Introduction to Gravitational Wave Data Analysis

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http://www.oit.ac.jp/is/shinkai/

https://scimeter.org says.

2020-2-20 山口大学セミナー
Gravitational-Wave Physics and Astronomy

An Introduction to Theory, Experiment and Data Analysis

Gravitational Wave Open Science Center

Tutorials

Each tutorial will lead you step-by-step through some common data analysis tasks. While GWOSC data can be analyzed using libraries in many software languages (C, C++, Matlab, etc.), most of these tutorials use Python. See also the software page for more examples.

See the tutorial setup page for help installing software to run these tutorials.

Tutorials shown here are not used to produce published results. For gravitational-wave software analysis packages that are used to produce LSC and Virgo Collaboration publications, see software page.

https://www.gw-openscience.org/tutorials/
重力波の直接検出とデータ解析

真貝 寿明*

日本物理学会誌 Vol. 72, No. 3, 2017

http://www.oit.ac.jp/is/~shinkai/

https://www.iscie.or.jp/pub/journal
1. GW Sources

重力波の波源 (GW sources)

supernovae  pulsars  black hole  binary neutron stars

超新星爆発 (写真出典: NASA)  パルサー (写真出典: NASA)  プラックホール (想像図)  連星中性子星合体 (想像図)

予測が難しい  振幅が小さい  振幅が小さい  連星合体をターゲットに

hard to predict  too small amplitude  too small amplitude  binary coalescence

http://gwcenter.icrr.u-tokyo.ac.jp
<table>
<thead>
<tr>
<th>sources</th>
<th>waveform prediction</th>
<th>data analysis</th>
<th>projects/codes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CBC</strong></td>
<td>binary BHBH/NSNS/BHNS</td>
<td>so so</td>
<td>so so</td>
</tr>
<tr>
<td><strong>Burst</strong></td>
<td>supernovae</td>
<td>hard</td>
<td>unknown</td>
</tr>
<tr>
<td><strong>CW</strong></td>
<td>pulsars, rotating stars</td>
<td>easy</td>
<td>hard</td>
</tr>
<tr>
<td><strong>Stochastic</strong></td>
<td>cosmological</td>
<td>model dependent</td>
<td>hard</td>
</tr>
<tr>
<td><strong>unknown</strong></td>
<td>unknown</td>
<td>unknown</td>
<td>unknown</td>
</tr>
</tbody>
</table>
1. GW Sources

**CBC: compact binary coalescence**

連星系のパラメータ。$s_1, s_2, n$はベクトル量。

- 2つの天体の質量 $m_1, m_2$
- 2つの天体の回転角運動量 $s_1, s_2$
- 連星軌道面の傾斜角 $i$
- 合体時刻と合体時の位相 $t_c, \varphi_c$
- 観測地点からの波源方向 $-\hat{n}$
- 2つの重力波モードの偏角 $\psi$
- 観測地点からの距離 $r$

GW151226  (S/N=13.0)

http://ligo.org/detections/GW170104.php
Introduction to GW data analysis

1. GW Sources

CBC: compact binary coalescence

- **Merger**
- **Ringdown**
- **Inspiral**

### Quadrupole Formula

- \( h_+ (t) = \frac{-G M}{c^2 r} \frac{1 + \cos^2 \iota}{2} \left( \frac{t_c - t}{5 G M / c^3} \right)^{-1/4} \cos 2\varphi \)

- \( h_\times (t) = \frac{-G M}{c^2 r} \cos \iota \left( \frac{t_c - t}{5 G M / c^3} \right)^{-1/4} \sin 2\varphi \)

### Chirp Mass

\[ M = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \]

### Waveform

\[ h(t + \tau) = F_+ (\bm{n}, \psi) h_+ (t) + F_\times (\bm{n}, \psi) h_\times (t) \]

\[ h(t) = -\frac{G M}{c^2 D} \left( \frac{t_c - t}{5 G M / c^3} \right)^{-1/4} \cos 2[\varphi(t) + \Delta \varphi] \]

where

\[ D \equiv r \left[ F_+^2 \left( \frac{1 + \cos^2 \iota}{2} \right)^2 + F_\times^2 \cos^2 \iota \right]^{-1/2} \]

and

\[ 2\Delta \varphi \equiv -\tan^{-1} \left( \frac{F_\times}{F_+} \frac{2 \cos \iota}{1 + \cos^2 \iota} \right) \]
## 1. GW Sources

**CBC: compact binary coalescence**

- **2nd PN近似までで:**
  \[
  \tilde{g}(f) = -\sqrt{\frac{5\pi G^2 M^2}{24 \frac{c^5}{c^3}}} \left(\frac{\pi G M f}{c^3}\right)^{-7/6} e^{-i\Psi(f)}
  \]
  where
  \[
  \Psi(f) = -\frac{\pi}{4} + \frac{3}{128\eta} \left[ x^{-5/2} + \left(\frac{3715}{756} + \frac{55}{9}\eta\right) x^{-3/2} - 16\pi x^{-1} \right.
  
  \left. + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2\right) x^{-1/2} \right]
  \]

- Fourier Transform

- **h(t + \tau) =**
  \[
  h(t + \tau) = F_+ (\hat{n}, \psi) h_+(t) + F_\times (\hat{n}, \psi) h_\times(t)
  \]

- **h(t) =**
  \[
  h(t) = -\frac{GM}{c^2 D} \left(\frac{t_c - t}{5GM/c^3}\right)^{-1/4} \cos 2[\varphi(t) + \Delta \varphi]
  \]
  where
  \[
  D \equiv r \left[ F_+^2 \left(\frac{1 + \cos^2 t}{2}\right)^2 + F_\times^2 \cos^2 t \right]^{-1/2}
  \]
  and
  \[
  2\Delta \varphi \equiv -\tan^{-1} \left(\frac{F_\times}{F_+} \frac{2 \cos t}{1 + \cos^2 t} \right)
  \]
2. Basics

Sensitivity curves

<table>
<thead>
<tr>
<th>BH-BH 質量</th>
<th>$50R_g$</th>
<th>$10R_g$</th>
<th>$5R_g$</th>
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</thead>
<tbody>
<tr>
<td>$10^5M_{\odot} + 10^8M_{\odot}$</td>
<td>244 yr</td>
<td>142 day</td>
<td>8.91 day</td>
</tr>
<tr>
<td>$10^4M_{\odot} + 10^7M_{\odot}$</td>
<td>24.4 yr</td>
<td>14.3 day</td>
<td>21.4 hr</td>
</tr>
<tr>
<td>$10^8M_{\odot} + 10^8M_{\odot}$</td>
<td>2.44 yr</td>
<td>1.43 day</td>
<td>2.14 hr</td>
</tr>
<tr>
<td>$10^5M_{\odot} + 10^5M_{\odot}$</td>
<td>89.1 day</td>
<td>3.42 hr</td>
<td>12.8 min</td>
</tr>
<tr>
<td>$10^4M_{\odot} + 10^4M_{\odot}$</td>
<td>8.91 day</td>
<td>20.5 min</td>
<td>77.0 sec</td>
</tr>
<tr>
<td>$10^3M_{\odot} + 10^3M_{\odot}$</td>
<td>21.4 hr</td>
<td>2.05 min</td>
<td>7.70 sec</td>
</tr>
<tr>
<td>$10^2M_{\odot} + 10^2M_{\odot}$</td>
<td>2.14 hr</td>
<td>12.3 sec</td>
<td>0.77 sec</td>
</tr>
<tr>
<td>$10M_{\odot} + 10M_{\odot}$</td>
<td>12.8 min</td>
<td>1.23 sec</td>
<td>77 msec</td>
</tr>
</tbody>
</table>

各質量の BH 連星の separation について，合体までの時間は

Characteristic Strain – frequency

$h_n(f) = \sqrt{f S_n(f)}$, $h_c(f)$

$f$ [Hz]
Figure A1. A plot of characteristic strain against frequency for a variety of detectors and sources.
Sensitivity curve with Power Spectral Density

\[ /\sqrt{\text{Hz}} \]

Figure A2. A plot of the square root of PSD against frequency for a variety of detectors and sources.
Sensitivity curve with dimensionless energy density

(dimensionless)

