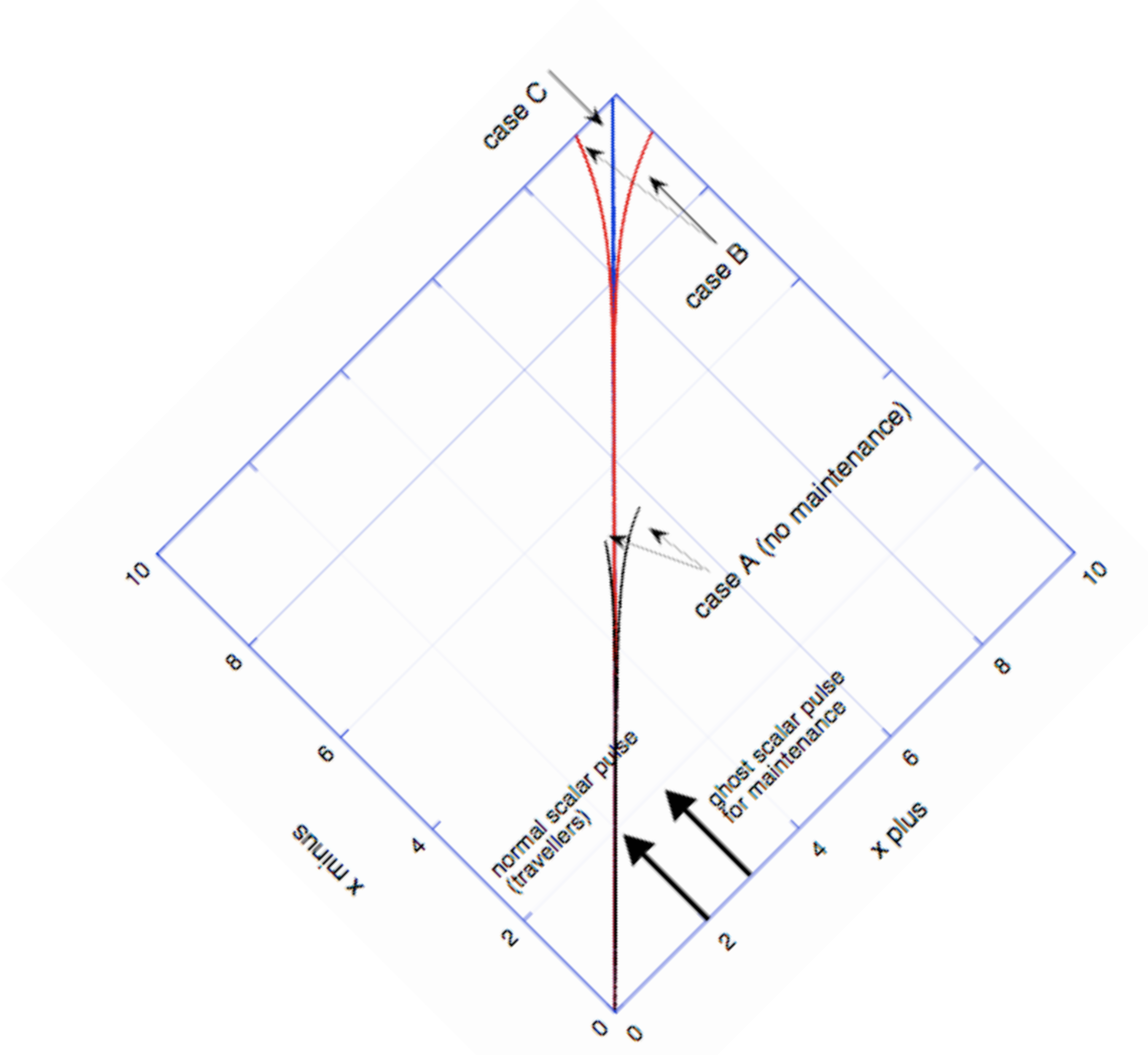
Wormhole evolutionの結果







Travel through a Wormhole
-- with Maintenance Operations!



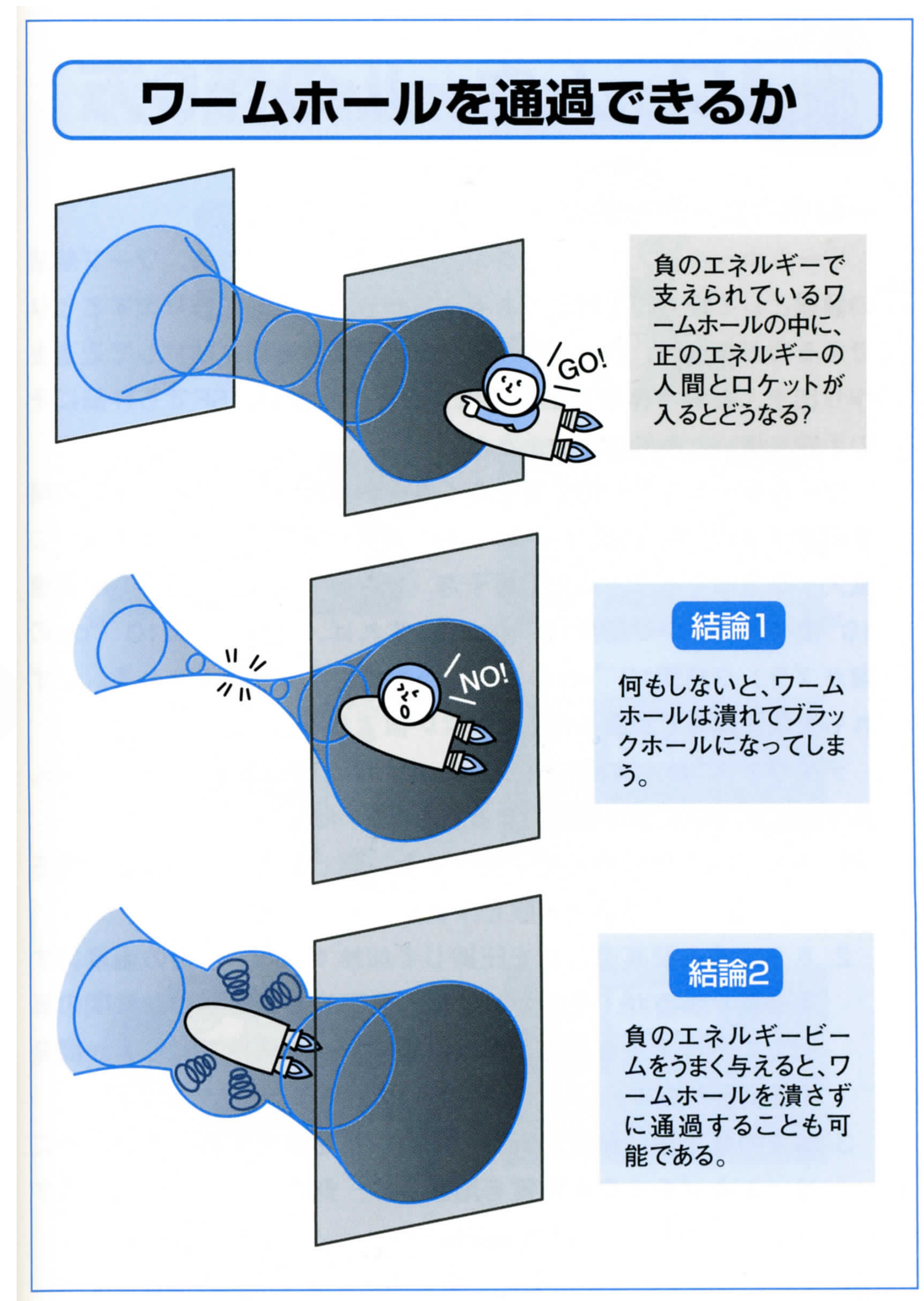


Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. The travellers pulse are commonly expressed with a normal scalar field pulse, $(\tilde{c}_a, \tilde{c}_b, \tilde{c}_c) = (+0.1, 6.0, 2.0)$. Horizon locations $\vartheta_+ = 0$ are plotted for three cases:

(A) no maintenance case (results in a black hole),

(B) with maintenance pulse of $(c_a, c_b, c_c) = (0.02390, 6.0, 3.0)$ (results in an inflationary expansion),

(C) with maintenance pulse of $(c_a, c_b, c_c) = (0.02385, 6.0, 3.0)$ (keep stationary structure upto the end of this range).

Summary of Part I

HS & Hayward, PRD66 (2002) 044005

Dynamics of Ellis (Morris-Thorne) traversible WH

WH is Unstable

(A) with positive energy pulse ---> BH

---> confirms duality conjecture between BH and WH.

(B) with negative energy pulse ---> Inflationary expansion

---> provides a mechanism for enlarging a quantum WH to macroscopic size

(C) can be maintained by sophisticated operations

---> a round-trip is available for our hero/heroine

The basic behaviors has been confirmed by

A Doroshkevich, J Hansen, I Novikov, A Shatskiy, IJMPD 18 (2009) 1665

J A Gonzalez, F S Guzman & O Sarbach, CQG 26 (2009) 015010, 015011

J A Gonzalez, F S Guzman & O Sarbach, PRD80 (2009) 024023

O Sarbach & T Zannias, PRD 81 (2010) 047502

(1) Exact Solution : Basic eqns.

Torii & HS, PRD88 (2013) 064027

- general relativity, *n-dimension*

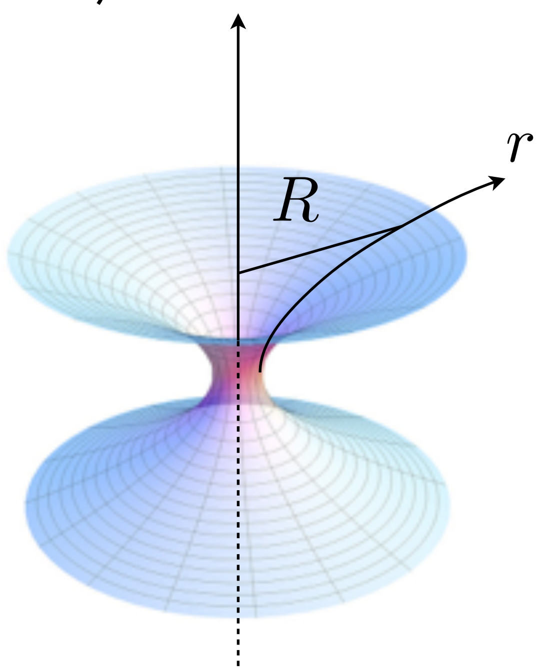
massless scalar field (ghost)

$$S = \int d^n x \sqrt{-g} \left[\frac{1}{2\kappa_n^2} R - \frac{1}{2} \epsilon (\partial\phi)^2 - \cancel{V(\phi)} \right], \qquad \epsilon = -1$$

- static, spherical sym., asymptotically flat

$$ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 \underline{h_{ij} dx^i dx^j}$$

$(k = 1)$



- Basic equations

$$(t,t): \quad -\frac{n-2}{2} f^2 \left[\frac{2R''}{R} + \frac{f'R'}{fR} + \frac{(n-3)R'^2}{R^2} \right] + \frac{(n-2)(n-3)kf}{2R^2} = \kappa_n^2 f \left[\frac{1}{2} \epsilon f \phi'^2 + \cancel{V(\phi)} \right],$$

$$(r,r): \quad \frac{n-2}{2} \frac{R'}{R} \left[\frac{f'}{f} + \frac{(n-3)R'}{R} \right] - \frac{(n-2)(n-3)k}{2fR^2} = \frac{\kappa_n^2}{f} \left[\frac{1}{2} \epsilon f \phi'^2 - \cancel{V(\phi)} \right],$$

$$(i,j): \quad \frac{f''}{2} + (n-3) f \left(\frac{R''}{R} + \frac{f'R'}{fR} + \frac{n-4}{2} \frac{R'^2}{R^2} \right) - \frac{(n-3)(n-4)k}{2R^2} = \kappa_n^2 \left[\frac{1}{2} \epsilon f \phi'^2 + \cancel{V(\phi)} \right],$$

$$(KG): \quad \frac{1}{R^{n-2}} \left(R^{n-2} f \phi' \right)' = -\epsilon \cancel{\frac{dV}{d\phi}}. \quad \Rightarrow \quad \phi' = \frac{C}{fR^{n-2}}$$

C : constant

Solution

$$ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 h_{ij}dx^i dx^j$$

- regularity at the throat ($r = 0$)

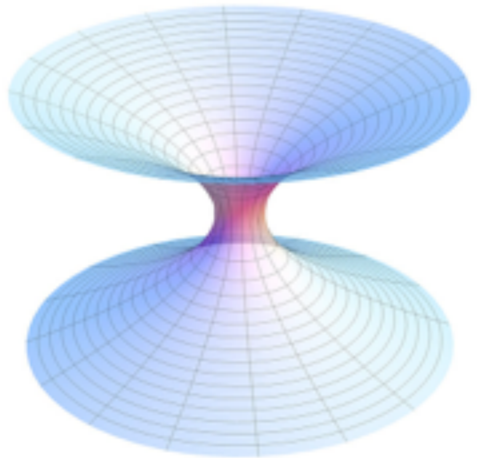
$R = a$ ← throat radius

$R' = 0, \quad f = f_0, \quad f' = 0, \quad \phi = 0$

★ from the scaling rule

$a = 1 \qquad f_0 = 1$

Basics eqns. → $\kappa_n^2 C^2 = (n-2)(n-3)a^{2(n-3)}$



- Exact solution

$$f \equiv 1$$

$$r(R) = -mB_z\left[-m, \frac{1}{2}\right] - \frac{\sqrt{\pi}\Gamma[1-m]}{\Gamma[m(n-4)]}$$

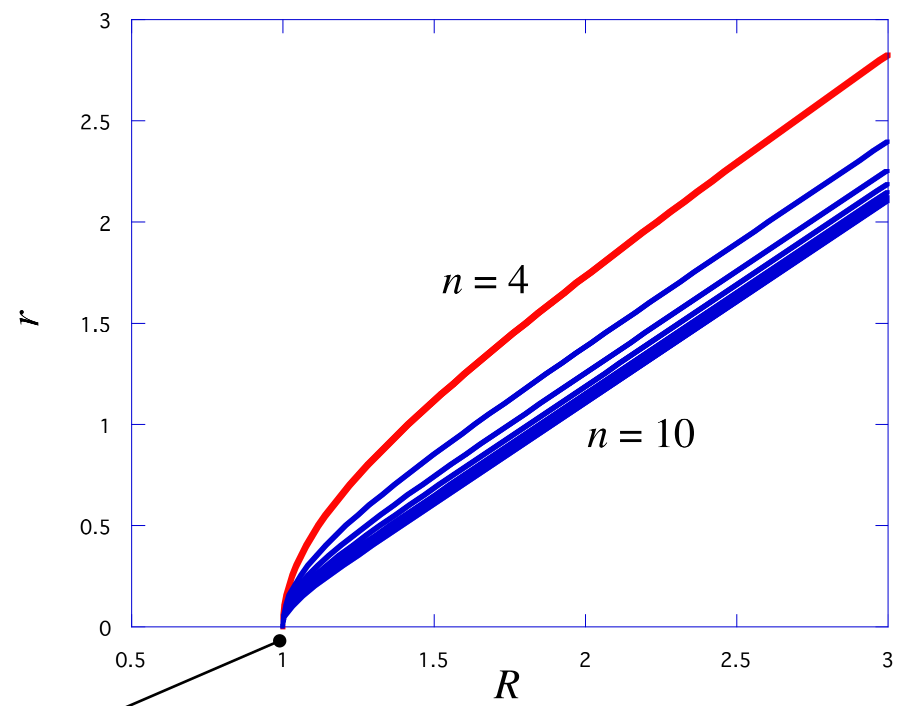
$$\phi = \frac{\sqrt{(n-2)(n-3)}}{\kappa_n}a^{n-3}\int \frac{1}{R(r)^{n-2}}dr$$

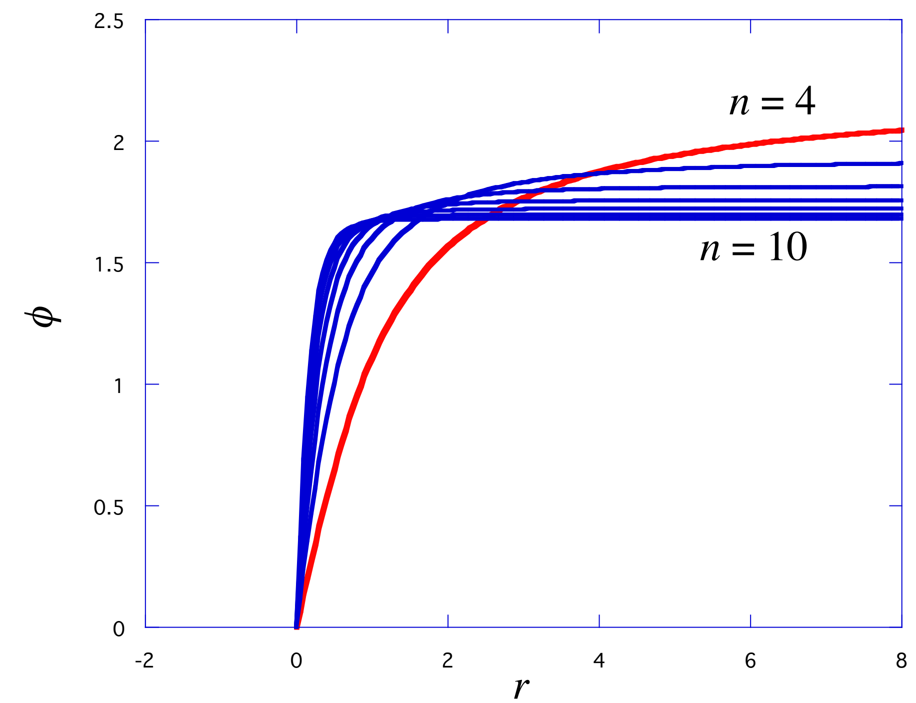
$$m = \frac{1}{2(n-3)}, \quad z = R^m \quad B_z(p,q) := \int_0^z t^{p-1}(1-t)^{q-1}dt$$ Incomplete Beta func.

★ in another metric form: V. Dzhunushaliev+, 2013

Configurations

- configurations $ds_n^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R(r)^2 h_{ij}dx^i dx^j$





expansion is 0

trapping horizon

★ large curvature near the throat.