Figure A3. A plot of the dimensionless energy density in GWs against frequency for a variety of detectors and sources.
Ideal vs Reality  
(Theory vs Data Analysis)

GW150914  (S/N=23.7)

challenging for data analysis
GW data is with noise
signal quickly decays
(M=60M_{\odot}, a=0.75 \rightarrow 300Hz, \tau = 3 \text{ ms})
Power spectrum of Noise

Parseval id.\[\int_{-\infty}^{\infty} [x(t)]^2 \, dt = \int_{-\infty}^{\infty} [\tilde{x}(f)]^2 \, df\]

\[\langle x^2 \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} P(f) \, df\]

if stationary prob. process

Wiener-Khinchin theorem \[P(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-2\pi i f \tau} \, d\tau\]

\[R_x(\tau) = \langle x(t) x(t + \tau) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x(t + \tau) \, dt\]

auto-correlation func.

\[\langle x^2 \rangle = 2 \int_{0}^{\infty} P(f) \, df = \int_{0}^{\infty} S_x(f) \, df\]

if noise is Gaussian

\[S_x(f) = 2 \int_{-\infty}^{\infty} R_x(\tau) e^{-2\pi i f \tau} \, d\tau = \lim_{\Delta t \to 0} 2\sigma^2 \Delta t\]
2. Basics

Power spectrum of Noise

$$\langle x^2 \rangle = 2 \int_0^\infty P(f) \, df = \int_0^\infty S_x(f) \, df$$

if noise is Gaussian

$$S_x(f) = 2 \int_{-\infty}^\infty R_x(\tau) e^{-2\pi i f \tau} \, d\tau = \lim_{\Delta t \to 0} 2\sigma^2 \Delta t$$

$$p_x[x(t)] \propto \exp \left[ -\frac{1}{2\sigma^2} \sum_j x_j^2 \right] \propto \exp \left[ -\int_{-\infty}^\infty \frac{\tilde{x}(f)^2}{S_x} \, df \right]$$

define the noise-weighed inner product

$$(a, b) \equiv 4 \, \Re \left\{ \int_0^\infty \frac{\tilde{a}(f) \tilde{b}^*(f)}{S(f)} \, df \right\}$$

$$= 2 \int_{-\infty}^\infty \frac{\tilde{a}(f) \tilde{b}^*(f)}{S(|f|)} \, df$$

$$= \int_{-\infty}^\infty \frac{\tilde{a}(f) \tilde{b}^*(f) + \tilde{a}^*(f) \tilde{b}(f)}{S(|f|)} \, df$$
### 2. Basics

#### Matched Filter

\[ \text{Matched Filter (signal-noise ratio)} \]

\[ (s, h) = 4 \text{Re} \int_0^\infty \frac{\tilde{s}(f) \tilde{h}^*(f)}{S_n(f)} \, df \]

#### Odds Ratio

\[ O(\mathcal{H}_1|s) = \frac{P(\mathcal{H}_1|s)}{P(\mathcal{H}_0|s)} \]

#### Likelihood

\[ \Lambda(\mathcal{H}_1|s) = \frac{p(s|\mathcal{H}_1)}{p(s|\mathcal{H}_0)} \]

\[ p(s|\mathcal{H}_1) = p_n[s(t) - h(t)] \propto e^{-(s-h,s-h)/2} \]

\[ p(s|\mathcal{H}_0) = p_n[s(t)] \propto e^{-(s,s)/2} \]

#### Hypotheses

\[ \mathcal{H}_0: s(t) = n(t) \]

\[ \mathcal{H}_1: s(t) = n(t) + h(t) \]

\[ \text{signal} = \text{gw} + \text{noise} \]

\[ s(t) = h(t) + n(t) \]
2. Basics

Bayes Theorem

\[ P(B_k|A) = \frac{P(A \cap B_k)}{P(A)} \]

\[ P(A \cap B_k) = P(A|B_k)P(B_k) \]

\[ P(A) = \sum_k P(A \cap B_k) \]

\[ P(B_k|A) = \frac{P(A|B_k)P(B_k)}{\sum_k P(A|B_k)P(B_k)} \]

A→B

原因→結果

B→A

結果→原因

応用例

インターネットでの個別広告の実現
本屋や音楽ダウンロードサイトで「おすすめ」
迷惑メールフィルタ
ユーザに合わせた設定
Introduction to GW data analysis

2. Basics

Matched Filter

\[ \text{signal} = \text{gw} + \text{noise} \]

\[ s(t) = h(t) + n(t) \]

- gw template \( h(t) \)

\[ \rho = 2 \int_{-\infty}^{\infty} \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_h(|h|)} df \]

signal/noise ratio

GW151226 \((S/N=13.0)\)
2. Basics

Parameter Estimation

**Likelihood**

\[
\Lambda(\mathcal{H}_1 | s) = \frac{p(s | \mathcal{H}_1)}{p(s | \mathcal{H}_0)}
\]

\[
s(t) = n(t) + h_\theta(t)
\]

\[
\Lambda(\mathcal{H}_\theta | s) = \frac{p(s | \mathcal{H}_\theta)}{p(s | \mathcal{H}_0)}
\]

\[
\log \Lambda(\mathcal{H}_\theta | s) = (s, h_\theta) - \frac{1}{2} (h_\theta, h_\theta)
\]

**Fisher matrix**

\[
\Gamma_{ij} \equiv \bar{v}_i \bar{v}_j = (n, \frac{\partial h_{\theta_{\text{max}}}}{\partial \theta^i})(\frac{\partial h_{\theta_{\text{max}}}}{\partial \theta^j}, n) = (\frac{\partial h_{\theta_{\text{max}}}}{\partial \theta^i}, \frac{\partial h_{\theta_{\text{max}}}}{\partial \theta^j}) \quad v_i \equiv (n, \frac{\partial h_{\theta_{\text{max}}}}{\partial \theta^i})
\]

\[
p(v) = \frac{1}{\sqrt{2\pi \det \Gamma}} \exp \left[ -\frac{1}{2} v_i v_j \right]
\]

\[
V^{ij} \equiv (\Gamma^{-1})^{ij}
\]

**uncertainty**

\[
(\Delta \theta^i)_{\text{rms}} = \sqrt{V^{ii}} \quad \text{(no summation)}
\]

**correlation coef.**

\[
c_{ij} = \frac{\Delta \theta^i \Delta \theta^j}{V^{ii} V^{jj}} = \frac{V^{ij}}{\sqrt{V^{ii} V^{jj}}}
\]
Markov Chain Monte Carlo (MCMC)

確率分布を見つける力業
マルコフ連鎖モンテカルロ法（MCMC）の考え方

問題発生
山田君は誤って大きな洞穴に落ちて気を失ってしまった。気付けば夜で、中は真っ暗に。落ちた穴をどう見つければいいのか。

ランダムに歩いて（乱数発生）、小石を投げる（計測）
そこで山田君は、ランダムに歩き、近くの小石を拾って真上に投げ、小石が天井にぶつかるまで時間の測った。「ここが高いな」と思った、その付近に集中的に小石を投げた。合計で20回小石を投げた。

足跡を数える（確率分布（P(x)）の推測）
暗闇の中、山田君が注目したのは足元だった。スマートフォンの明かりで足跡を調べ、その分布を見て洞穴の形状を推測。右から2番目の足跡までの確率を2/20、すなわち0.1とみた。

本質
天井ではなく足元を見る。これがMCMCの発想だ。小石を投げる回数を20回から1000回にすれば、推測の精度は上がる。体力勝負の力ではない、コンピュータとの相性が抜群だ。

2. Basics

今週のテーマは、フランスの数学家オーギュスト・ボアロンが提唱したモンテカルロ法の基礎であるMCMC（マルコフ連鎖モンテカルロ法）について説明する。MCMCは、確率分布を効率的に探索する手法として広く使われている。

MCMCの原理を理解するには、まず確率論の基礎を理解する必要がある。確率論は、試行の結果をどのように解析するかを研究する数学の一分野である。

MCMCの具体的な応用例として、統計学、物理学、経済学など、多くの分野で使われている。

今週の講義では、MCMCの原理を理解するための基礎的な概念を解説し、実際の応用例を紹介する予定である。
LIGO Computing Latencies

LIGO Laboratory Tier 1

- Detector Characterisation
- Low-latency BNS & NSBH Search
  - gstlal
- Rapid Sky Localisation
  - BAYESTAR
- Rapid Parameter Estimation
  - lalinference
- Rapid GRB Followup
  - grb_dhope

XSEDE Resources

- Deep offline search for BNS, NSBH and BBH sources
  - offline_dhope
- Offline Triggers
- Detection Candidates
- Full Parameter Estimation
  - lalinference

Latency

- To Electromagnetic Observing Partners
  - Seconds - Minutes
  - 1 Week
  - 2 Weeks
- 3 months

Sharon Brunett, 2015/10
**CBC: compact binary coalescence**

$$h(t) = -\frac{G M}{c^2 D} \left( \frac{t_c - t}{5G M/c^3} \right)^{-1/4} \cos 2[\varphi(t) + \Delta \varphi]$$

where

$$D \equiv r \left[ F_+^2 \left( \frac{1 + \cos^2 \iota}{2} \right)^2 + F_\times \cos^2 \iota \right]^{-1/2}$$

and

$$2\Delta \varphi \equiv -\tan^{-1} \left( \frac{F_\times}{F_+ \left( 1 + \cos^2 \iota \right)} \right)$$

Horizon distance (Observational range)

$$D_{\text{horizon}} = \frac{2}{5} \sqrt{\frac{5}{6}} \frac{c}{\pi^{2/3}} \left( \frac{G M}{c^3} \right)^{5/6} \left[ \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{f^{7/3}}{S_n(f)} df \right]^{1/2} \frac{1}{\rho}$$
Horizon distance

LVK, 1304.0670 (2020/1 update)

Horizon distance (Observational range)

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<tr>
<th></th>
<th>O1</th>
<th>O2</th>
<th>O3</th>
<th>O4</th>
<th>O5</th>
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<td><strong>BNS Range (Mpc)</strong></td>
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<tr>
<td>aLIGO</td>
<td>80</td>
<td>100</td>
<td>110–130</td>
<td>160–190</td>
<td>330</td>
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<td>AdV</td>
<td>-</td>
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<td>50</td>
<td>90–120</td>
<td>150–260</td>
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<td>-</td>
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<td><strong>BBH Range (Mpc)</strong></td>
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<td><strong>Burst Range (Mpc)</strong></td>
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<tr>
<td>(\sqrt{\varepsilon_\text{GW} = 10^{-2} M_\odot c^2})</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>aLIGO</td>
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<td>60</td>
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<td>110–120</td>
<td>210</td>
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<tr>
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<td>-</td>
<td>0–10</td>
<td>10–30</td>
<td>30+</td>
</tr>
</tbody>
</table>

Table 2: Achieved and projected detector sensitivities for a 1.4\(M_\odot\)+1.4\(M_\odot\) BNS system, a 30\(M_\odot\)+30\(M_\odot\) BBH system, a 1.4\(M_\odot\)+10\(M_\odot\) NSBH system, and for an unmodeled burst signal. The quoted ranges correspond to the orientation-averaged spacetime volumes surveyed per unit detector time. For the burst ranges, we assume an emitted energy in GWs at 140 Hz of \(E_\text{GW} = 10^{-2} M_\odot c^2\) and of \(E_\text{GW} = 10^{-9} M_\odot c^2\). The later is consistent with the order of magnitude of the energy expected from core-collapse of massive stars (see footnote 4). Both CBC and burst ranges are obtained using a single-detector SNR threshold of 8. The O1 and O2 numbers are representative of the best ranges for the LIGO detectors: Hanford in O1 and Livingston in O2. The O3 numbers for aLIGO and AdV reflect recent average performance of each of the three detectors. Range intervals are quoted for future observing runs due to uncertainty about the sequence and impact of upgrades.

\[
D_{\text{horizon}} = \frac{2}{\rho} \sqrt{\frac{5}{6}} \frac{c}{\pi^{2/3}} \left(\frac{GM}{c^3}\right)^{5/6} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{f^{7/3}}{S_n(f)} df^{1/2} \frac{1}{\rho}
\]

\(\rho = 8\)

Hisaaki Shinkai (真貝 命明) 2020/2/20 @ Yamaguchi Univ.
3. waveforms

**CBC: compact binary coalescence**

$F(t) = A e^{-(t-t_0)/\tau} \cos(2\pi f_R(t-t_0) - \phi_0)$

$h(t) \sim A e^{i2\pi f_{qnm}t} = A e^{i2\pi(f_R + if_I)t}$

**Fitting formula**

\[
\begin{align*}
    f_R &= f_1 + f_2(1 - a)^{f_3} \\
    Q &= \frac{f_R}{2f_I} = q_1 + q_2(1 - a)^{q_3}
\end{align*}
\]

\[
\begin{align*}
    f_1 &= 1.5251, \quad f_2 = -1.1568, \quad f_3 = 0.1292 \\
    q_1 &= 0.7000, \quad q_2 = 1.4187, \quad q_3 = -0.4990.
\end{align*}
\]

\[
\begin{align*}
    a &= 1 - \left( \frac{Q - q_1}{q_2} \right)^{1/q_3} \\
    M[M_\odot] &= \frac{c^3}{2\pi G} \frac{f_1 + f_2(1 - a)^{f_3}}{f_{qnm}[Hz]}
\end{align*}
\]

BH mass and spin.
### CBC: compact binary coalescence

**Figure 8.** Detectable distance $D$ of the ring-down signal at KAGRA. S/N is set to (a) 10 and (b) 100.

$$h(t) = A e^{-\frac{(t-t_0)}{\tau}} \cos \left(2\pi f_R (t - t_0) - \phi_0\right)$$

**Horizon distance (Observational range)**

$$\rho^2 = \frac{8}{5} \frac{\epsilon_r(a)}{f_R^2} \frac{(1 + z)M}{S_h(f_R/(1 + z))}$$

$$\times \left(\frac{(1 + z)M}{d_L(z)}\right)^2 \left(\frac{4\mu}{M}\right)^2.$$
GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs

B. P. Abbott et al.
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 14 December 2018; revised manuscript received 27 March 2019; published 4 September 2019)

**O1: September 12, 2015 -- January 19, 2016**
- GW150914 BHBH

**O2: November 30, 2016 -- August 25, 2017**
- GW170817 NSNS
- **GWTC-1 catalogue paper [arXiv:1811.12907]**
- data released to public Feb, 2019

**O3a: April 1, 2019 -- September 30, 2019**
- data released to public April, 2021

**O3b: November 1, 2019 -- May 1, 2020**
GWTC-1: A GRAVITATIONAL-WAVE TRANSIENT CATALOG … PHYS. REV. X 9, 031040 (2019)

TABLE I. Search results for the 11 GW events. We report a false-alarm rate for each search that found a given event; otherwise, we display ⋄. The network SNR for the two matched-filter searches is that of the template ranked highest by that search, which is not necessarily the template with the highest SNR. Moreover, the network SNR is the quadrature sum of the detectors coincident in the highest-ranked trigger; in some cases, only two detectors contribute, even if all three are operating nominally at the time of that event.