★ scalar field goes steep if n is large.

★ In the $n \to \infty$ limit

$$R = r + 1 \qquad \phi = 0 \ \ (r = 0) \quad \frac{\pi}{2} \ \ (r > 0)$$

(2) Linear Stability: Master eqn.

Torii & HS, PRD88 (2013) 064027

- metric

$$ds_n^2 = -f(t,r)e^{-2\delta(t,r)}dt^2 + f(t,r)^{-1}dr^2 + R(t,r)^2 h_{ij}dx^i dx^j$$

- linear perturbation

static solution

$$f = f_0(r) + f_1(r)e^{i\omega t}, \quad R = R_0(r) + R_1(r)e^{i\omega t},$$
$$\delta = \delta_0(r) + \delta_1(r)e^{i\omega t}, \quad \phi = \phi_0(r) + \phi_1(r)e^{i\omega t}.$$

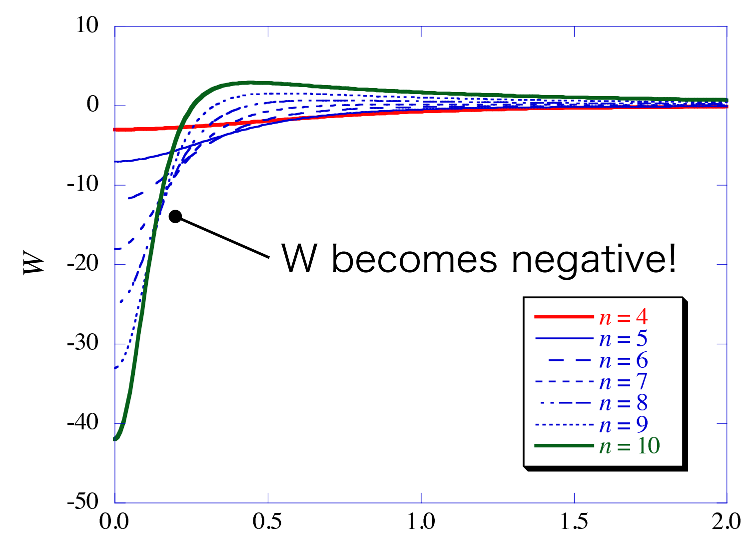
- master equation

$$-\Psi_1'' + W(r)\Psi_1 = \omega^2 \Psi_1,$$

$$W(r) = -\frac{1}{4R_0^2}\left[\frac{3(n-2)^2}{R_0^{2(n-3)}} - (n-4)(n-6)\right].$$

$$\Psi_1 = \mathcal{D}_+\psi_1 \quad \mathcal{D}_+ = \frac{d}{dr} - \frac{\bar{\psi}_1'}{\bar{\psi}_1} \quad \psi_1 = R_0^{\frac{n-2}{2}}\left(\phi_1 - \frac{\phi_0'}{R_0'}R_1\right),$$

★ Ψ_1 : Gauge invariant in spherical sym.



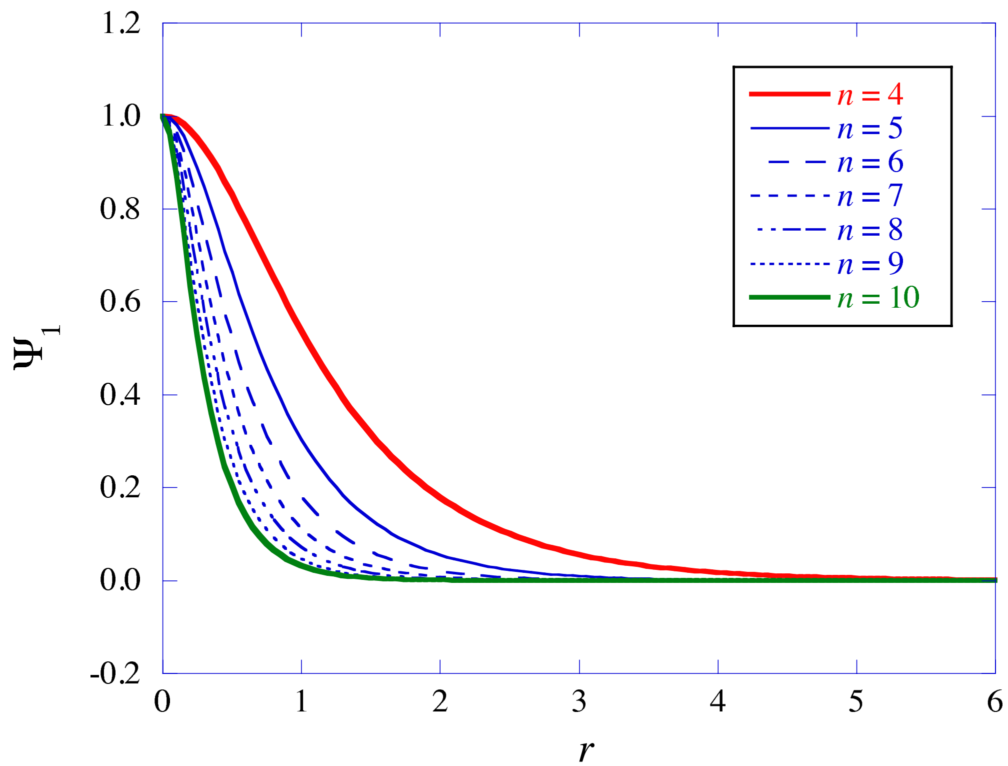
★ potential W

Unstable!

- exist negative mode

n	ω^2
4	-1.39705243371511
5	-2.98495893027790
6	-4.68662054299460
7	-6.46258414126318
8	-8.28975936306259
9	-10.1535530451867
10	-12.0442650147438
11	-13.9552091676647
20	-31.5751101285105
50	-91.3457759137153
100	-191.283017729717

eigenvalues of negative mode



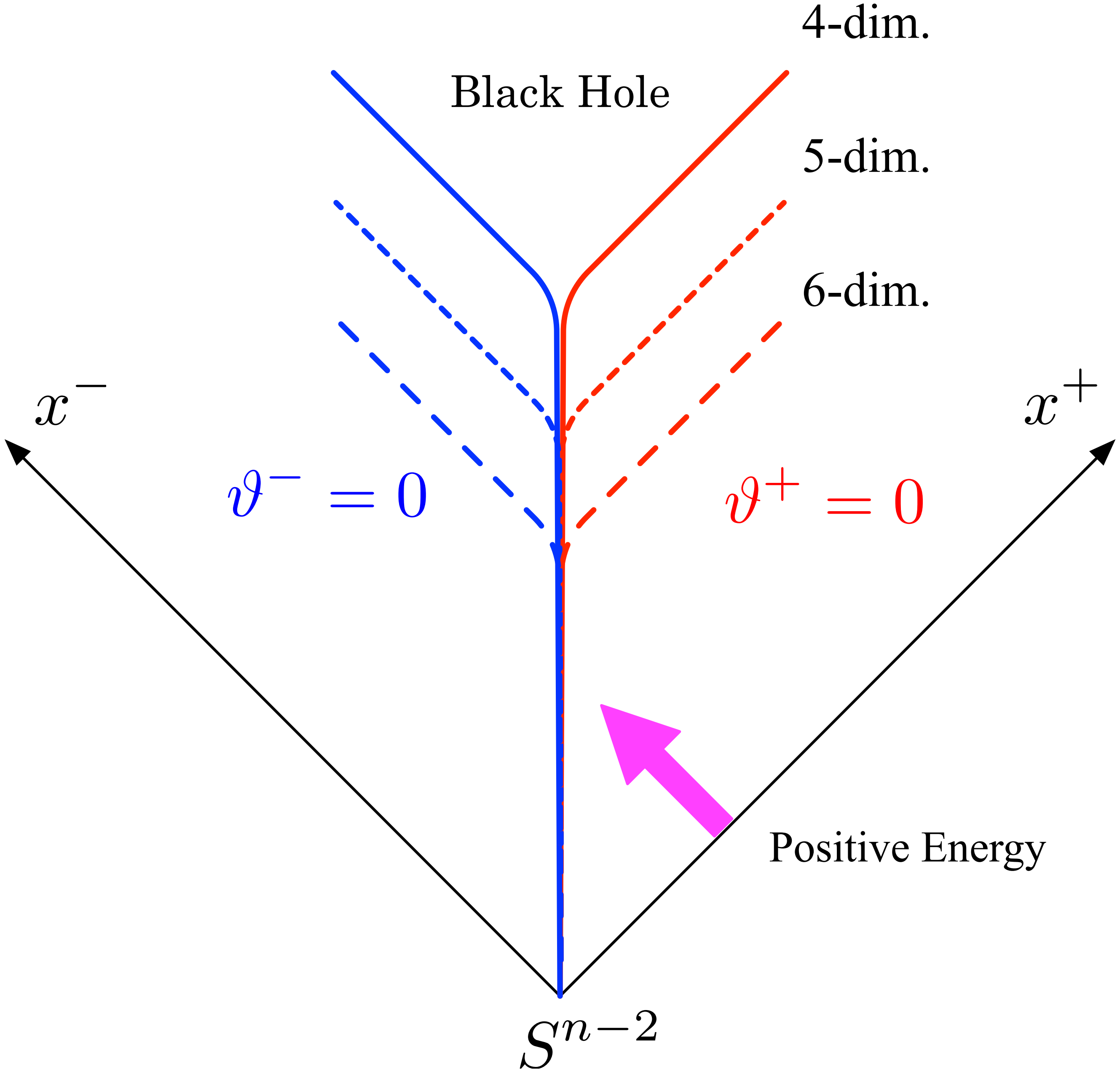
eigenfunction of negative mode

★ In all dimensions, we found negative modes.

➡ Ellis's wormhole is unstable

★ Higher dimension, instability appears in short time scale

Wormhole evolution in n-dim の線形解析結果



PHYSICAL REVIEW D **88,** 064027 (2013)

TABLE I. The negative eigenvalues ω^2.

n	ω^2
4	−1.39705243371511
5	−2.98495893027790
6	−4.68662054299460
7	−6.46258414126318
8	−8.28975936306259
9	−10.1535530451867
10	−12.0442650147438
11	−13.9552091676647
20	−31.5751101285105
50	−91.3457759137153
100	−191.283017729717

$$f(t,r) = f_0(r) + \varepsilon f_1(r) e^{i\omega t}, \quad (3.1)$$

$$\delta(t,r) = \delta_0(r) + \varepsilon \delta_1(r) e^{i\omega t}, \quad (3.2)$$

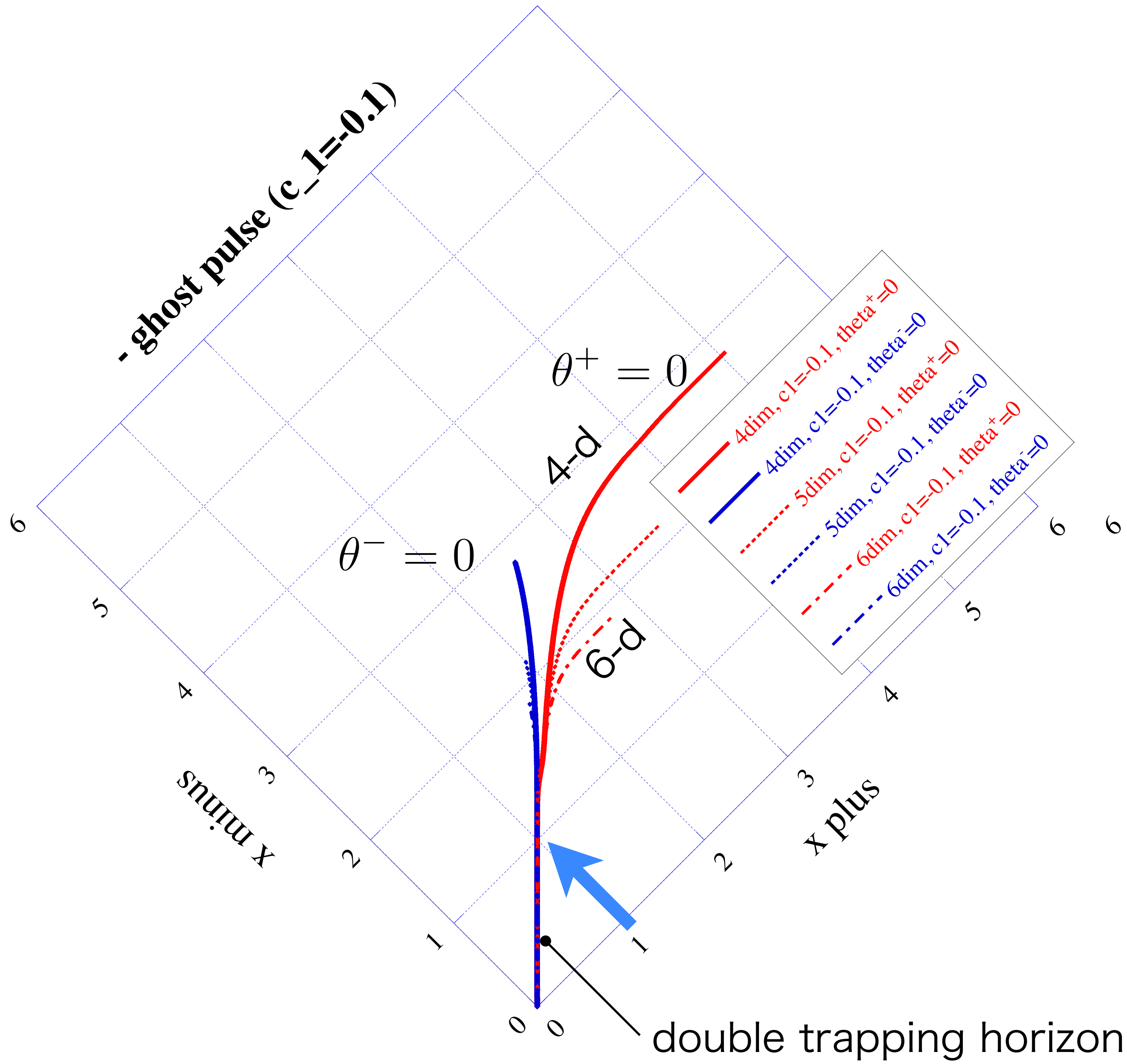
$$R(t,r) = R_0(r) + \varepsilon R_1(r) e^{i\omega t}, \quad (3.3)$$

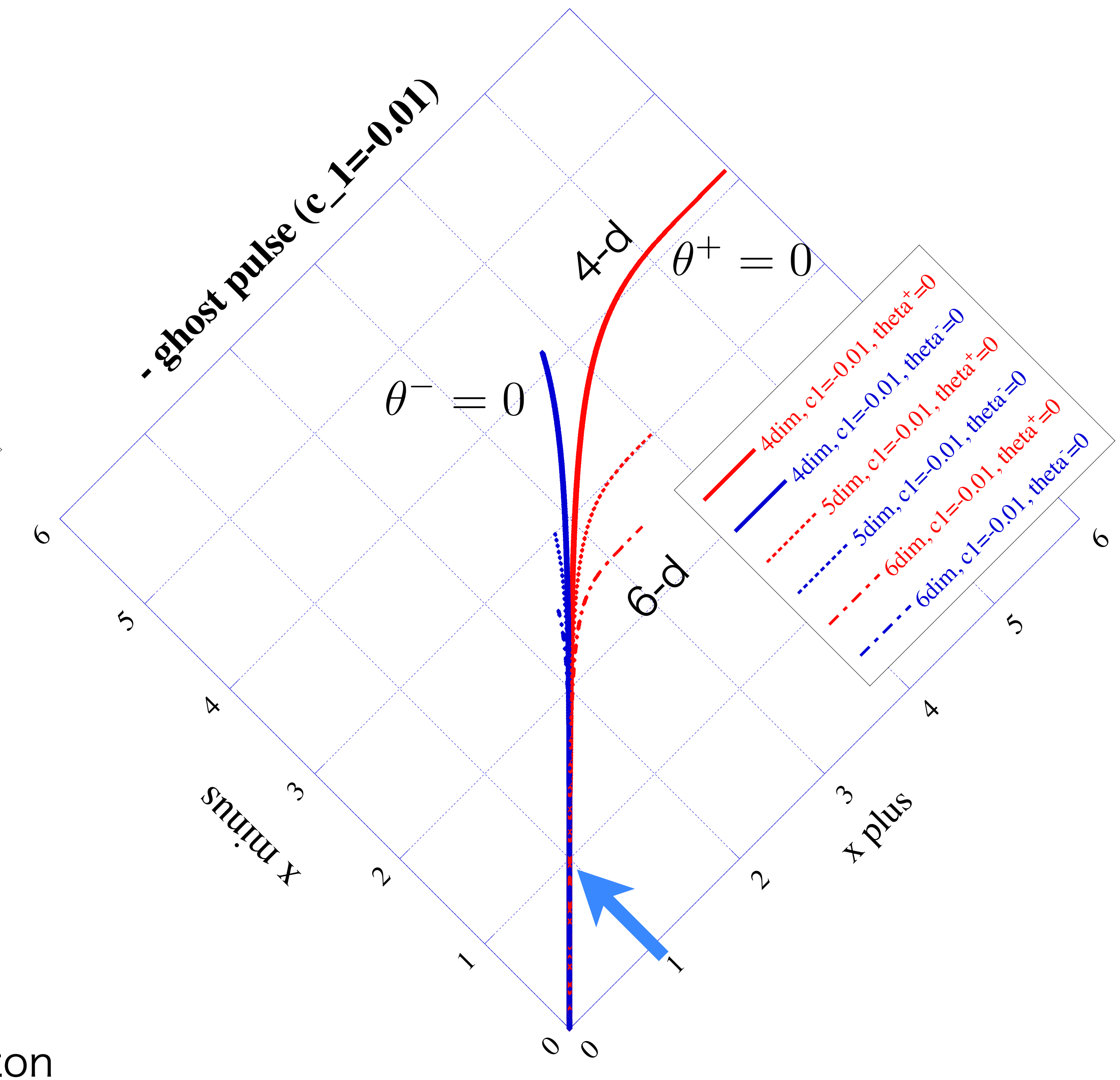
$$\phi(t,r) = \phi_0(r) + \varepsilon \phi_1(r) e^{i\omega t}. \quad (3.4)$$

次元が大きいほど，不安定モードを拾う.