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<th>GstLAL</th>
<th>cWB</th>
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<td>&lt;1.00 × 10^{-7}</td>
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<td>10.0</td>
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<td>13.0</td>
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<td>⋄</td>
<td>⋄</td>
<td>11.3</td>
<td>⋄</td>
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<tr>
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<td>2.14 × 10^{-3}</td>
<td>11.1</td>
<td>11.5</td>
<td>10.8</td>
</tr>
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</table>
TABLE III. Selected source parameters of the 11 confident detections. We report median values with 90\% credible intervals that include statistical errors and systematic errors from averaging the results of two waveform models for BBHs. For GW170817, credible intervals and statistical errors are shown for IMRPhenomPv2NRT with a low spin prior, while the sky area is computed from TaylorF2 samples. The redshift for NGC 4993 from Ref. [94] and its associated uncertainties are used to calculate source-frame masses for GW170817. For BBH events, the redshift is calculated from the luminosity distance and assumed cosmology as discussed in Appendix B. The columns show source-frame component masses \( m_i \) and chirp mass \( \mathcal{M} \), dimensionless effective aligned spin \( \chi_{\text{eff}} \), final source-frame mass \( M_f \), final spin \( a_f \), radiated energy \( E_{\text{rad}} \), peak luminosity \( L_{\text{peak}} \), luminosity distance \( d_L \), redshift \( z \), and sky localization \( \Delta \Omega \). The sky localization is the area of the 90\% credible region. For GW170817, we give conservative bounds on parameters of the final remnant discussed in Sec. V E.
FIG. 10. Time-frequency maps and reconstructed signal waveforms for the ten BBH events. Each event is represented with three panels showing whitened data from the LIGO detector where the higher SNR is recorded. The first panel shows a normalized time-frequency power map of the GW strain. The remaining pair of panels shows time-domain reconstructions of the whitened signal, in units of the standard deviation of the noise. The upper panels show the 90% credible intervals from the posterior probability density functions of the waveform time series, inferred using CBC waveform templates from Bayesian inference (LALInference) with the PhenomP model (red band) and by the BAYESWAVE wavelet model (blue band) [53]. The lower panels show the point estimates from the cWB search (solid lines), along with a 90% confidence interval (green band) derived from cWB analyses of simulated waveforms from the LALInference CBC parameter estimation injected into data near each event. Visible differences between the different reconstruction methods are verified to be consistent with the noise origin (see the text for details).
GWTC-1 : Signal-Noise-Ratio of 3-detectors

<table>
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<tr>
<th>Event</th>
<th>GW150914</th>
<th>GW151012</th>
<th>GW151226</th>
<th>GW170104</th>
<th>GW170608</th>
<th>GW170729</th>
<th>GW170809</th>
<th>GW170814</th>
<th>GW170817</th>
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Sky Localization

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<th>$\Delta\Omega$ (deg$^2$)</th>
<th>IFOs</th>
<th>Refined analysis</th>
<th>$d_c$ (Mpc)</th>
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<th>IFOs</th>
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<td>HL</td>
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<td>—</td>
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<td>320$^{+120}_{-110}$</td>
<td>392</td>
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<td>2840$^{+1400}_{-1360}$</td>
<td>1041</td>
<td>HLV</td>
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<td>1155</td>
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Fig. 3 Cumulative histograms of triggers obtained by the offline searches plotted versus the IFAR. The top panel shows results for the matched-filter searches: on the left the PyCBC (Dal Canton et al. 2014b; Usman et al. 2016) search pipeline, and on the right the GstLAL (Cannon et al. 2012; Privitera et al. 2014; Messick et al. 2017; Sachdev et al. 2019) search pipeline. The bottom panels show unmodeled searches performed by the cWB (Klimenko et al. 2016, 2008) pipeline; on the left looking for stellar-mass BBHs mergers, and on the right for generic transients. The dashed lines show the expected background, given the analysis time. Shaded regions denote the sigma uncertainty bounds for the Poisson statistic. The blue dots are the confident GW events found by each search. Any events with a measured or bounded inverse false alarm rate greater than 3000 yrs are shown with a right pointing arrow. The values of the FARS of the confident events can be found in Abbott et al. (2018d), Abbott et al. (2019b), and Abbott et al. (2017b).
Fig. 8 Alert timeline. The *Preliminary GCN Notice* is sent autonomously within 1-10 minutes after the GW candidate trigger time. Some preliminary alerts may be retracted after human inspection for data quality, instrumental conditions, and pipeline behavior. The human vetted *Initial GCN Notice* or *Retraction GCN Notice* and associated *GCN Circular* are distributed within a few hours for BNS or NSBH sources and within one day for BBH. Update notices and circulars are sent whenever the estimate of the parameters of the signal significantly improves. Figure adapted from the LIGO/Virgo Public Alerts User Guide (see footnote 17)

https://emfollow.docs.ligo.org/userguide/
### GraceDB — Gravitational-Wave Candidate Event Database

#### Latest — as of 15 February 2020 13:15:11 UTC

Test and MDC events and superevents are not included in the search results by default; see the [query help](https://gracedb.ligo.org/later/) for information on how to search for events and superevents in those categories.

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[https://gracedb.ligo.org/latest/](https://gracedb.ligo.org/latest/)
LV event categories

Fig. 9 The four astrophysical categories in terms (BNS, NSBH, BBH, and MassGap) of component masses $m_1$ and $m_2$, which are used to define the source classification. By convention, the component masses are defined such that $m_1 \geq m_2$, so that the primary compact object in the binary (i.e., component 1), is always more massive than the secondary compact object (i.e., component 2). Figure adapted from the LIGO/Virgo Public Alerts User Guide (see footnote [17]).

https://emfollow.docs.ligo.org/userguide/
## Catalogue History

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https://www.gw-openscience.org/about/

https://github.com/gwastro/2-ogc
New search pipeline for compact binary mergers: Results for binary black holes in the first observing run of Advanced LIGO

Tejaswi Venunadav, 1, a Barak Zackay, 1 Javier Roulet, 2 Liang Dai, 1 and Matias Zaldarriaga 1
1 School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, New Jersey 08540, USA
2 Department of Physics, Princeton University, Princeton, New Jersey 08540, USA

(Received 21 March 2019; published 24 July 2019)

In this paper, we report on the construction of a new and independent pipeline for analyzing the public data from the first observing run of Advanced LIGO for mergers of compact binary systems. The pipeline incorporates different techniques and makes independent implementation choices in all its stages including the search design, the method to construct template banks, the automatic routines to detect bad data segments (“glitches”) and to insulate good data from them, the procedure to account for the nonstationary nature of the detector noise, the signal-quality vetoes at the single-detector level and the methods to combine results from multiple detectors. Our pipeline enabled us to identify a new binary black hole merger GW151216 in the public LIGO data. This paper serves as a bird’s eye view of the pipeline’s important stages. Full details and derivations underlying the various stages will appear in accompanying papers.

DOI: 10.1103/PhysRevD.100.023011

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| Name    | Bank  | \(M(M_\odot)^a\) | GPS time\(^b\) | \(\rho_H^2\) | \(\rho_L^2\) | FAR\(^{-1}\) (O1)\(^c\) | \(\frac{W}{R(\text{event}|H_0)^d}\) (days) | \(R_{>100}\) (days\(^{-1}\)) | \(p_{\text{astro}}\) |
|---------|-------|------------------|----------------|--------------|--------------|-----------------|-------------------------------|---------------------|------------|
| GW151226| BBH 1 | 9.74             | 1135136350.585 | 120.0        | 52.1         | >20000          | \(\ldots\) \(^d\)                | \(\ldots\)          | 1\(^d\)    |
| GW151012| BBH 2 | 18               | 1128678900.428 | 55.66        | 46.75        | >20000          | \(7 \times 10^5\) \(^e\)         | 0.01                | 0.9998\(^e\) |
| GW150914| BBH 3 | 28               | 1126259462.411 | 396.1        | 184.3        | >20000          | \(\ldots\) \(^d\)                | \(\ldots\)          | \( \ldots\) |
| GW151216\(^f\) | BBH 3 | 29               | 1134293073.164 | 39.4         | 34.8         | 52              | 74 \(\pm\) 2                   | 0.033               | 0.71       |
| 151231  | BBH 3 | 30               | 1135557647.145 | 37.5         | 25.2         | 0.98            | 5.4 \(\pm\) 0.4                 | 0.033               | 0.15       |
| 151011  | BBH 4 | 58               | 1128626886.595 | 24.5         | 39.9         | 1.1             | 16 \(\pm\) 1                   | 0.01                | 0.14       |

\(^a\)Posterior samples from parameter estimation runs for all the O1 and O2 events can be found at https://github.com/jroulet/O2_samples.

\(^b\)Times are given as the linear free times, that is, the times corresponding to when the waveforms generated by the bank were
Highly spinning and aligned binary black hole merger in the Advanced LIGO first observing run

Barak Zackay,¹ Tejaswi Venumadhav,¹ Liang Dai,¹ Javier Roulet,² and Matias Zaldarriaga¹

¹School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, New Jersey 08540, USA
²Department of Physics, Princeton University, Princeton, New Jersey 08540, USA

(Received 21 March 2019; published 17 July 2019)

We report a new binary black hole merger in the publicly available LIGO first observing run (O1) data release. The event has a false alarm rate of one per six years in the detector-frame chirp-mass range $\mathcal{M}_{\text{det}} \in [20, 40] M_\odot$ in a new independent analysis pipeline that we developed. Our best estimate of the probability that the event is of astrophysical origin is $P_{\text{astro}} \sim 0.71$. The estimated physical parameters of the event indicate that it is the merger of two massive black holes, $\mathcal{M}_{\text{det}} = 31_{-2}^{+3} M_\odot$ with an effective spin parameter $\chi_{\text{eff}} = 0.81_{-0.15}^{+0.25}$, making this the most highly spinning merger reported to date. It is also among the two highest redshift mergers observed so far. The high aligned spin of the merger supports the hypothesis that merging binary black holes can be created by binary stellar evolution.
New Binary Black Hole Mergers in the Second Observing Run of Advanced LIGO and Advanced Virgo

Tejaswi Vennumadhav,1 Barak Zackay,1 Javier Roulet,2 Liang Dai,1 and Matias Zaldarriaga1
1School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540, USA
2Department of Physics, Princeton University, Princeton, NJ, 08540, USA
(Dated: April 16, 2019)

TABLE I: Events already reported by the LIGO–Virgo Collaboration [2] as detected with our pipeline. The rate distributions used to compute $p_{\text{astro}}$ are shown in Fig. 3. The maximum likelihood rates are $R_{\text{max}} = 8/02$ and $5/02$ in banks BBH 3 and BBH 4, respectively.

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<tr>
<th>Name</th>
<th>Bank</th>
<th>GPS time</th>
<th>$\rho_H^2$</th>
<th>$\rho_L^2$</th>
<th>FAR$^{-1}(O2)^b$</th>
<th>$W(\text{event})_{R(\text{event}/N)}(O2)$</th>
<th>$p_{\text{astro}}$</th>
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<td>GW170104</td>
<td>BBH</td>
<td>1167559936.582</td>
<td>85.1</td>
<td>104.3</td>
<td>$&gt; 2 \times 10^4$</td>
<td>&gt; 100</td>
<td>&gt; 0.99</td>
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<tr>
<td>GW170809</td>
<td>BBH</td>
<td>1186302519.740</td>
<td>40.5</td>
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<tr>
<td>GW170814</td>
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<td>170</td>
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<td>1187058327.075</td>
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<td>95.1</td>
<td>1.7</td>
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<td>GW170729</td>
<td>BBH</td>
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<td>GW170823</td>
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<td>$&gt; 2 \times 10^4$</td>
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a The times given are the ‘linear-free’ times of the best fit templates in our bank; with this time as the origin, the phase of the template is orthogonal to shifts in time, given the fiducial PSD.

b The FARs given are computed within each bank; our BBH analysis has 5 chirp-mass banks. The inverse FAR is given in terms of “O2” to reflect the volumetric weighting of events. Under the approximation of constant sensitivity of the detectors during the observing run, the unit “O2” corresponds to ≈ 118 days.

c See discussion in §III.

TABLE II: New events with astrophysical probability > 50% in all of the BBH banks. The rate distributions used to compute $p_{\text{astro}}$ are shown in Fig. 3, the maximum-likelihood rates in banks BBH 3 and BBH 4 are $R_{\text{max}} = 8/02$ and $5/02$, respectively.

<table>
<thead>
<tr>
<th>Name</th>
<th>Bank</th>
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<th>$\chi_{\text{eff}}$</th>
<th>$\chi$</th>
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<td>1169069154.565</td>
<td>29.4</td>
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<td>&gt; 0.99</td>
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<td>GW170304</td>
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<td>47$^{+5}_{-3}$</td>
<td>0.2$^{+0.3}_{-0.3}$</td>
<td>0.5$^{+0.2}_{-0.2}$</td>
<td>1172680691.356</td>
<td>24.9</td>
<td>55.9</td>
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<td>42$^{+6}_{-5}$</td>
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<td>0.4$^{+0.3}_{-0.3}$</td>
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<td>25.4</td>
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<td>1175295989.221</td>
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<td>31.0</td>
<td>4.7</td>
<td>0.23</td>
<td>0.56</td>
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a The times given are the ‘linear-free’ times of the best fit templates in our bank; with this time as the origin, the phase of the template is orthogonal to shifts in time, given the fiducial PSD.

b The FARs given are computed within each bank; our BBH analysis has 5 chirp-mass banks. The inverse FAR is given in terms of “O2” to reflect the volumetric weighting of events. Under the approximation of constant sensitivity of the detectors during the observing run, the unit “O2” corresponds to ≈ 118 days.
FIG. 4: Source-frame total mass and effective spin for the BBH events found in Hanford–Livingston coincidence, over O1 and O2. We recovered all the previously reported events with high confidence, $p_{\text{astro}} \approx 1$, except for GW170608 and GW170818, see §III. We found seven additional events ranging from marginal triggers to confident detections: one in O1 [17] and six in O2 (this work). The dots and error bars show median and 90% confidence intervals, respectively. The spin $\chi_{\text{eff}}$ and the mass can be correlated (not shown). The full posteriors can be found in Appendix [A] The prior used was uniform in $m_1$, $m_2$, $\chi_{\text{eff}}$, and luminosity volume.
1-OGC: The first open gravitational-wave catalog of binary mergers from analysis of public Advanced LIGO data

ALEXANDER H. NITZ,1,2 COLLIN CAPANO,1,2 ALEX B. NIELSEN,1,2 STEVEN REYES,3 REBECCA WHITE,4,3 DUNCAN A. BROWN,3 AND BADRI KRISHNAN1,2

1Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), D-30167 Hannover, Germany
2Leibniz Universität Hannover, D-30167 Hannover, Germany
3Department of Physics, Syracuse University, Syracuse NY 13244, USA
4Fayetteville-Manlius High School, Manlius, NY 13104, USA

ABSTRACT

We present the first Open Gravitational-wave Catalog (1-OGC), obtained by using the public data from Advanced LIGO’s first observing run to search for compact-object binary mergers. Our analysis is based on new methods that improve the separation between signals and noise in matched-filter searches for gravitational waves from the merger of compact objects. The three most significant signals in our catalog correspond to the binary black hole mergers GW150914, GW151226, and LVT151012. We assume a common population of binary black holes for these three signals by defining a region of parameter space that is consistent with these events. Under this assumption, we find that LVT151012 has a 97.6% probability of being astrophysical in origin. No other significant binary black hole candidates are found, nor did we observe any significant binary neutron star or neutron star–black hole candidates. We make available our complete catalog of events, including the sub-threshold population of candidates.

Figure 1. The component masses and spins of the templates used to search for compact binary mergers. Due to the exclusion of short duration templates, there is a dependency on the total mass searched and its effective spin. For binary black holes with negligible spin, this implies that this study only probes sources with total mass less than $200 M_\odot$. Visible artifacts due to the procedure for constructing the template bank do not impact performance. Templates which we conservatively consider to produce binary black hole (BBH) candidates consistent with known observations are shown in red as discussed in Sec. 3. The upper mass boundary of the analysis performed by the LVC in Abbott et al. (2016a) is shown as a black dotted line.
2-OGC: Open Gravitational-wave Catalog of binary mergers from analysis of public Advanced LIGO and Virgo data

ALEXANDER H. NITZ,1,2 THOMAS DENT,3 GARETH S. DAVIES,3 SUMIT KUMAR,1,2 COLLIN D. CAPPANO,1,2 IAN HARRY,4,5 SIMONE MOZZON,4 LAURA NUTTALL,4 ANDREW LUNDGREN,4 AND MARTON TÁPAI6

1 Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), D-30167 Hannover, Germany
2 Leibniz Universität Hannover, D-30167 Hannover, Germany
3 Instituto Galego de Física de Altas Enerxías, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Galicia, Spain
4 University of Portsmouth, Portsmouth, PO1 3FX, United Kingdom
5 Kavli Institute of Theoretical Physics, UC Santa Barbara, CA
6 Department of Experimental Physics, University of Szeged, Szeged, 6720 Dóm tér 9., Hungary

ABSTRACT

We present the second Open Gravitational-wave Catalog (2-OGC) of compact-binary coalescences, obtained from the complete set of public data from Advanced LIGO’s first and second observing runs. For the first time we also search public data from the Virgo observatory. Our analysis is based upon updated methods to improve detection of compact binary mergers by incorporating corrections for short time variations in the detectors’ noise power spectral density and instantaneous network sensitivity. We identify a population of 14 binary black hole mergers with $p_{astro} > 0.5$ along with the GW170817 binary neutron star merger. We confirm the binary black hole merger observations of GW170121, GW170304, and GW170727. We also report GW151205, a new marginal binary black hole merger during the first observing run. No other individually significant binary neutron star candidates are found, nor did we observe any significant neutron star–black hole candidates. We make available our comprehensive catalog of events, including the sub-threshold population of candidates to enable deeper follow-up as our understanding of the underlying populations evolves.