（線形解析）

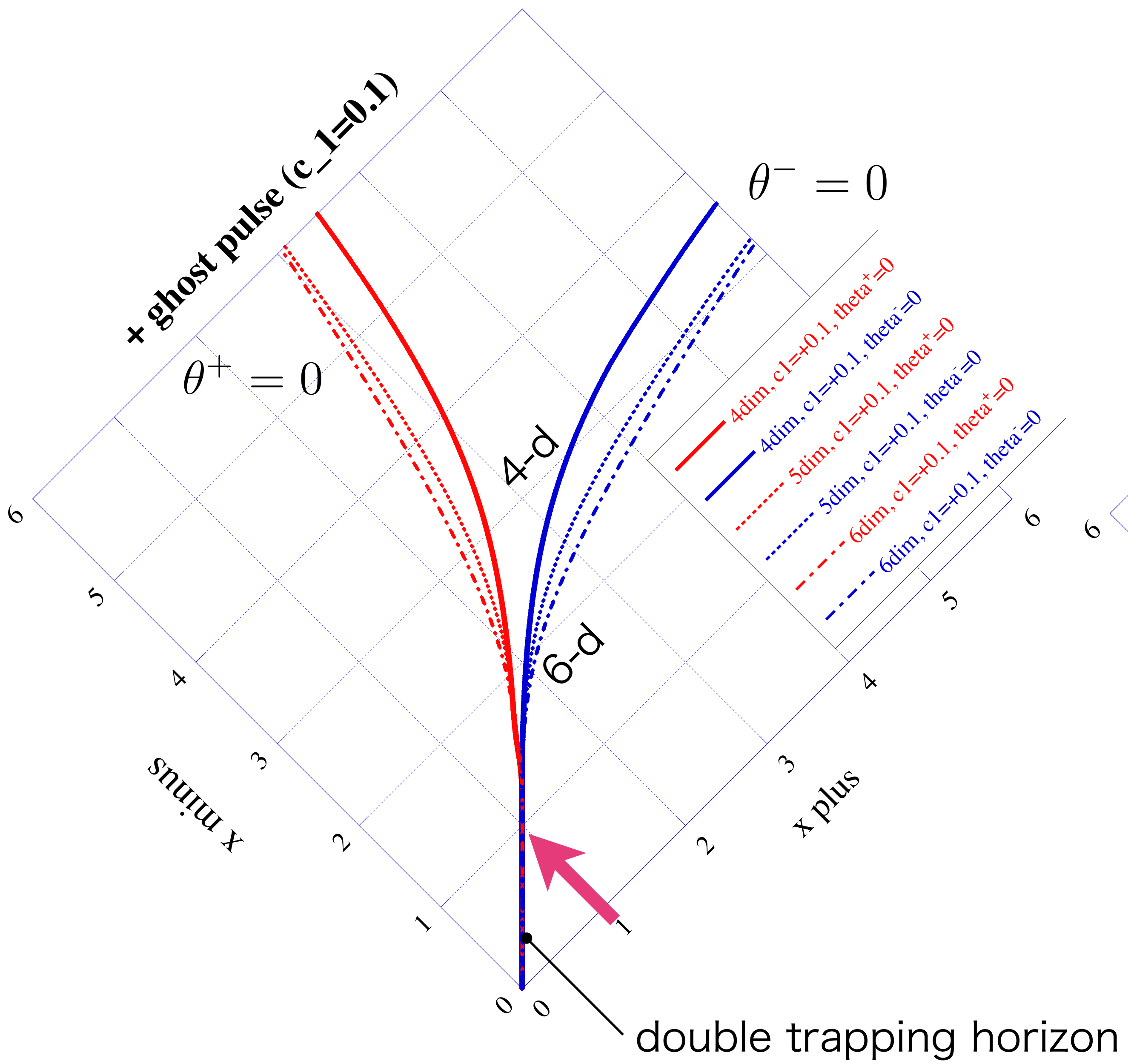
ghost pulse (negative amp.) input

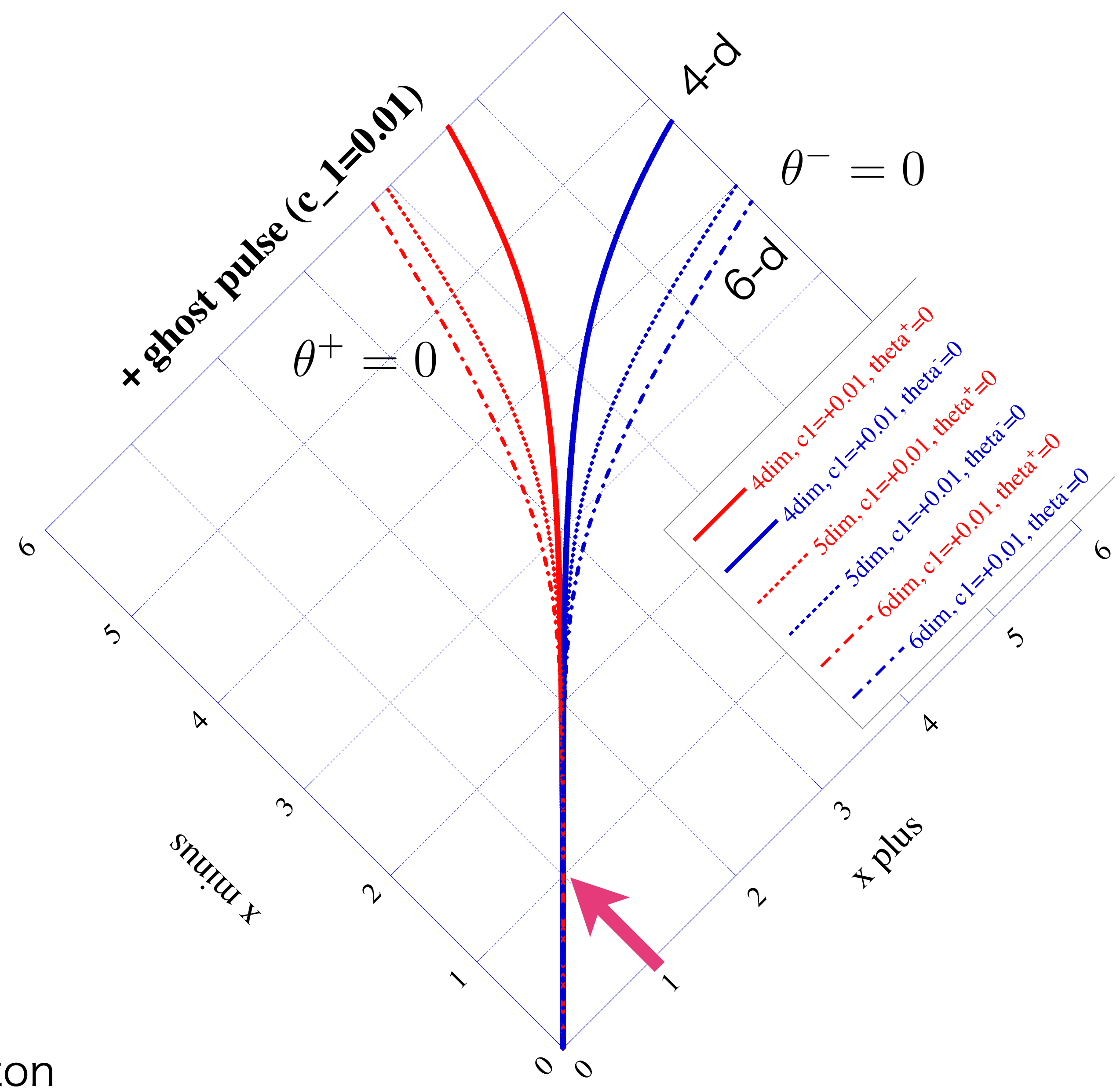




positive energy input --> BH formation

ghost pulse (positive amp.) input





negative energy input --> throat inflates

Dynamics in ~~5~~ n-dim GR gravity?

2. *Spheroidal matter collapse*
   Initial data analysis, Evolutions

Yamada & HS, CQG 27 (2010) 045012
Yamada & HS, PRD 83 (2011) 064006

3. *Wormhole dynamics in GR*
   linear stability,
   dynamical stability

Torii & HS, PRD 88 (2013) 064027
HS & Torii, in preparation

Ellis (Morris-Thorne) traversable WH解　線形摂動 & 時間発展

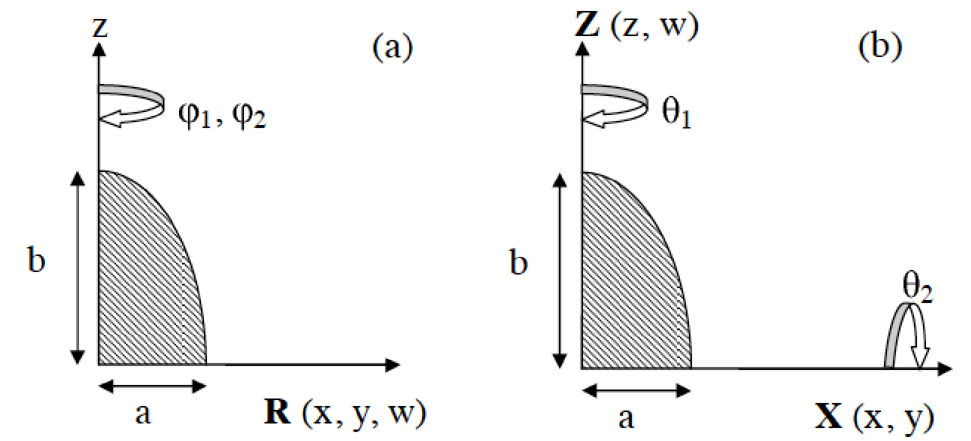
WH は 不安定である　　高次元ほど不安定

(A) 正のエネルギーパルス ---> BH
(B) 負のエネルギーパルス ---> Inflationary expansion
(C) 頑張ればメンテナンス可能

Dynamics in 5dim GR gravity?

Yamada & HS, CQG 27 (2010) 045012
Yamada & HS, PRD 83 (2011) 064006

2. Spheroidal matter collapse
Initial data analysis, Evolutions



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HS & Torii, in preparation

Dynamics in Gauss-Bonnet gravity?

4. Wormhole dynamics in GB
5. Plane-wave collision in GB

HS & Torii, in preparation

Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\right]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **has minimum mass for static spherical BH solution**

  T Torii & H Maeda, PRD 71 (2005) 124002
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated in full gravity.)
- **new topic in numerical relativity.**

  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496
- **much attentions in WH community**

  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

Formulation for evolution [dual null]

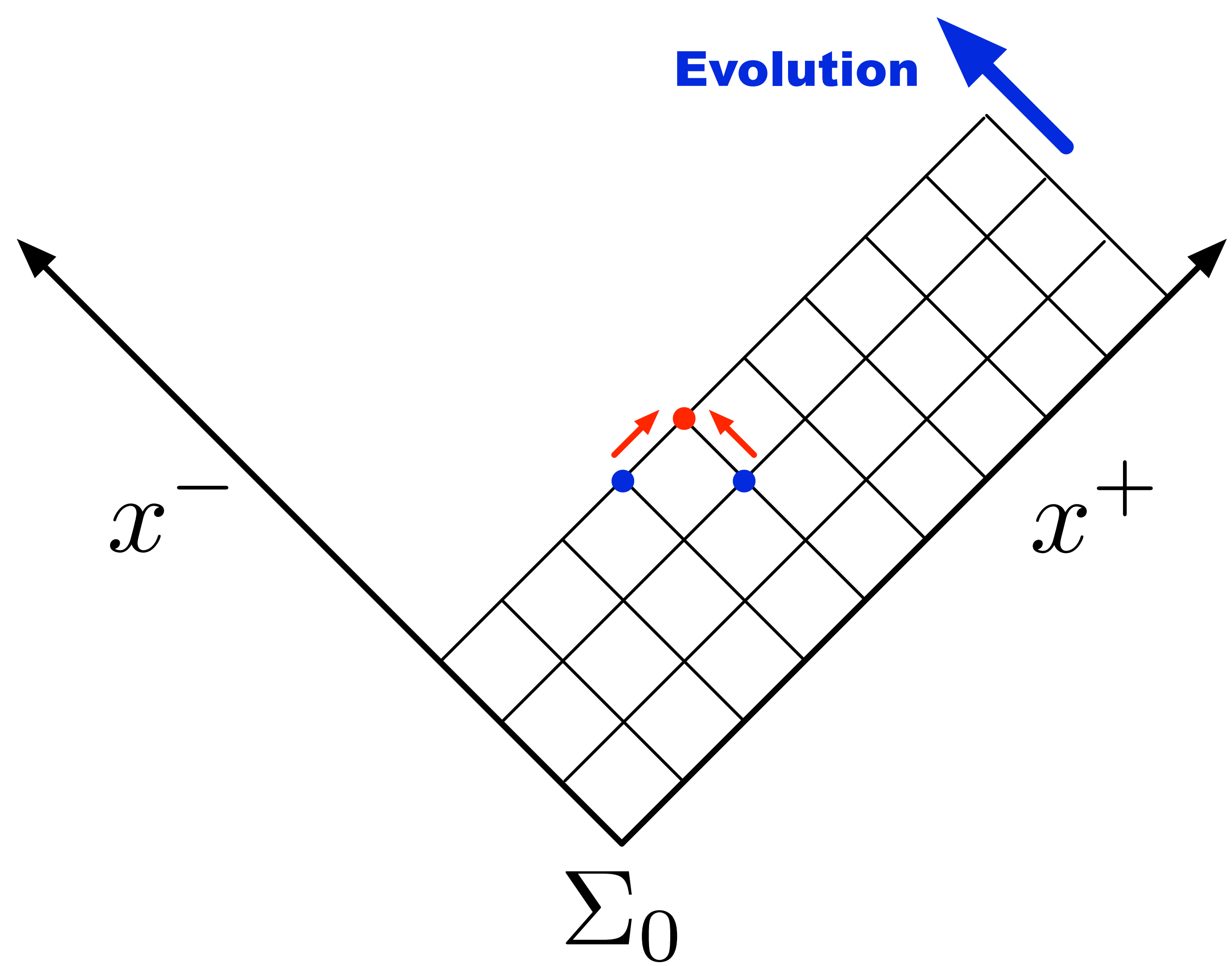
Metric n-dimensional, dual-null coordinate, $2+(n-2)$ decomposition

$$ds^2 = -2e^{-f(x^+,x^-)}dx^+dx^- + r^2(x^+,x^-)\gamma_{ij}dx^i dx^j \quad (1)$$

Variables

$\Omega = \frac{1}{r}$	Conformal factor
$\vartheta_\pm = (n-2)\partial_\pm r$	expansion
f	lapse function
$\nu_\pm = \partial_\pm f$	inaffinity (shift)

ψ	scalar field (normal)
$\pi_\pm = r\partial_\pm\psi$	scalar momentum
ϕ	scalar field (ghost)
$p_\pm = r\partial_\pm\phi$	scalar momentum



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$p_\pm = r\partial_\pm\phi$	scalar momentum

Parameters

n	dimension
k	curvature
Λ	cosmological constant

For simplicity, we define

$$\tilde{\alpha} = (n-3)(n-4)\alpha_2, \quad (2)$$

$$A = \alpha_1 + 2\tilde{\alpha}\Omega^2 Z, \quad (3)$$

$$W = \frac{2e^f}{(n-2)^2}\vartheta_+\vartheta_-, \quad (4)$$

$$Z = k + W, \quad (5)$$

$$\eta = \Omega^2\frac{(n-2)(n-3)}{2}e^{-f}Z, \quad (6)$$

matter variables

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) \\ &= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \end{aligned}$$

this derives Klein-Gordon equations

$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}.$$

Scalar field variables

$$\begin{aligned} \pi_\pm &\equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi \\ p_\pm &\equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi \end{aligned}$$

Klein-Gordon eqs.

$$\begin{aligned} \Box\phi &= -\frac{e^f}{r}\left(2r\phi_{uv} + (n-2)r_u\phi_v + (n-2)r_v\phi_u\right) \\ &= -2e^f\phi_{uv} - e^f\Omega^2\left(\vartheta_- p_+ + \vartheta_+ p_-\right) \end{aligned}$$

Energy-momentum tensor

$$\begin{aligned} T_{++} &= \Omega^2(\pi_+^2 - p_+^2) \\ T_{--} &= \Omega^2(\pi_-^2 - p_-^2) \\ T_{+-} &= -e^{-f}\left(V_1(\psi) + V_2(\phi)\right) \\ T_{zz} &= e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}\left(V_1(\psi) - V_2(\phi)\right) \end{aligned}$$

evolution equations (1)

Equations for x^+ direction

$$\partial_+\Omega = -\frac{1}{n-2}\vartheta_+\Omega^2 \quad (7)$$

$$\partial_+\vartheta_+ = -\vartheta_+\nu_+ - \frac{1}{\Omega A}\kappa^2 T_{++} = -\vartheta_+\nu_+ - \frac{1}{A}\kappa^2\Omega(\pi_+^2 - p_+^2) \quad (8)$$

$$\partial_+\vartheta_- = \frac{1}{A}\frac{e^{-f}}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}Z + \Lambda + \kappa^2(V_1+V_2)\right] - \frac{\tilde{\alpha}}{A}\Omega^3 e^{-f}\frac{(n-2)(n-5)}{2}\left[Z^2 + W\right] \quad (9)$$

$$\partial_+ f = \nu_+ \quad (10)$$

$\partial_+\nu_+ =$ no evolution eq. exists

$$\begin{aligned}\partial_+\nu_- =\ & \frac{\alpha_1}{A}Ze^{-f}\Omega^2\frac{(n-3)}{2}\left\{-\frac{\alpha_1}{A}2(n-3)+n-4\right\} \\ & +\frac{1}{A}\Omega^2 e^{-f}\kappa^2(\pi_+\pi_- - p_+p_-) + \frac{1}{A}e^{-f}\left\{\frac{\alpha_1}{A}\frac{2(n-3)}{(n-2)} - 1\right\}\{\Lambda + \kappa^2(V_1+V_2)\} \\ & -\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{\alpha_1}{A}\Omega^2(n-3)\left\{k^2+2WZ+2Z^2\right\} + \frac{\tilde{\alpha}}{A}\Omega^4 2(n-5)\left\{k^2+2WZ\right\}Z\right] \\ & +\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{1}{2}\Omega^2\left\{(n-2)k^2+2WZ-4Z^2\right\} + \frac{1}{A}\frac{4}{n-2}Z\left\{\Lambda+\kappa^2(V_1+V_2)\right\}\right] \\ & -\frac{\tilde{\alpha}}{A}e^f\Omega^2\frac{4}{(n-2)^2}\left\{\nu_+\vartheta_+(\partial_-\vartheta_-) + \nu_-\vartheta_-(\partial_+\vartheta_+) + (\partial_+\vartheta_+)(\partial_-\vartheta_-) + \nu_+\nu_-\vartheta_+\vartheta_- - (\partial_-\vartheta_+)^2\right\} \quad (11)\end{aligned}$$

$$\partial_+\psi = \Omega\pi_+ \quad (12)$$

$$\partial_+\phi = \Omega p_+ \quad (13)$$

$\partial_+\pi_+ =$ no evolution eq. exists

$$\partial_+\pi_- = \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \quad (14)$$

$\partial_+ p_+ =$ no evolution eq. exists

$$\partial_+ p_- = \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_+ p_- - \frac{1}{2}\Omega\vartheta_- p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \quad (15)$$

evolution equations (2)

Equations for x^- direction

$$
\begin{aligned}
\partial_- \Omega &= -\frac{1}{n-2}\vartheta_- \Omega^2 && (16)\\
\partial_- \vartheta_+ &= (9) && (17)\\
\partial_- \vartheta_- &= -\vartheta_- \nu_- - \frac{1}{\Omega A}\kappa^2 T_{--} = -\vartheta_- \nu_- - \frac{1}{A}\Omega \kappa^2 (\pi_-^2 - p_-^2) && (18)\\
\partial_- f &= \nu_- && (19)\\
\partial_- \nu_+ &= (11) && (20)\\
\partial_- \nu_- &= \text{no evolution eq. exists} &&\\
\partial_- \psi &= \Omega \pi_- && (21)\\
\partial_- \phi &= \Omega p_- && (22)\\
\partial_- \pi_+ &= -\frac{1}{2}\Omega \vartheta_+ \pi_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega \vartheta_- \pi_+ - \frac{1}{2e^f \Omega}\frac{dV_1}{d\psi} && (23)\\
\partial_- \pi_- &= \text{no evolution eq. exists} &&\\
\partial_- p_+ &= -\frac{1}{2}\Omega \vartheta_+ p_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega \vartheta_- p_+ - \frac{1}{2e^f \Omega}\frac{dV_2}{d\phi} && (24)\\
\partial_- p_- &= \text{no evolution eq. exists} &&
\end{aligned}
$$

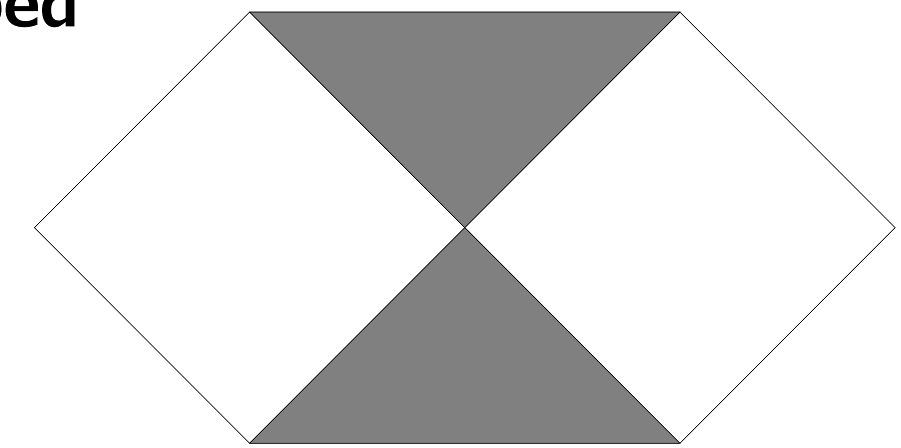
This constitutes the first-order dual-null form, suitable for numerical coding.

BH & WH are interconvertible?

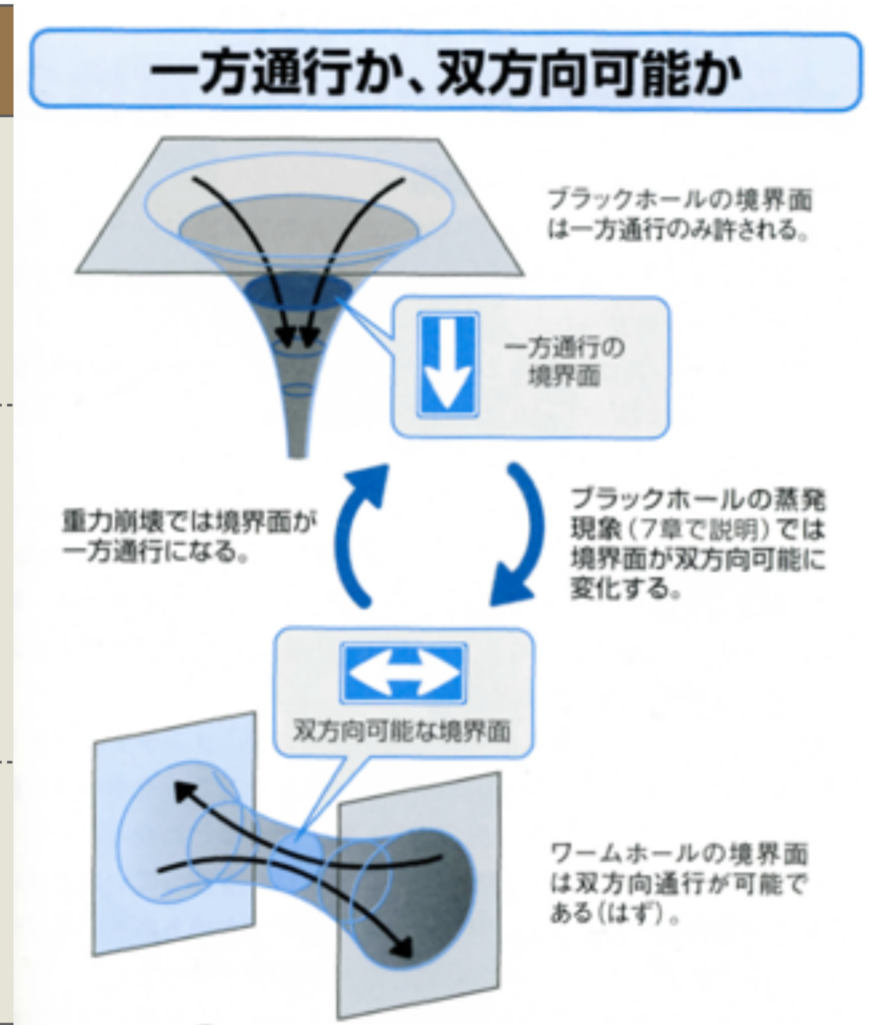
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)

Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.

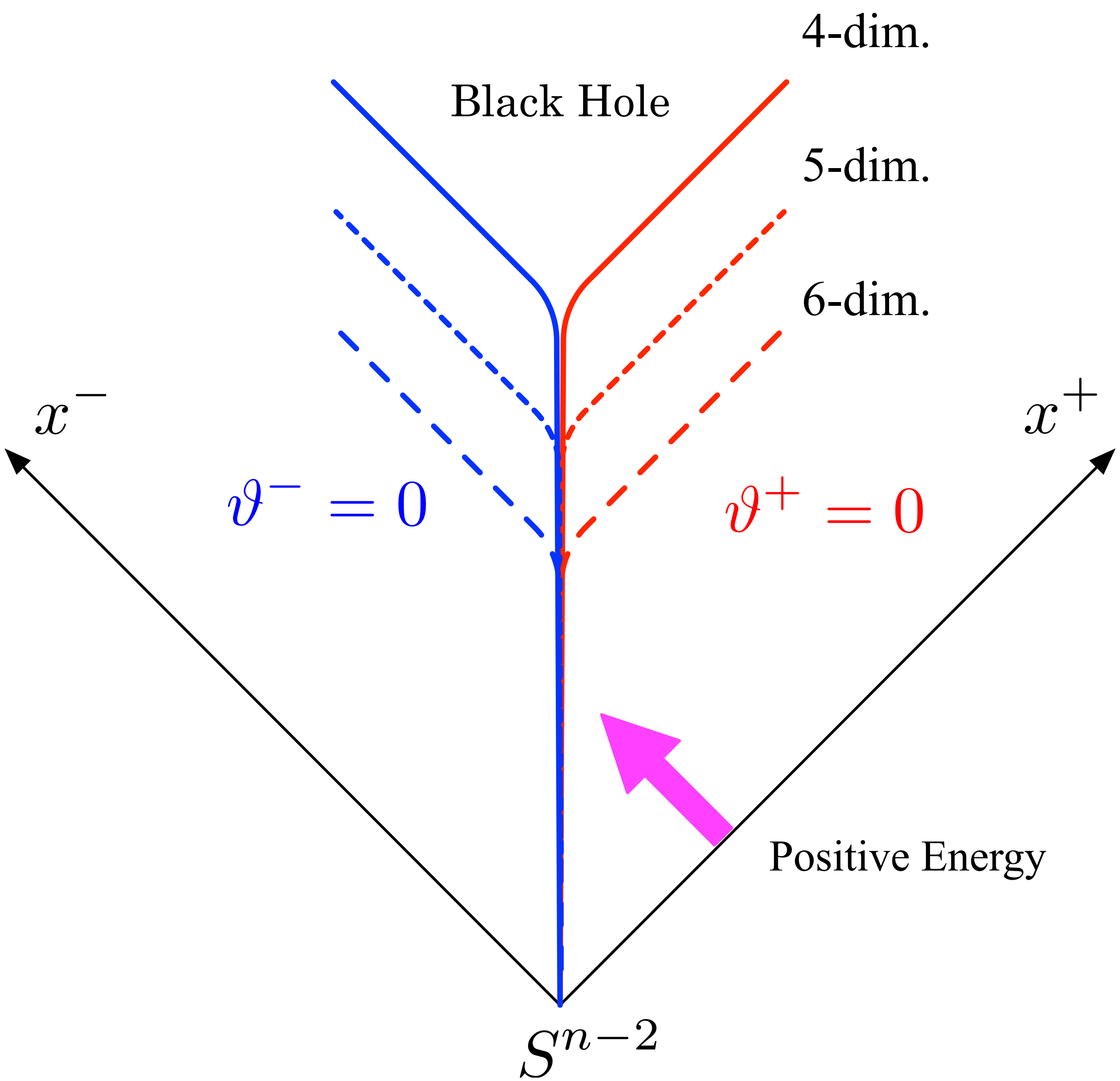


	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH ⇨ 1-way traversable	Temporal (timelike) outer THs ⇨ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density “exotic” matter
Appear-ance	occur naturally	Unlikely to occur naturally. but constructible??



n-dim GR $\alpha_{\rm GB} = 0$ Torii-HS PRD 88 (2013) 064027

Wormhole evolution in n-dim のおさらい



PHYSICAL REVIEW D **88**, 064027 (2013)

TABLE I. The negative eigenvalues ω^2.

n	ω^2
4	−1.39705243371511
5	−2.98495893027790
6	−4.68662054299460
7	−6.46258414126318
8	−8.28975936306259
9	−10.1535530451867
10	−12.0442650147438
11	−13.9552091676647
20	−31.5751101285105
50	−91.3457759137153
100	−191.283017729717

$$f(t, r) = f_0(r) + \varepsilon f_1(r) e^{i\omega t}, \tag{3.1}$$

$$\delta(t, r) = \delta_0(r) + \varepsilon \delta_1(r) e^{i\omega t}, \tag{3.2}$$

$$R(t, r) = R_0(r) + \varepsilon R_1(r) e^{i\omega t}, \tag{3.3}$$

$$\phi(t, r) = \phi_0(r) + \varepsilon \phi_1(r) e^{i\omega t}. \tag{3.4}$$

次元が大きいほど，不安定モードを拾う.

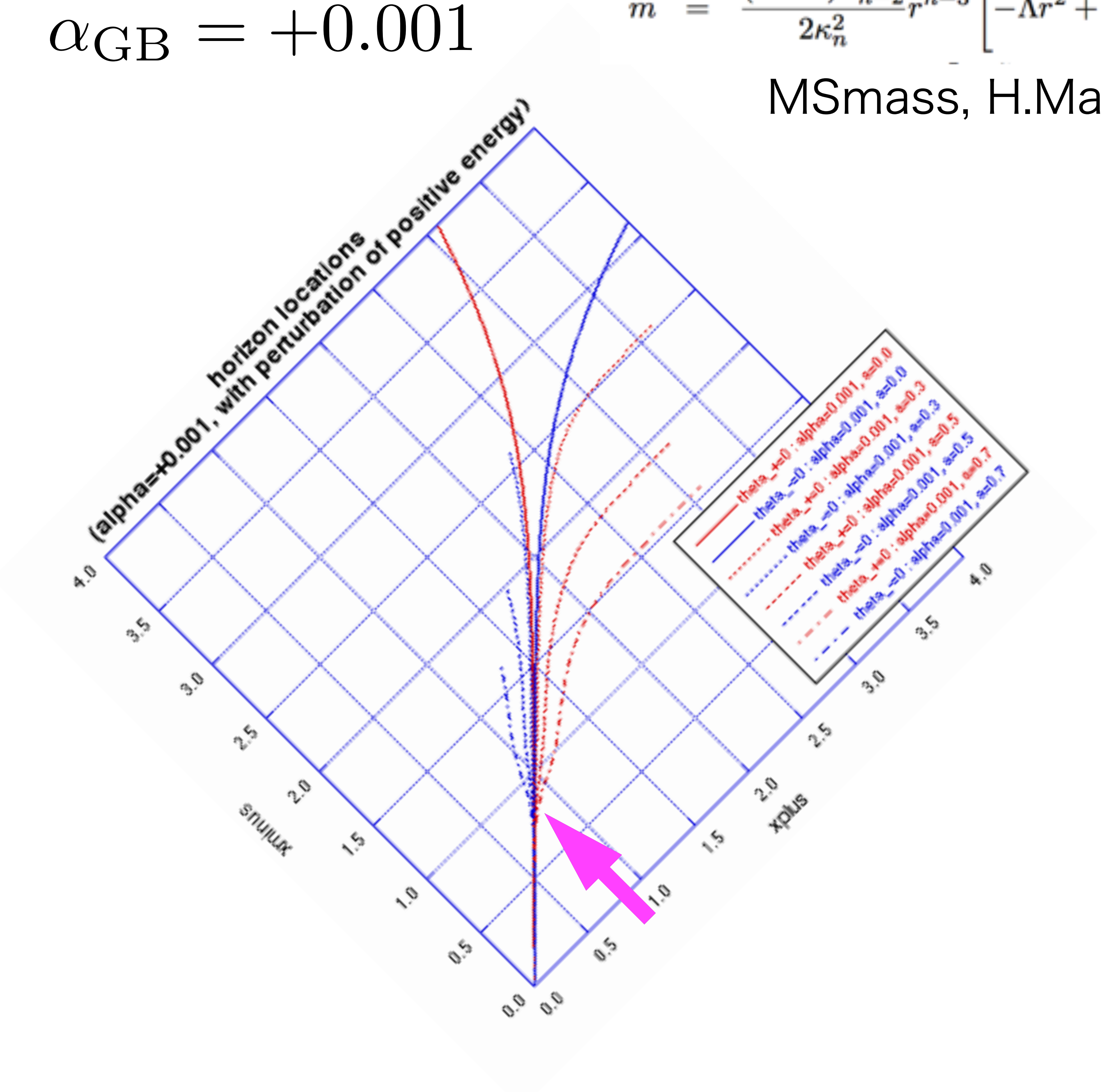
（線形解析）

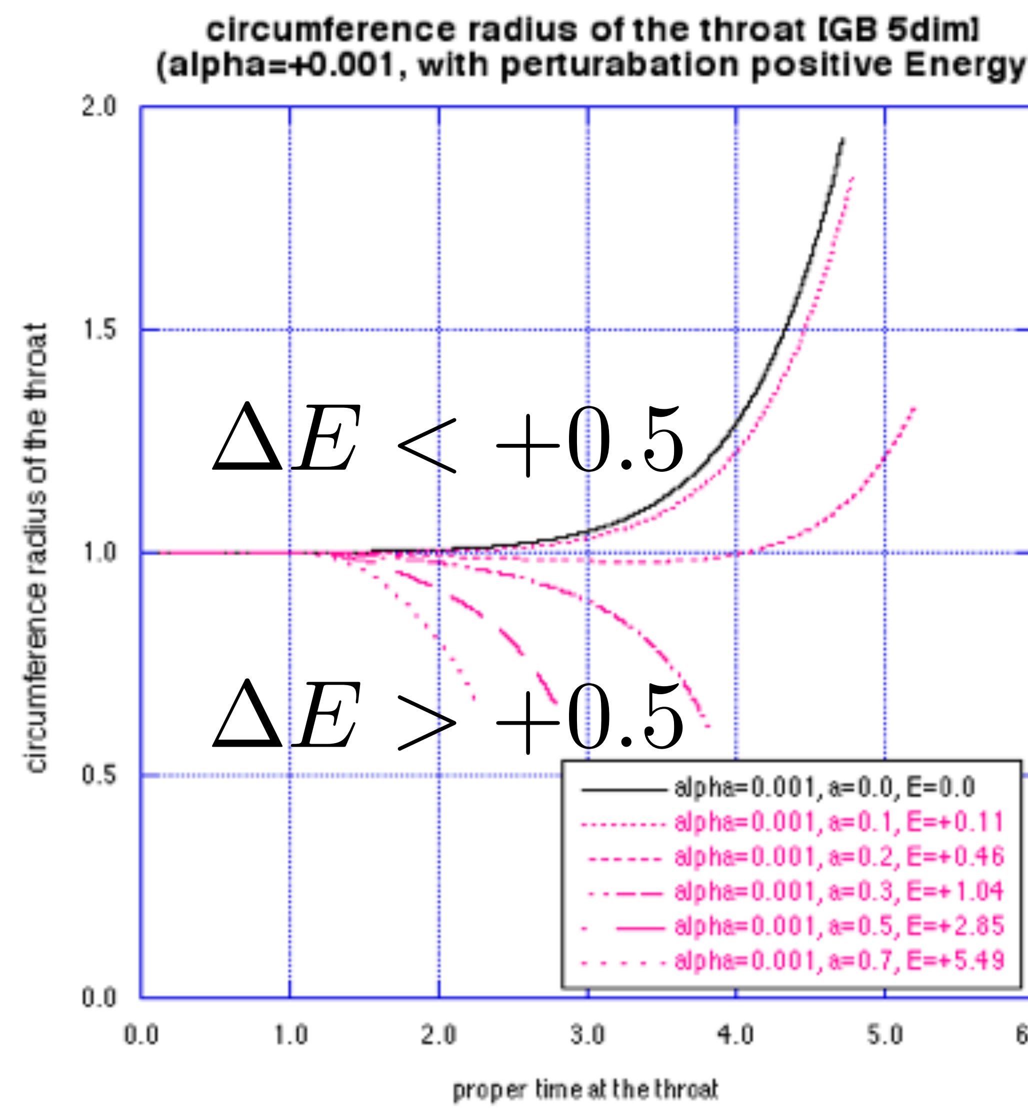
5d Gauss-Bonnet WH : positive energy injection (1)