O1/O2
new 4 events
151205
170121
170304
170727

Figure 2. The component masses of templates used to search for compact binary mergers. Templates which define the targeted-binary black hole region are colored in red. Candidates which fall in the selected BNS-like region are discussed in 4.2.
自己回帰モデルを用いた重力波データ解析(2)：LIGO/Virgo O2までのカタログデータ解析

ブラックホールリングダウン波形の

真見寿明（大阪工大）

BH 準固有モード（quasi-normal modes）
⇔ BH 摂動理論
⇔ (M, a)

強い重力場の表れ
⇔ 一般相対論の検証ができる

テンプレートを使わず、データから波形を再構築。

2019/9/20 物理学会 @ 山形大学
**O1/O2 カタログ**

GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs

B. P. Abbott et al.

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 14 December 2018; revised manuscript received 27 March 2019; published 4 September 2019)

**O1**: September 12, 2015 -- January 19, 2016
- GW150914 BHBH

**O2**: November 30, 2016 -- August 25, 2017
- GW170817 NSNS
- GWTC-1 catalogue paper [arXiv:1811.12907]
- data released to public Feb, 2019

**O3a**: April 1, 2019 -- September 30, 2019
- 28 alerts so far
- data released to public April, 2021

**O3b**: November 1, 2019 -- May 1, 2020
- KAGRA will join, “LVK collaboration”

FIG. 10: Time-frequency maps and reconstructed signal waveforms for the ten BBH events. Each event is represented with three panels showing whitened data from the LIGO detector where the higher SNR is recorded. The first panel shows a normalized time-frequency power map of the GW strain. The remaining pair of panels shows time-domain reconstructions of the whitened signal, in units of the standard deviation of the noise. The upper panels show the 90% credible intervals from the posterior probability density functions of the waveform time series, inferred using CBC waveform templates from Bayesian inference (LALInference) with the PhenomP model (red band) and by the BAYES wavelet model (blue band) [53]. The lower panels show the point estimates from the cWB search (solid lines), along with a 90% confidence interval (green band) derived from cWB analyses of simulated waveforms from the LALInference CBC parameter estimation injected into data near each event. Visible differences between the different reconstruction methods are verified to be consistent with a noise origin (see the text for details).
### Table V

KL divergences (in bits) between the prior and posterior for the effective aligned spin $\chi_{\text{eff}}$ and the effective precession spin $\chi_{p}$. For the computation of the KL divergence for $\chi_{p}$, we quote the KL divergence with the prior conditioned on the $\chi_{\text{eff}}$ posterior, $D_{\text{KL}}^{\chi_{p}}(\chi_{\text{eff}})$, and without conditioning, $D_{\text{KL}}^{\chi_{p}}$. For GW170817, $D_{\text{KL}}^{\chi_{p}}$ is given for the high spin prior. The median and 90% interval for the KL divergences is estimated by computing the statistic for repeated draws of a subset of the posterior and prior PDFs. Single-detector optimal SNRs from parameter-estimation analyses for Hanford (H), Livingston (L), and Virgo (V).

<table>
<thead>
<tr>
<th>Event</th>
<th>$m_1/M_\odot$</th>
<th>$m_2/M_\odot$</th>
<th>$M/M_\odot$</th>
<th>$\chi_{\text{eff}}$</th>
<th>$M_f/M_\odot$</th>
<th>$a_f$</th>
<th>$E_{\text{rad}}/(M_\odot c^2)$</th>
<th>$\tau_{\text{peak}}/(\text{erg s}^{-1})$</th>
<th>$d_L$/Mpc</th>
<th>$z$</th>
<th>$\Delta\Omega$/deg$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GW150914</td>
<td>35.6$^{+4.7}_{-3.1}$</td>
<td>30.6$^{+3.0}_{-4.4}$</td>
<td>28.6$^{+1.7}_{-1.5}$</td>
<td>0.01$^{+0.12}_{-0.11}$</td>
<td>63.1$^{+3.4}_{-0.9}$</td>
<td>0.69$^{+0.05}_{-0.04}$</td>
<td>3.1$^{+0.4}_{-0.3}$</td>
<td>3.6$^{+0.4}_{-0.3}$ $\times 10^{56}$</td>
<td>440$^{+150}_{-170}$</td>
<td>0.09$^{+0.03}_{-0.03}$</td>
<td>182</td>
</tr>
<tr>
<td>GW151012</td>
<td>23.2$^{+14.9}_{-5.5}$</td>
<td>13.6$^{+4.8}_{-4.8}$</td>
<td>15.2$^{+2.1}_{-1.2}$</td>
<td>0.05$^{+0.31}_{-0.20}$</td>
<td>35.6$^{+10.8}_{-3.8}$</td>
<td>0.67$^{+0.13}_{-0.11}$</td>
<td>1.6$^{+0.6}_{-0.5}$</td>
<td>3.2$^{+0.8}_{-1.7}$ $\times 10^{56}$</td>
<td>1080$^{+550}_{-490}$</td>
<td>0.21$^{+0.09}_{-0.09}$</td>
<td>1523</td>
</tr>
<tr>
<td>GW151226</td>
<td>13.7$^{+8.8}_{-3.2}$</td>
<td>7.7$^{+2.2}_{-2.5}$</td>
<td>8.9$^{+0.3}_{-0.3}$</td>
<td>0.18$^{+0.20}_{-0.20}$</td>
<td>20.5$^{+6.4}_{-1.5}$</td>
<td>0.74$^{+0.07}_{-0.05}$</td>
<td>1.0$^{+0.2}_{-0.1}$</td>
<td>3.4$^{+0.7}_{-1.9}$ $\times 10^{56}$</td>
<td>450$^{+180}_{-190}$</td>
<td>0.09$^{+0.04}_{-0.04}$</td>
<td>1033</td>
</tr>
<tr>
<td>GW170104</td>
<td>30.8$^{+7.3}_{-5.6}$</td>
<td>20.0$^{+4.9}_{-4.6}$</td>
<td>21.4$^{+2.2}_{-1.8}$</td>
<td>0.04$^{+0.12}_{-0.12}$</td>
<td>48.9$^{+5.1}_{-4.0}$</td>
<td>0.66$^{+0.08}_{-0.08}$</td>
<td>2.2$^{+0.5}_{-0.5}$</td>
<td>3.3$^{+0.6}_{-1.0}$ $\times 10^{56}$</td>
<td>990$^{+440}_{-430}$</td>
<td>0.20$^{+0.08}_{-0.08}$</td>
<td>921</td>
</tr>
<tr>
<td>GW170608</td>
<td>11.0$^{+5.5}_{-1.7}$</td>
<td>7.6$^{+1.2}_{-2.2}$</td>
<td>7.9$^{+0.2}_{-0.2}$</td>
<td>0.03$^{+0.19}_{-0.17}$</td>
<td>17.8$^{+3.4}_{-0.7}$</td>
<td>0.69$^{+0.04}_{-0.04}$</td>
<td>0.9$^{+0.0}_{-0.1}$</td>
<td>3.5$^{+0.4}_{-1.3}$ $\times 10^{56}$</td>
<td>320$^{+120}_{-110}$</td>
<td>0.07$^{+0.02}_{-0.02}$</td>
<td>392</td>
</tr>
<tr>
<td>GW170729</td>
<td>50.2$^{+16.2}_{-10.2}$</td>
<td>34.0$^{+9.1}_{-10.1}$</td>
<td>35.4$^{+4.8}_{-4.8}$</td>
<td>0.37$^{+0.21}_{-0.25}$</td>
<td>79.5$^{+14.7}_{-10.2}$</td>
<td>0.81$^{+0.13}_{-0.13}$</td>
<td>4.8$^{+1.7}_{-1.7}$</td>
<td>4.2$^{+0.9}_{-1.5}$ $\times 10^{56}$</td>
<td>2840$^{+1400}_{-1360}$</td>
<td>0.49$^{+0.19}_{-0.21}$</td>
<td>1041</td>
</tr>
<tr>
<td>GW170809</td>
<td>35.0$^{+8.3}_{-5.9}$</td>
<td>23.8$^{+5.1}_{-5.2}$</td>
<td>24.9$^{+2.1}_{-1.7}$</td>
<td>0.08$^{+0.17}_{-0.17}$</td>
<td>56.3$^{+5.2}_{-3.8}$</td>
<td>0.70$^{+0.09}_{-0.09}$</td>
<td>2.7$^{+0.6}_{-0.5}$</td>
<td>3.5$^{+0.6}_{-0.9}$ $\times 10^{56}$</td>
<td>1030$^{+320}_{-390}$</td>
<td>0.20$^{+0.05}_{-0.07}$</td>
<td>308</td>
</tr>
<tr>
<td>GW170814</td>
<td>30.6$^{+5.6}_{-3.0}$</td>
<td>25.2$^{+2.8}_{-4.0}$</td>
<td>24.1$^{+1.4}_{-1.1}$</td>
<td>0.07$^{+0.12}_{-0.12}$</td>
<td>53.2$^{+3.2}_{-2.4}$</td>
<td>0.72$^{+0.07}_{-0.05}$</td>
<td>2.7$^{+0.4}_{-0.3}$</td>
<td>3.7$^{+0.4}_{-1.4}$ $\times 10^{56}$</td>
<td>600$^{+150}_{-220}$</td>
<td>0.12$^{+0.03}_{-0.04}$</td>
<td>87</td>
</tr>
<tr>
<td>GW170817</td>
<td>1.46$^{+0.12}_{-0.10}$</td>
<td>1.27$^{+0.09}_{-0.09}$</td>
<td>1.186$^{+0.001}_{-0.001}$</td>
<td>0.00$^{+0.02}_{-0.01}$</td>
<td>$\leq 2.8 \leq 0.89$</td>
<td></td>
<td>&amp;</td>
<td>&amp;</td>
<td>&amp;</td>
<td>&amp;</td>
<td></td>
</tr>
<tr>
<td>GW170817</td>
<td>35.4$^{+7.5}_{-4.7}$</td>
<td>26.7$^{+4.3}_{-4.7}$</td>
<td>26.5$^{+2.1}_{-1.7}$</td>
<td>0.09$^{+0.13}_{-0.13}$</td>
<td>59.4$^{+4.9}_{-3.8}$</td>
<td>0.67$^{+0.07}_{-0.07}$</td>
<td>2.7$^{+0.5}_{-0.5}$</td>
<td>3.4$^{+0.5}_{-1.7}$ $\times 10^{56}$</td>
<td>1060$^{+420}_{-380}$</td>
<td>0.21$^{+0.07}_{-0.07}$</td>
<td>87</td>
</tr>
<tr>
<td>GW170823</td>
<td>39.5$^{+11.2}_{-6.7}$</td>
<td>29.0$^{+7.8}_{-7.8}$</td>
<td>29.2$^{+4.6}_{-3.6}$</td>
<td>0.09$^{+0.22}_{-0.26}$</td>
<td>65.4$^{+10.1}_{-7.4}$</td>
<td>0.72$^{+0.06}_{-0.06}$</td>
<td>3.3$^{+1.0}_{-0.8}$</td>
<td>3.6$^{+0.4}_{-1.1}$ $\times 10^{56}$</td>
<td>1940$^{+970}_{-900}$</td>
<td>0.35$^{+0.15}_{-0.15}$</td>
<td>1666</td>
</tr>
</tbody>
</table>
Comparison of various methods to extract ringdown frequency from gravitational wave data