$\alpha_{\rm GB} = +0.001$

$$m = \frac{(n-2)V^k_{n-2}}{2\kappa_n^2} r^{n-3}\left[-\tilde{\Lambda} r^2 + \left(k+\frac{2}{(n-2)^2}r^2 e^f\theta_+\theta_-\right) + \tilde{\alpha} r^{-2}\left(k+\frac{2}{(n-2)^2}r^2 e^f\theta_+\theta_-\right)^2\right]$$

MSmass, H.Maeda-Nozawa, PRD77 (2008) 063031





coupling 正　（通常のGaussBonnet）$\Rightarrow$ BHを形成しにくい

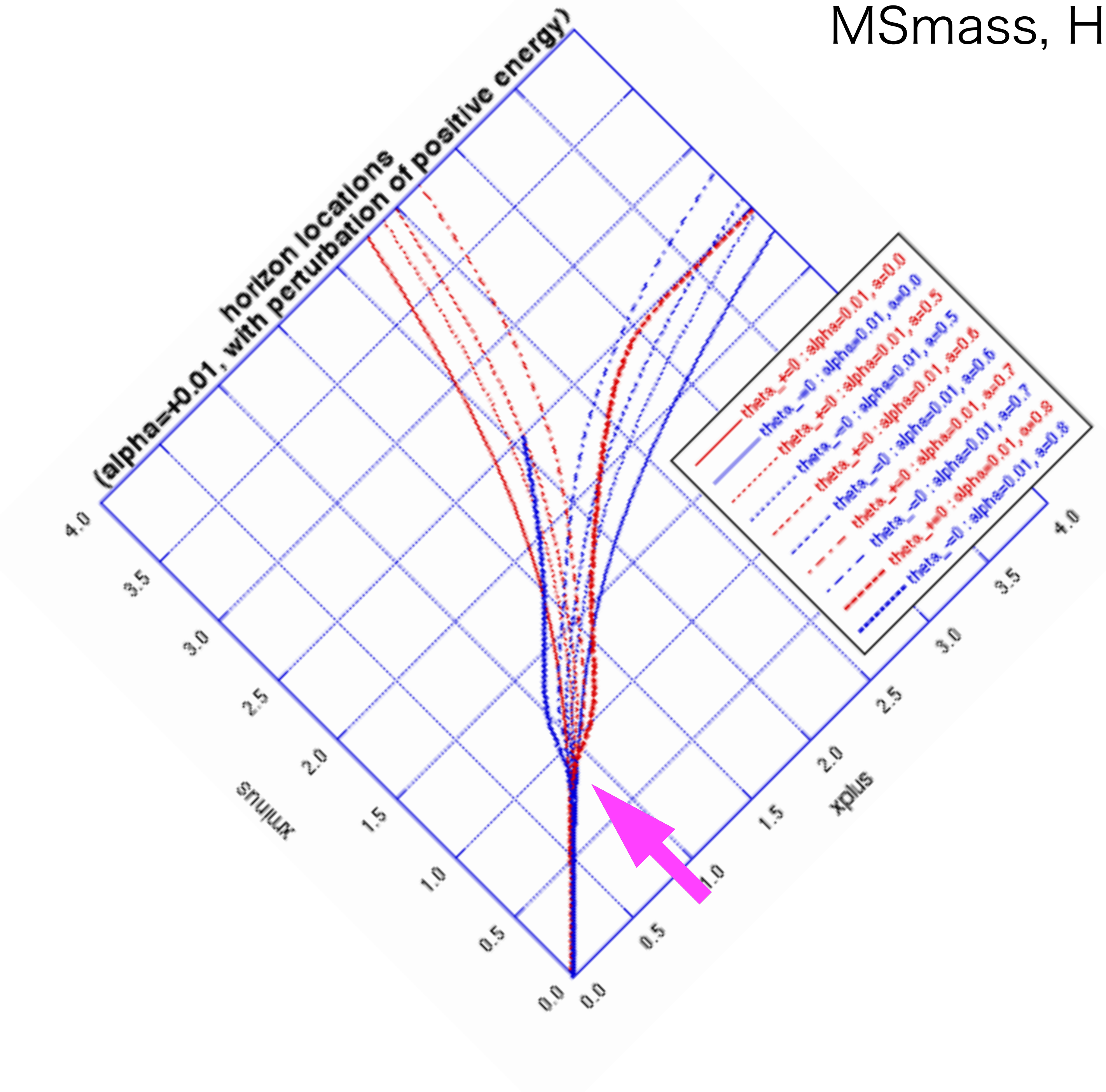
ある程度以上の正エネルギーを追加　$\Rightarrow$ BH形成 に転じる

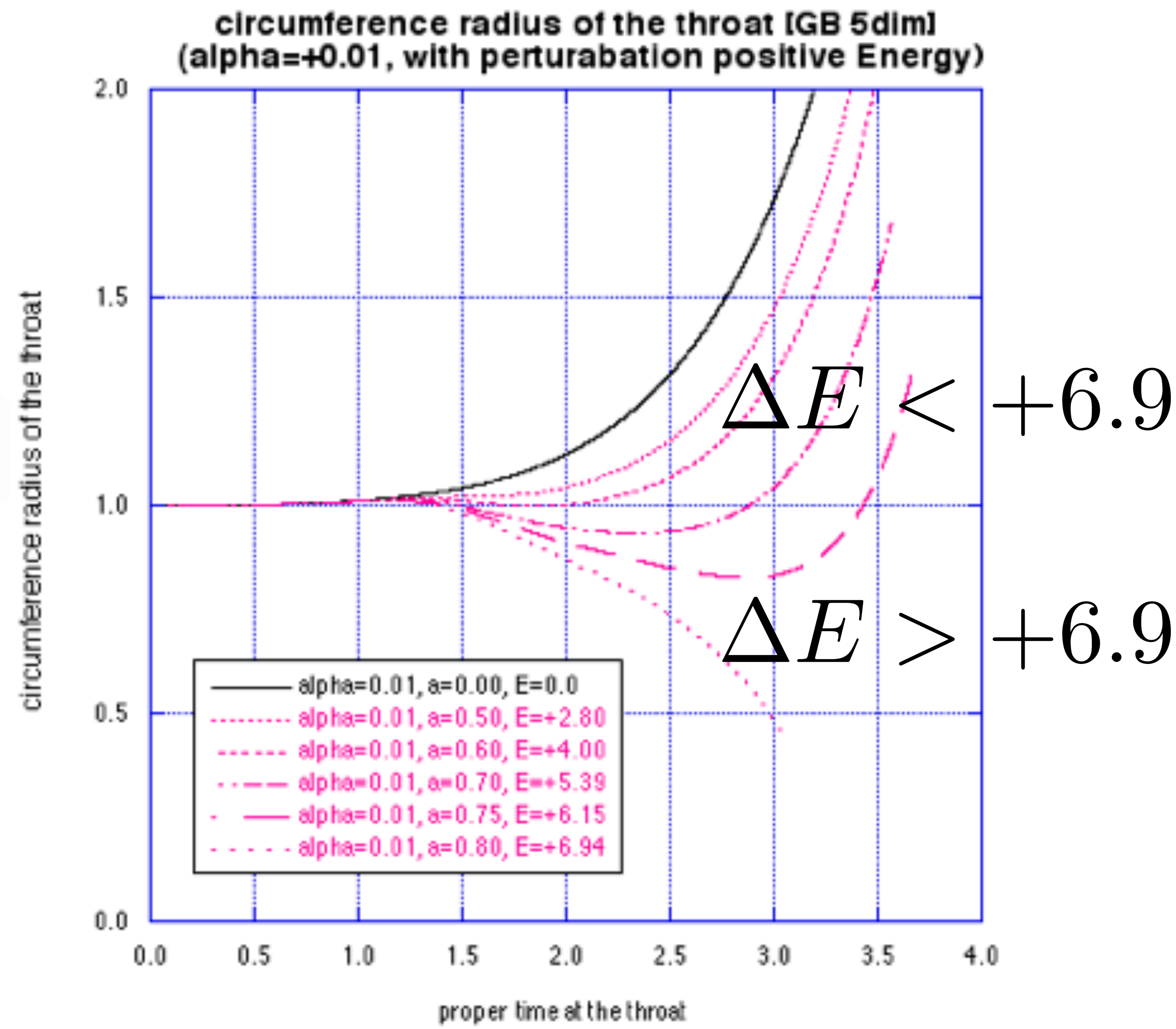
5d Gauss-Bonnet WH : positive energy injection (2)

$\alpha_{\rm GB} = +0.01$

$$m = \frac{(n-2)V^k_{n-2}}{2\kappa_n^2} r^{n-3}\left[-\tilde{\Lambda} r^2 + \left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+ \theta_-\right) + \tilde{\alpha} r^{-2}\left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+ \theta_-\right)^2\right]$$

MSmass, H.Maeda-Nozawa, PRD77 (2008) 063031





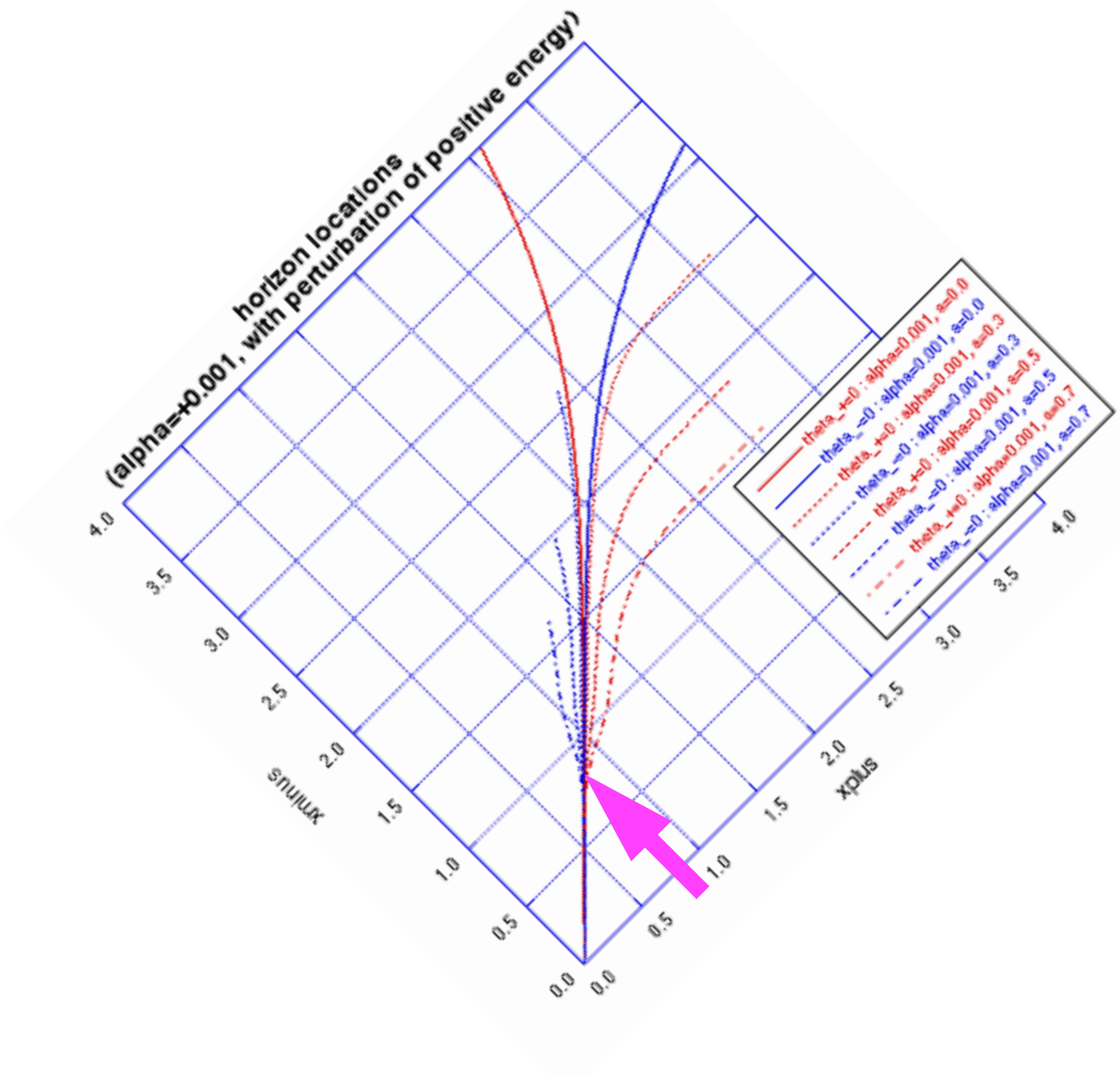
alpha 大きければ，閾値大きい

coupling 正 （通常のGaussBonnet） ⟹ BHを形成しにくい

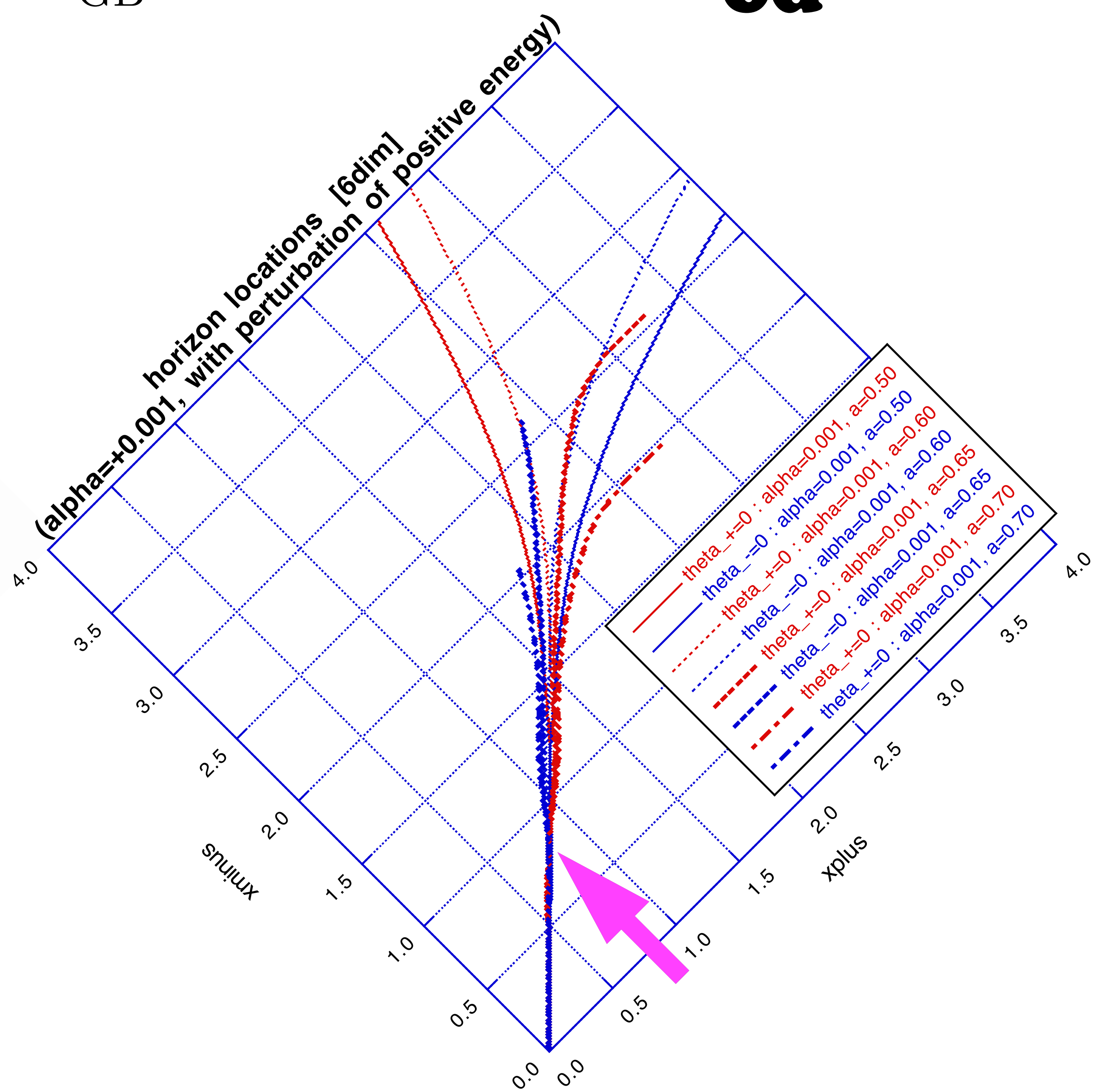
ある程度以上の正エネルギーを追加 ⟹ BH形成 に転じる

5d, 6d Gauss-Bonnet WH

$\alpha_{\rm GB} = 0.001$ **5d**



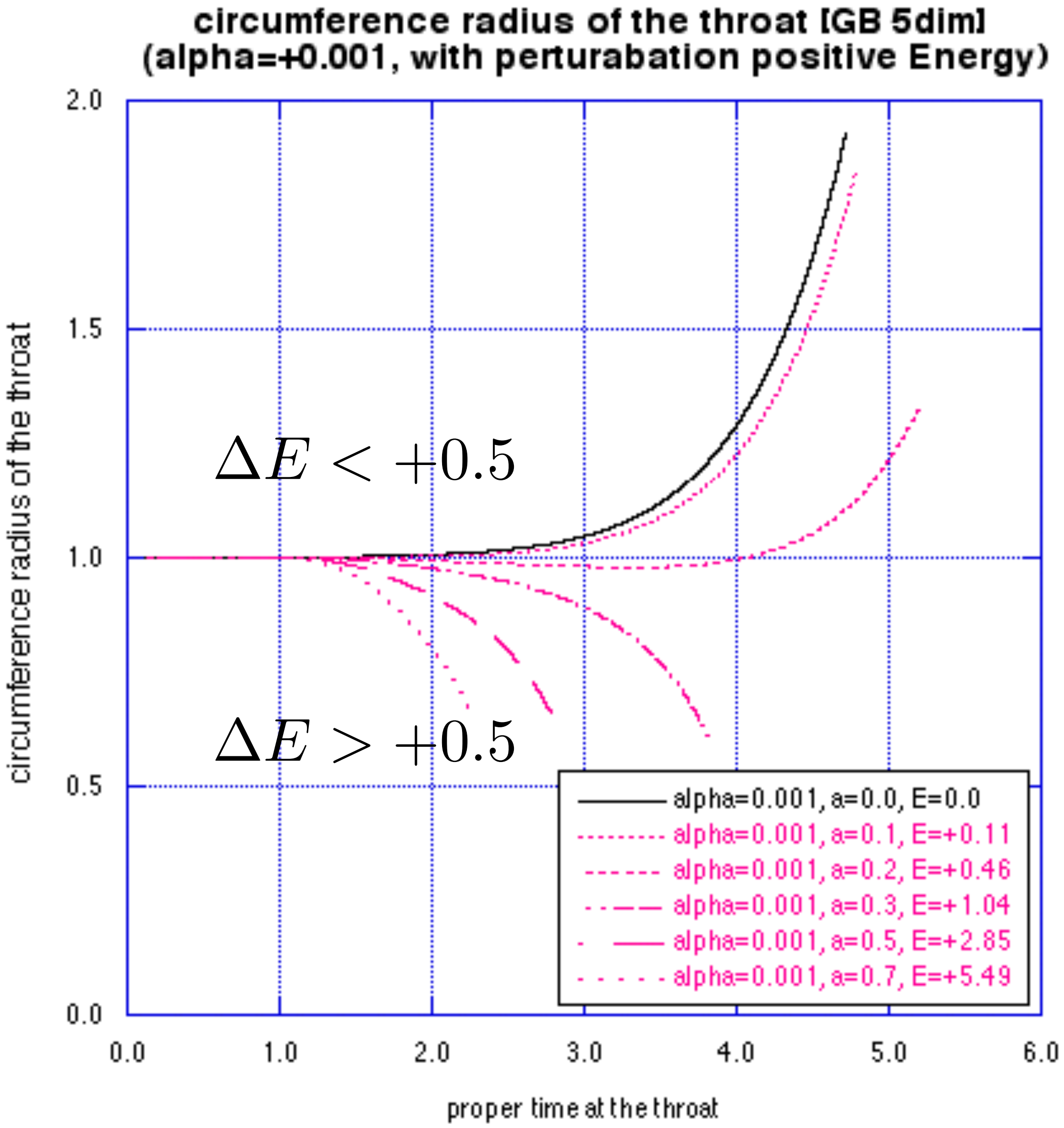
$\alpha_{\rm GB} = 0.001$ **6d**



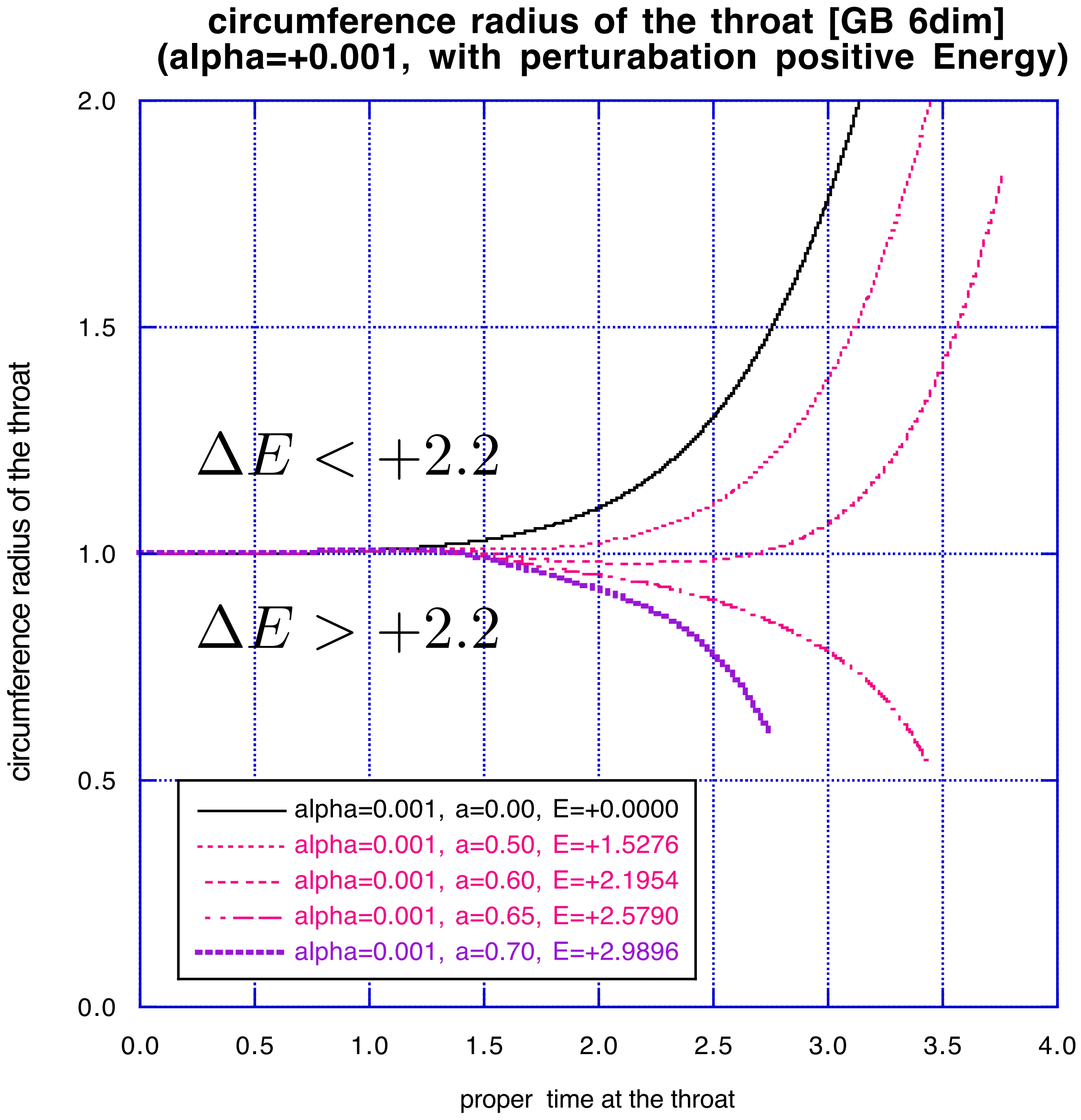
need more positive energy for transition to BH in 6dim

5d, 6d Gauss-Bonnet WH

$\alpha_{\rm GB} = 0.001$ **5d**



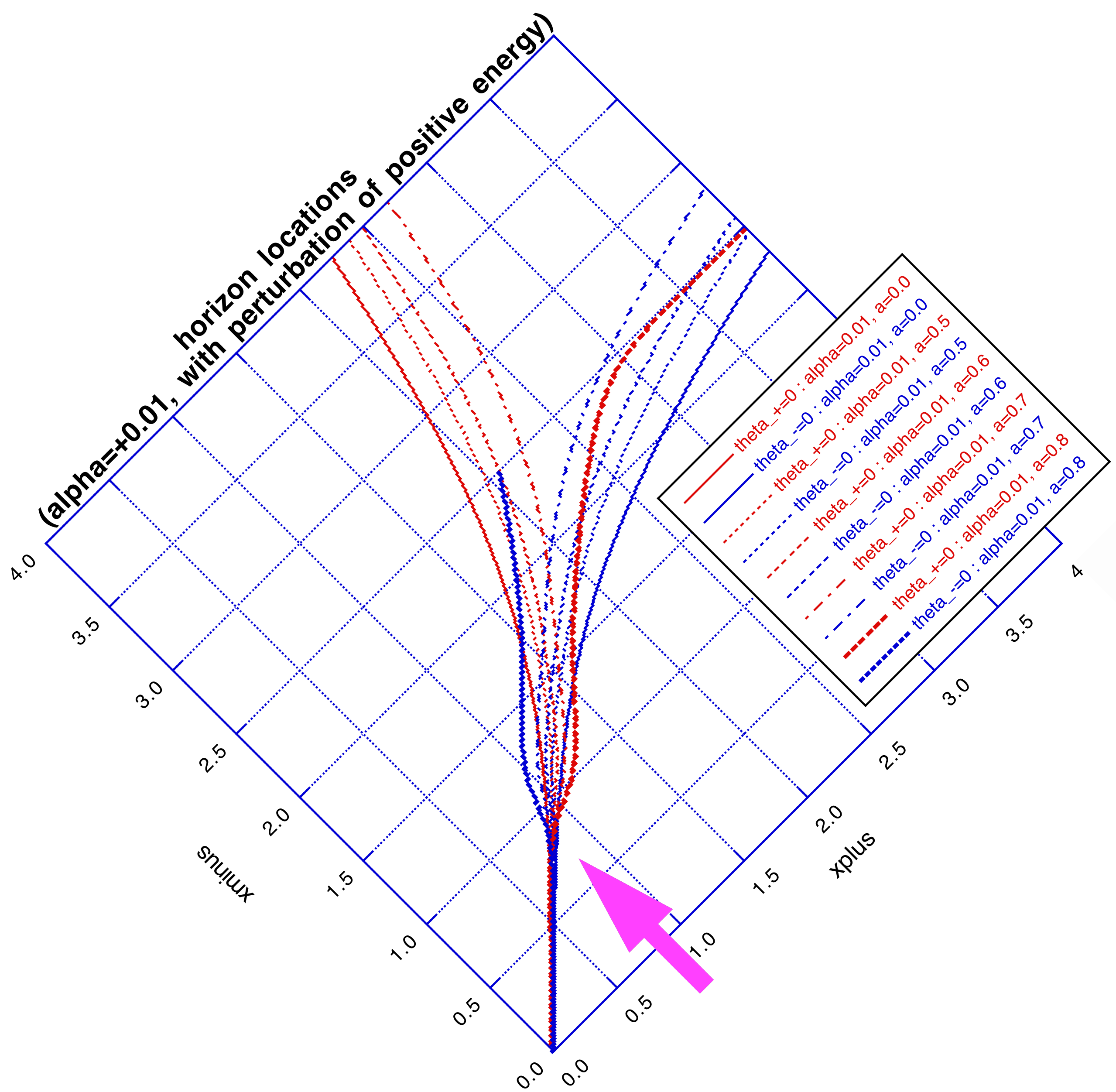
$\alpha_{\rm GB} = 0.001$ **6d**



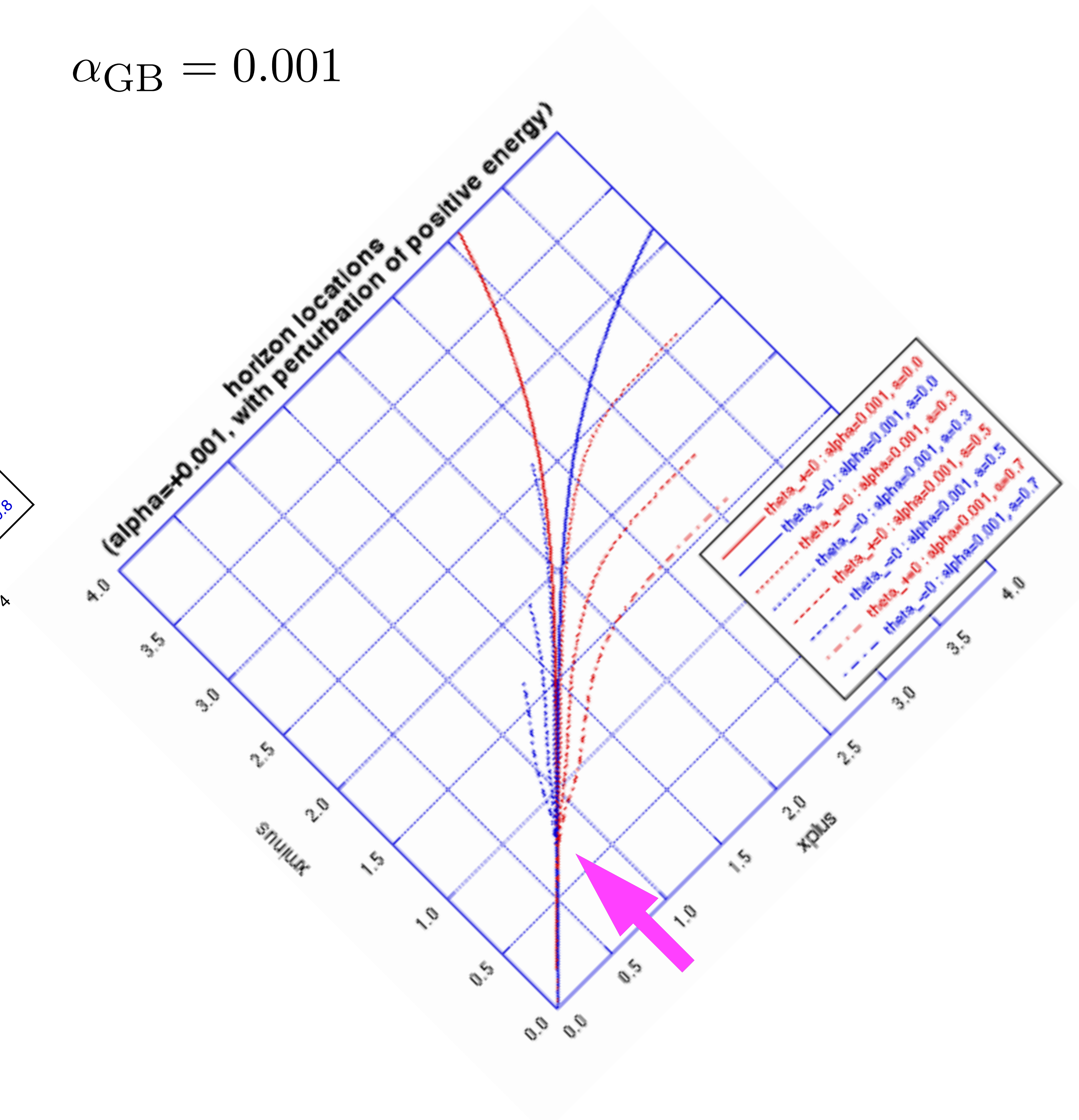
need more positive energy for transition to BH in 6dim