Hiroyuki Nakano,¹,⁎ Tatsuya Narikawa,²,³,† Ken-ichi Oohara,⁴,†‡ Kazuki Sakai,⁵,§ Hisa-aki Shinkai,⁶,† Hirotaka Takahashi,⁷,⁸,† Takahiro Tanaka,⁹,⁎⁎ Nami Uchikata,²,⁴,†† Shun Yamamoto,⁷ and Takahiro S. Yamamoto³,††

ringdown search
60 mockdata

matched filtering
Hilbert-Huan Transformation
Auto-Regression Method
Neural Network method

TABLE III. We show the values of $\delta \log f_R$, $\sigma(f_R)$, $\delta \log f_1$, and $\sigma(f_1)$ for various methods. The results limited to set A are given on the first law of each method, while those limited to set B are on the second.

<table>
<thead>
<tr>
<th>Method</th>
<th>Set</th>
<th>$\delta \log f_R$ (%)</th>
<th>$\sigma(f_R)$ (%)</th>
<th>$\delta \log f_1$ (%)</th>
<th>$\sigma(f_1)$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF-R</td>
<td>A</td>
<td>−12.88</td>
<td>28.36</td>
<td>−71.51</td>
<td>97.79</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>−0.82</td>
<td>27.53</td>
<td>−46.11</td>
<td>75.48</td>
</tr>
<tr>
<td>MF-MR</td>
<td>A</td>
<td>6.25</td>
<td>17.27</td>
<td>−12.62</td>
<td>37.9</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>2.47</td>
<td>10.41</td>
<td>7.18</td>
<td>27.61</td>
</tr>
<tr>
<td>HHT</td>
<td>A</td>
<td>−13.38</td>
<td>21.91</td>
<td>−44.11</td>
<td>61.58</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>−8.08</td>
<td>19.81</td>
<td>−28.78</td>
<td>49.61</td>
</tr>
<tr>
<td>AR</td>
<td>A</td>
<td>0.2</td>
<td>9.93</td>
<td>4.88</td>
<td>38.75</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1.91</td>
<td>8.57</td>
<td>6.2</td>
<td>34.64</td>
</tr>
<tr>
<td>NN</td>
<td>A</td>
<td>−6.64</td>
<td>16.48</td>
<td>−15.23</td>
<td>33.96</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>−6.65</td>
<td>11.97</td>
<td>9.96</td>
<td>23.76</td>
</tr>
</tbody>
</table>
1. Auto-Regressive model (Method, general) I

Fitting data with linear func.

\[ x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \]

\[ = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \]

e.g. \[ x_n = Ae^{-rn\Delta t} \cos(\omega n\Delta t) \]

\[ Z_1 = e^{-(r-j\omega)\Delta t} \]
\[ Z_2 = e^{-(r+j\omega)\Delta t} \]

\[ x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1 + Z_2)x_{n-1} - Z_1Z_2x_{n-2} \]

can be applied also to noisy data by adjusting \( M \)
1. Auto-Regressive model (Method, general) II

Fitting data with linear func.

\[ x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \]
\[ = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \]

- find \( a_j \) (Burg method)
- find \( M \) (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

\[ p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-i2\pi jf \Delta t} \right|^2} \]
Auto-Regressive model vs Short FFT

sampling rate = 4096

segment = 1/64 sec = 64 points

shift = 1/512 sec = 8 points

The order $M$ can be fixed at 2~8.

Even for short segment, AR model shows precise power-spectrum.
1. Auto-Regressive model (Method, general) III

Fitting data with linear func.

\[ x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \]

\[ = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \]

- find \( a_j \) (Burg method)
- find \( M \) (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

**power spectrum**

\[ p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-j2\pi j f \Delta t} \right|^2} \]

**characteristic eq.**

\[ f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0 \]

\(|z_k|\) says amplitude,
\(\arg(z_k)\) says frequency.
GW150914

LIGO paper

freq [Hz]

AR model

Hanford

4096 sampling rate
150-450 Hz filter
1 segment = 1/64 sec = 64 points
1 shift = 1/512 sec = 8 points

\[ f_{220} = 249.4 \text{ Hz}, f_{221} = 244.0 \text{ Hz}, f_{222} = 233.7 \text{ Hz} \]
\[ f_{210} = 349.3 \text{ Hz}, f_{211} = 207.1 \text{ Hz}, f_{200} = 231.9 \text{ Hz} \]
\[ f_{330} = 395.3 \text{ Hz}, f_{331} = 392.1 \text{ Hz}, f_{332} = 386.3 \text{ Hz} \]
\[ f_{320} = 355.9 \text{ Hz}, f_{310} = 322.1 \text{ Hz}, f_{300} = 293.9 \text{ Hz} \]
GW150914

Hanford (SNR=20.6)

Livingston (SNR=14.2)
Ringdown Search by Auto-Regressive Approach (O1/O2 public data, GWTC1 catalogue)

**GW150914**

**Hanford (SNR=20.6)**

**Livingston (SNR=14.2)**

(LV paper) \( (M, a, z) = \left( 63.1^{+3.4}_{-3.0}, 0.69^{+0.05}_{-0.04}, 0.09^{+0.03}_{-0.03} \right) \)

\[ f_{220} = 249.4 \text{ Hz}, f_{221} = 244.0 \text{ Hz}, f_{222} = 233.7 \text{ Hz} \]
\[ f_{210} = 349.3 \text{ Hz}, f_{211} = 207.1 \text{ Hz}, f_{200} = 231.9 \text{ Hz} \]
\[ f_{330} = 395.3 \text{ Hz}, f_{331} = 392.1 \text{ Hz}, f_{332} = 386.3 \text{ Hz} \]
\[ f_{320} = 355.9 \text{ Hz}, f_{310} = 322.1 \text{ Hz}, f_{300} = 295.9 \text{ Hz} \]

**fQNM**

**detector frame**

**source frame**
Ringdown Search by Auto-Regressive Approach (O1/O2 public data, GWTC1 catalogue)

GW170104

(M, a, z) = (48.9^{+5.1}_{-4.0}, 0.66^{+0.08}_{-0.11}, 0.2^{+0.08}_{-0.08})

f_{QNM}: f_{220} = 285.7 \text{ Hz}, f_{221} = 279.3 \text{ Hz}, f_{222} = 266.0 \text{ Hz}
f_{210} = 412.5 \text{ Hz}, f_{211} = 393.3 \text{ Hz}, f_{200} = 272.7 \text{ Hz}
f_{330} = 453.5 \text{ Hz}, f_{331} = 449.5 \text{ Hz}, f_{332} = 442.1 \text{ Hz}
f_{320} = 411.1 \text{ Hz}, f_{310} = 374.4 \text{ Hz}, f_{300} = 343.4 \text{ Hz}
Ringdown Search by Auto-Regressive Approach (O1/O2 public data, GWTC1 catalogue)

$$(M, a, z) = (79.5^{+14.7}_{-10.2}, 0.81^{+0.07}_{-0.13}, 0.49^{+0.19}_{-0.21})$$

$f_{200} = 161.4$ Hz, $f_{221} = 159.4$ Hz, $f_{222} = 155.3$ Hz

$f_{210} = 195.6$ Hz, $f_{211} = 128.1$ Hz, $f_{200} = 132.6$ Hz

GW170729

Hanford (SNR=5.9)

Virgo (SNR=1.7)

Livingston (SNR=8.3)

Kerr param

source frame

LV paper

(AR) Hanford

(AR) Livingston

(AR) Virgo

Mass

$20 \quad 40 \quad 60 \quad 80 \quad 100$
GW170809

\[(M, a, z) = (56.3^{+5.2}_{-3.8}, 0.7^{+0.08}_{-0.09}, 0.2^{+0.05}_{-0.07})\]

\[f_{200} = 255.9 \text{ Hz}, f_{201} = 250.6 \text{ Hz}, f_{202} = 240.3 \text{ Hz}\]

\[f_{210} = 354.7 \text{ Hz}, f_{211} = 211.9 \text{ Hz}, f_{200} = 235.8 \text{ Hz}\]
GW170814

\[ (M, a, z) = (53.2^{+3.2}_{-2.4}, 0.72^{+0.07}_{-0.05}, 0.12^{+0.03}_{-0.04}) \]

\[ f_{20} = 294.9 \text{ Hz}, f_{21} = 289.2 \text{ Hz}, f_{22} = 278.1 \text{ Hz} \]
\[ f_{20} = 400.0 \text{ Hz}, f_{211} = 242.6 \text{ Hz}, f_{200} = 266.8 \text{ Hz} \]

---

**Hanford (SNR=9.3)**

**Virgo (SNR=4.1)**

**Livingston (SNR=14.3)**

source frame

LV paper

(AR) Hanford

(AR) Livingston

(AR) Virgo
GW170818

\[(M, a, z) = (59.4^{+4.9}_{-3.8}, 0.67^{+0.07}_{-0.08}, 0.21^{+0.07}_{-0.07})\]

\[f_{220} = 235.0 \text{ Hz}, f_{221} = 229.6 \text{ Hz}, f_{222} = 219.3 \text{ Hz}, f_{210} = 336.0 \text{ Hz}, f_{211} = 196.3 \text{ Hz}, f_{200} = 222.4 \text{ Hz}\]

**Hanford (SNR=4.6)**

**Virgo (SNR=4.2)**

**Livingston (SNR=9.7)**

---

**source frame**

**LV paper**

**(AR) Hanford**

**(AR) Livingston**

---

**GW170818**

**Kerr parameter a**

**Mass**
GW170823

**Hanford (SNR=6.8)**

**Livinston (SNR=9.2)**

LV paper \( (M, a, z) = (65.4^{+10.1}_{-7.4}, 0.72^{+0.09}_{-0.12}, 0.35^{+0.15}_{-0.15}) \)

\( f_{220} = 199.0 \text{ Hz}, f_{221} = 195.2 \text{ Hz}, f_{222} = 187.7 \text{ Hz} \)

\( f_{210} = 269.9 \text{ Hz}, f_{211} = 163.7 \text{ Hz}, f_{200} = 180.1 \text{ Hz} \)

\( f_{330} = 315.0 \text{ Hz}, f_{331} = 312.8 \text{ Hz}, f_{332} = 308.6 \text{ Hz} \)

\( f_{320} = 281.5 \text{ Hz}, f_{310} = 253.2 \text{ Hz}, f_{300} = 229.7 \text{ Hz} \)
Summary & Outlook

自己回帰モデル $x(t)$

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$

$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

短いデータ (~ 60 pts) に対しても精度よく周波数・減衰率を特定できる。シグナルを見つけるのにテンプレートは不要。

LIGO/Virgo の O1/O2イベントデータに適用し、リングダウン部分の抽出を試みた。SN比が高ければ、独立にリングダウン部分が取り出せそうだ。

★ノイズ除去の方法や、他の方法と組み合わせ、より精密な周波数特定法を検討中。
★higher modesの検出へ、BHの特長量の特定へ、相対論検証へ。
★テンプレートを使わない方法は、今後、未知の重力波シグナルの候補検出に役立つかも。
Evan Hall, MIT

EM counterparts (certain)

10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4}

Total source-frame mass \([M_\odot]\)

10^{2} 10^{3} 10^{4}

Redshift

Horizon 10% detected 50% detected

Cosmic Explorer 40km L-shape

LIGO-India

Einstein Telescope 10km Triangle

USA

aLIGO

A+

Voyager

Cosmic Explorer ?

Europe

aVirgo

aVirgo+

Einstein Telescope ?

Japan

KAGRA

? DECGO?

early 2020s late 2020s early 2030s

late 2010s

aLIGO

A+

Voyager

Cosmic Explorer ?

Europe

aVirgo

aVirgo+

Einstein Telescope ?

Japan

KAGRA

? DECGO?

early 2020s late 2020s early 2030s

late 2010s

O3 O4 O5 O6

introduction to GW data analysis

重力波観測の将来計画

Cosmic Explorer 40km L-shape

Einstein Telescope 10km Triangle

Hisaaki Shinkai (真貝 命明) 2020/2/20 @ Yamaguchi Univ.
総まとめ

1. 重力波観測時代がはじまった
BH-BH, NS-NS. 次はBH-NS? SN?

2. 日本のKAGRAも2020年2月25日から実観測開始
LIGO/Virgoとの共同観測体制構築。

3. データ解析には、まだまだ試すべきアイデアがたくさんある。
将来計画もたくさんある。
新規参入、大歓迎！