5d Gauss-Bonnet WH : trapped surface

$\alpha_{\rm GB} = 0.01$



$\alpha_{\rm GB} = 0.001$

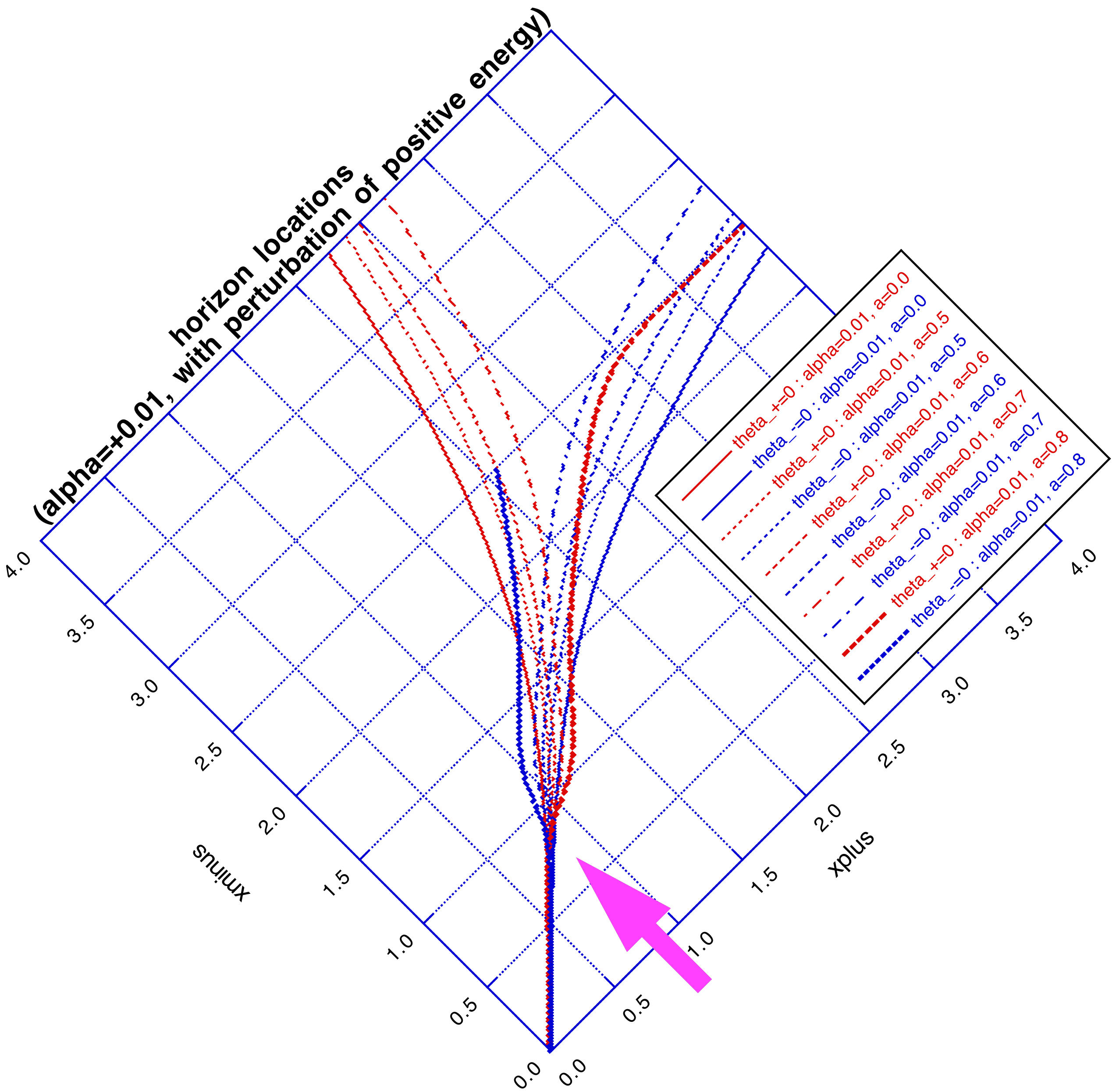


existence of trapped surface

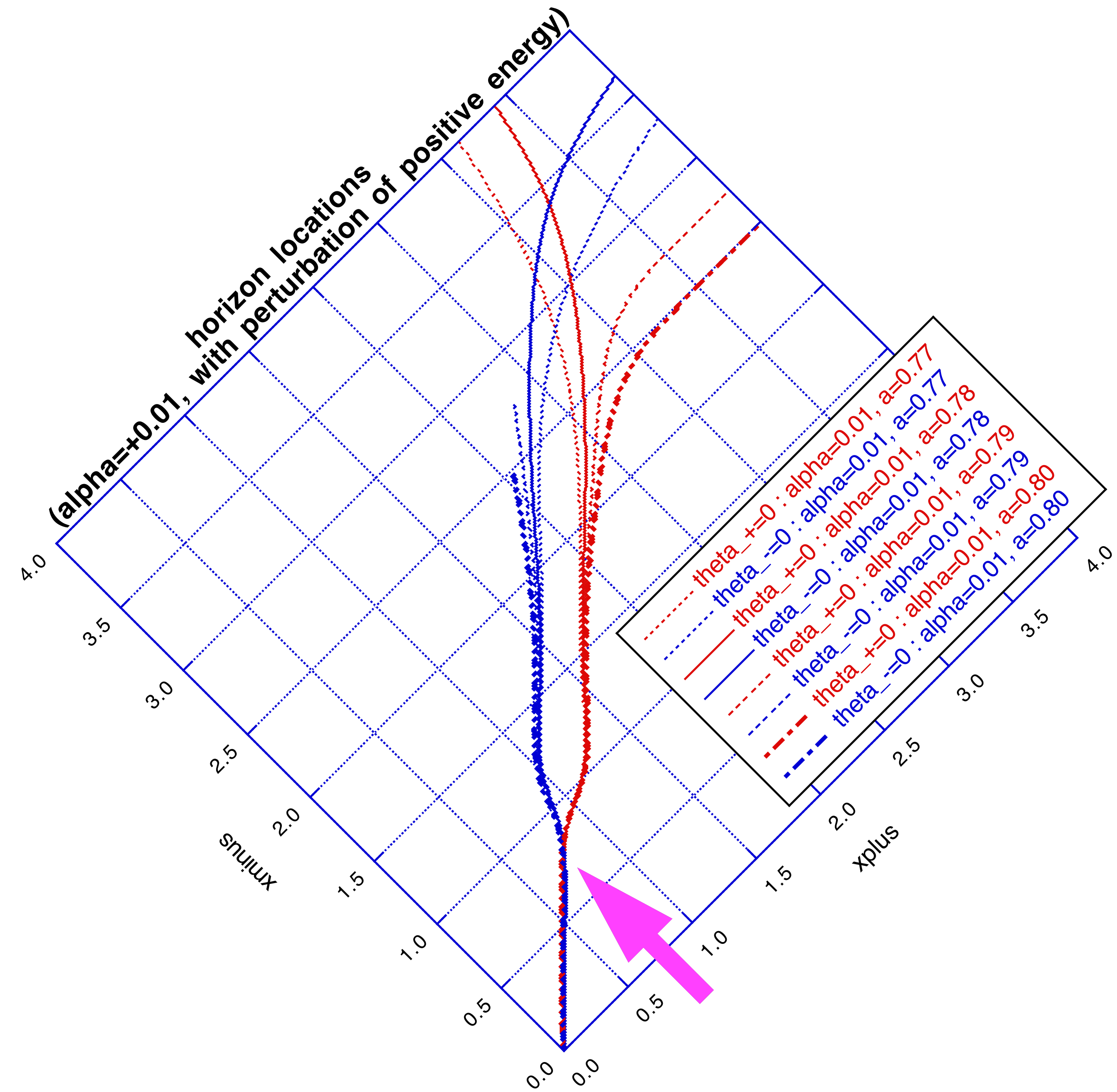
—> not necessary to form BH

5d Gauss-Bonnet WH : trapped surface

$\alpha_{\rm GB} = 0.01$



critical behavior



existence of trapped surface

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Torii & HS, PRD 88 (2013) 064027
HS & Torii, in preparation

Dynamics in Gauss-Bonnet gravity?

4. Wormhole dynamics in GB

HS & Torii, in preparation

Gauss-Bonnet重力の特色

正のcouplingでは，同じ初期条件でも特異点形成は遅くなる.
正のcouplingでは，同じ初期条件でもBHは形成しにくい.
エネルギー底上げ・特異点回避の傾向がある.
高次元になるほど，不安定性は拡大する.
trapped surfaceの存在は，必ずしもBH形成を意味しない.

Dynamics in 5dim GR gravity?

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HS & Torii, in preparation

5. Plane-wave collision in GB

GR 5d: **small amplitude waves**

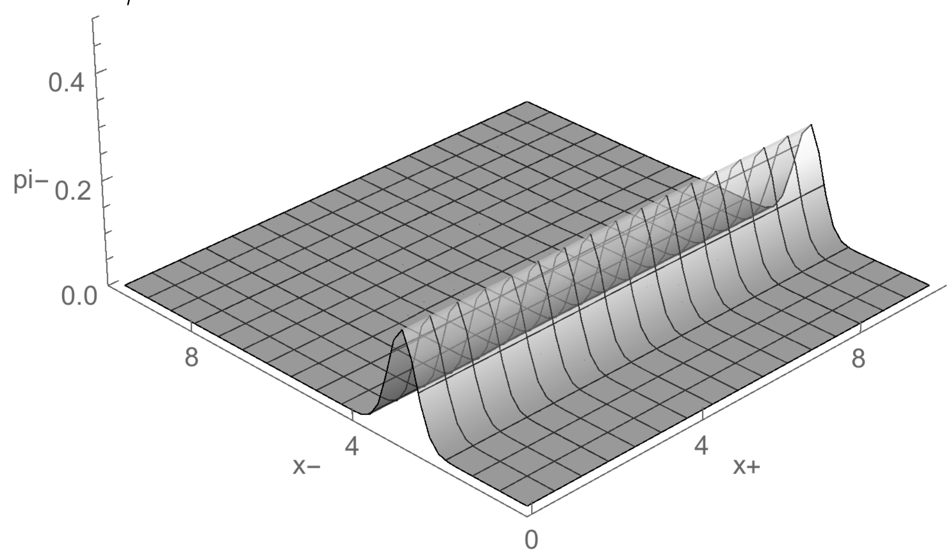
flat background, normal scalar field

Initial data:

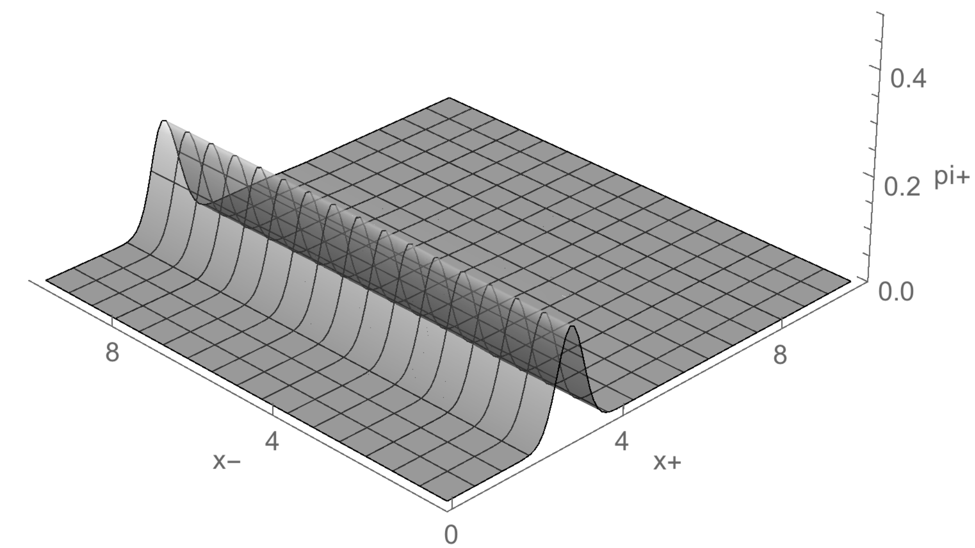
$\psi = 0,\ \pi_+ = a\exp(-b(z-c)^2)$ on $x_- = 0$ surface, where $z = x^+/\sqrt{2}$

$\psi = 0,\ \pi_- = a\exp(-b(z-c)^2)$ on $x_+ = 0$ surface, where $z = x^-/\sqrt{2}$

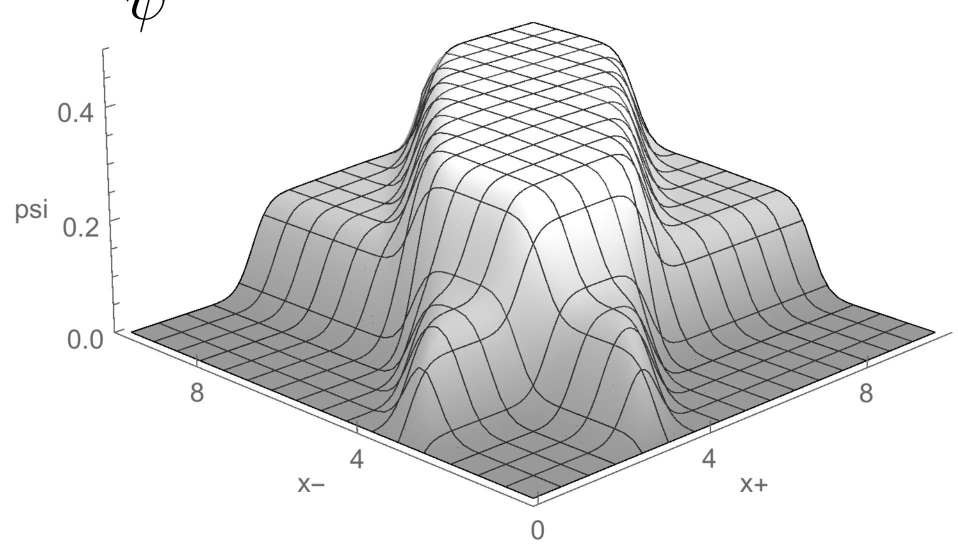
$\partial_-\psi$



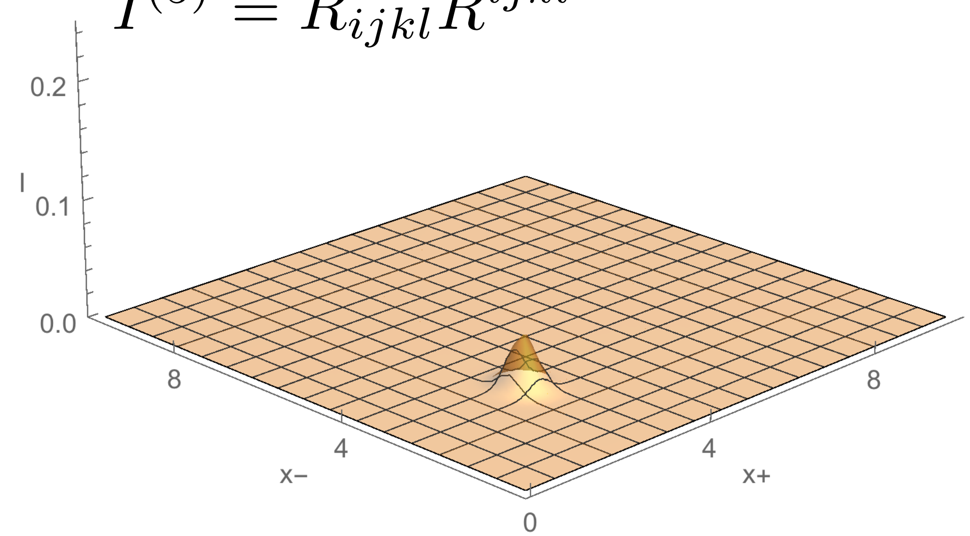
$\partial_+\psi$



ψ

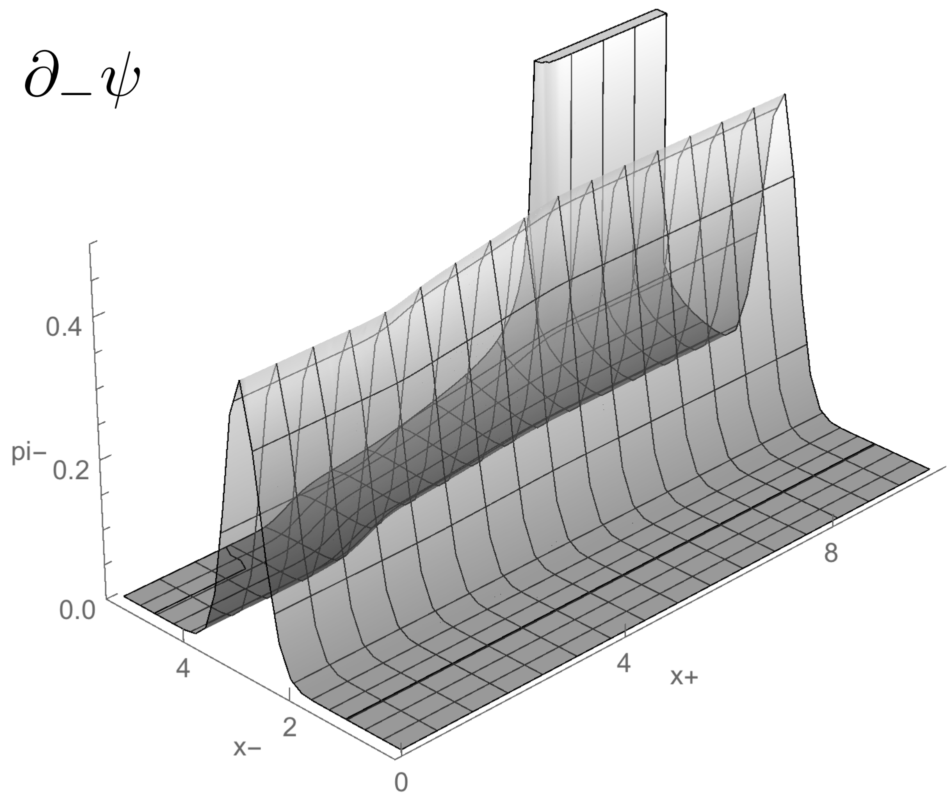


$I^{(5)} = R_{ijkl}R^{ijkl}$

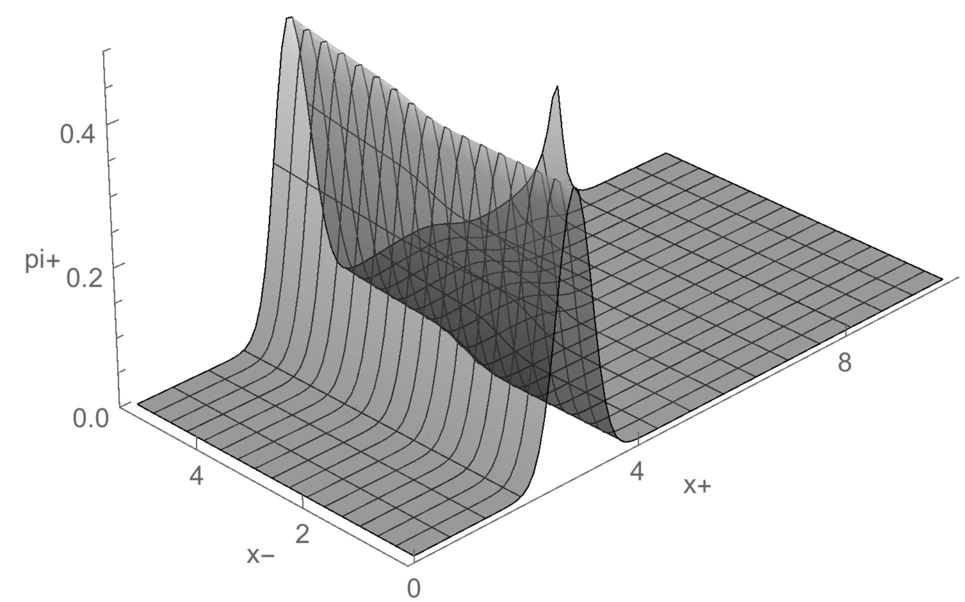


GR 5d: **large amplitude waves**

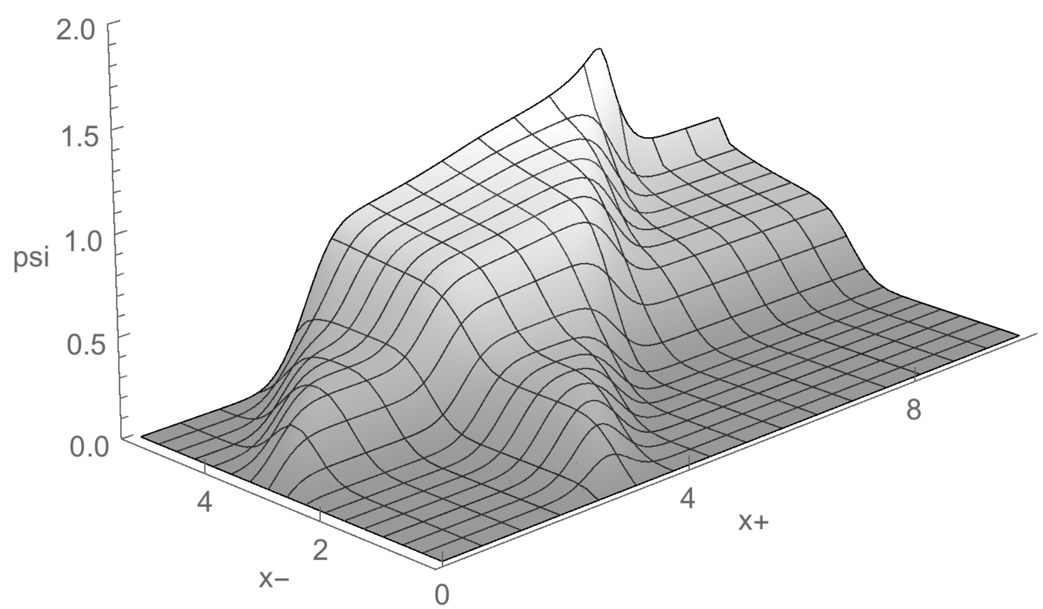
$\partial_- \psi$



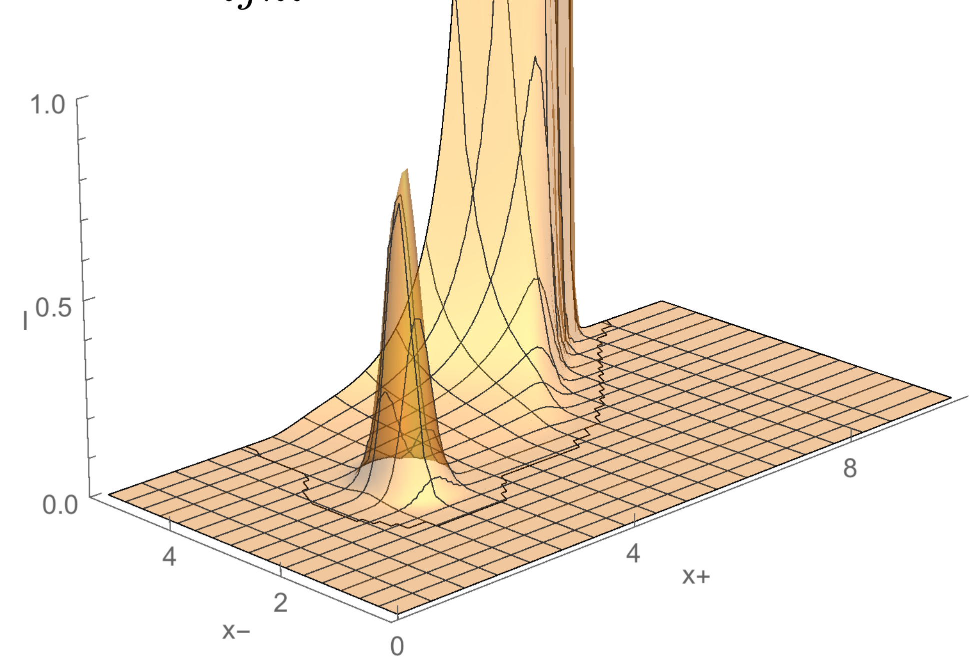
$\partial_+ \psi$

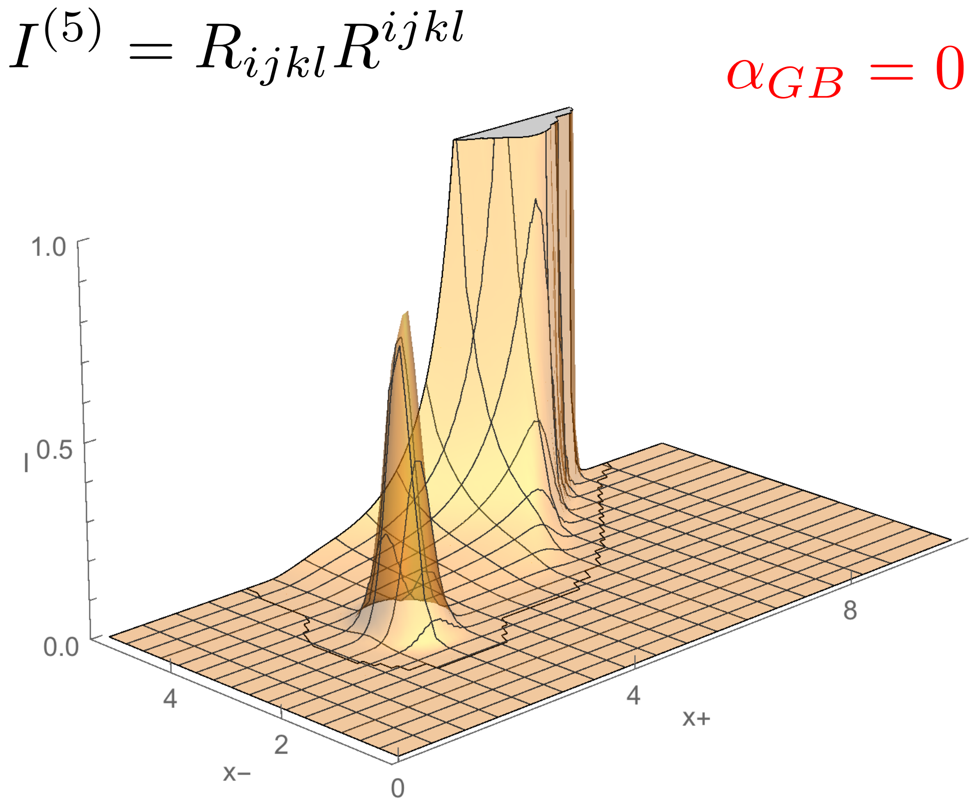


ψ

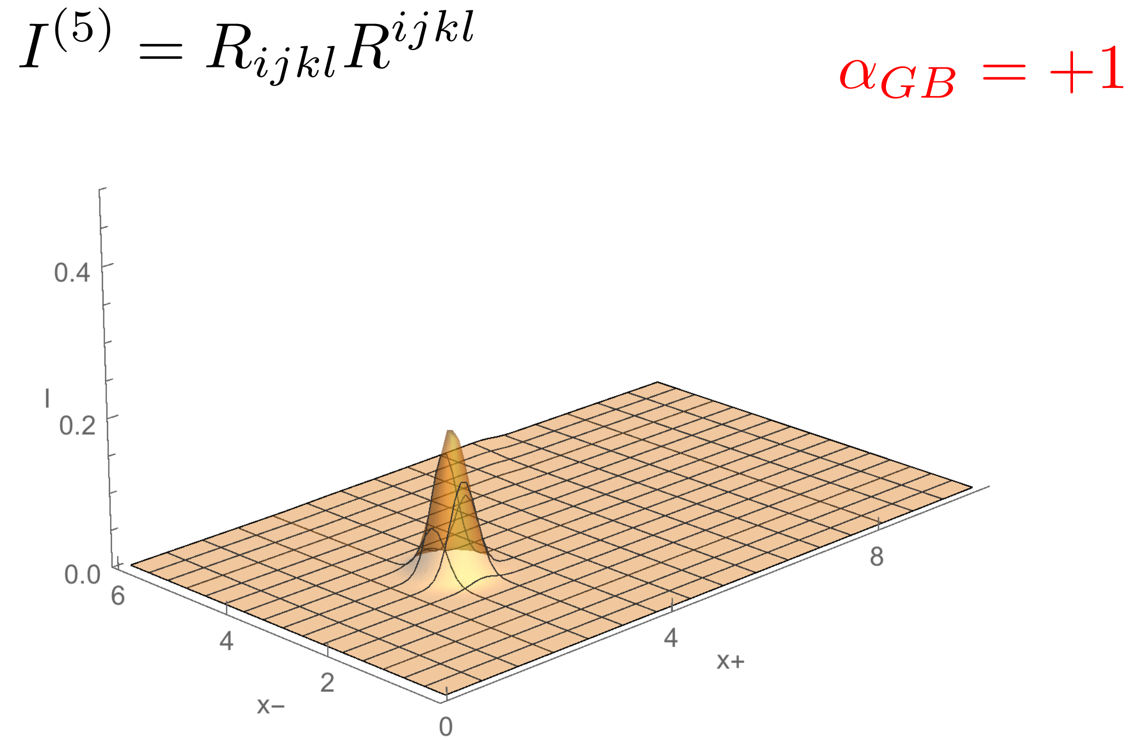


$I^{(5)} = R_{ijkl} R^{ijkl}$

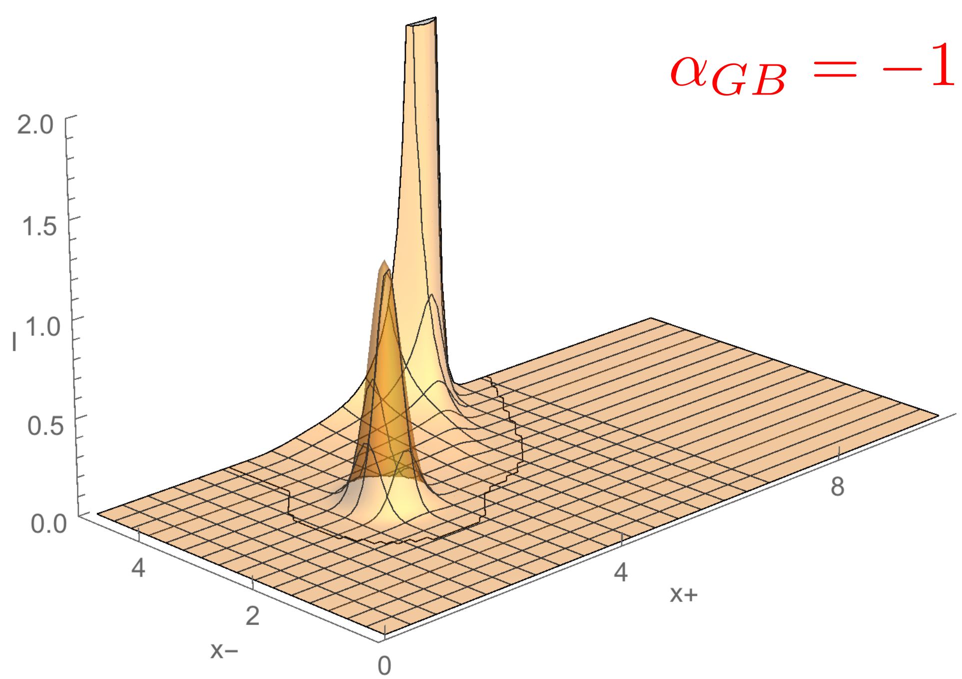




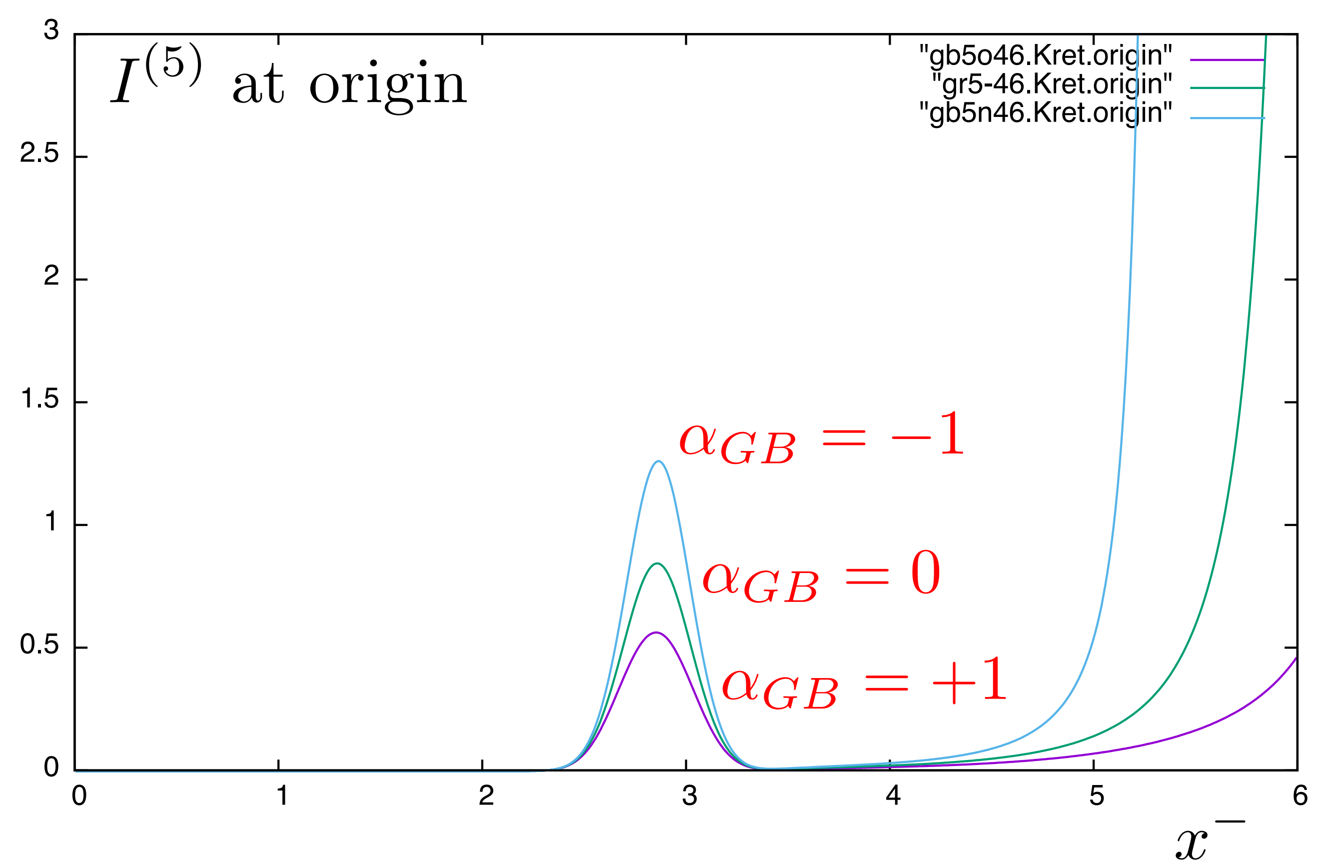
GR 5d



GaussBonnet 5d

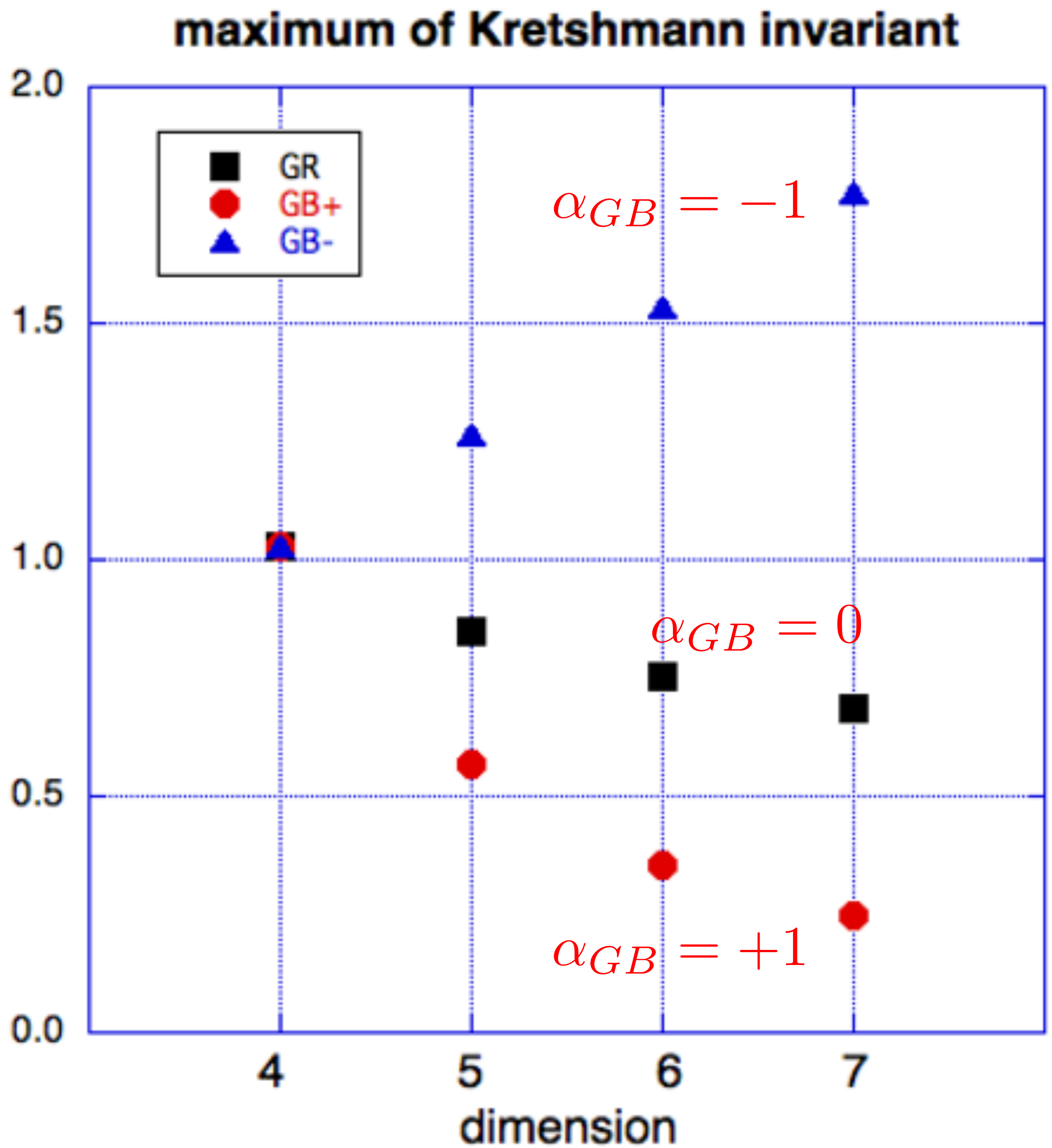


GaussBonnet 5d (negative α)



Colliding Scalar Waves massless scalar waveの衝突による特異点形成

$\max\ (R_{ijkl}R^{ijkl})$



5,6,7次元 Gauss-Bonnet

＊4dim, 5dim, 6dim,…　高次元化

＊Gauss-Bonnet項（正αの項）

は，どちらも特異点形成条件を緩くさせる

Summary

✅ 平面スカラー波の衝突

✅ 球対称ワームホールのBHへの転移現象

4dim, 5dim, 6dim, …

高次元になるほど，同じ初期条件でもBHは形成しにくい

Yamada-HS (2011) [naked singularity形成]とconsistent

高次元になるほど，不安定性は拡大する

Torii-HS (2013) [WH不安定性]とconsistent

$$F \sim \frac{1}{r^{n-2}}$$

Gauss-Bonnet重力の特色

正のcouplingでは，同じ初期条件でも特異点形成は遅くなる.

正のcouplingでは，同じ初期条件でもBHは形成しにくい.

エネルギー底上げ・特異点回避の傾向がある.

高次元になるほど，不安定性は拡大する.

trapped surfaceの存在は，必ずしもBH形成を意味しない.

（面積定理が成立しない解系列があることに対応か）