thesis08

Gravitational Wave extraction using Independent Component Analysis

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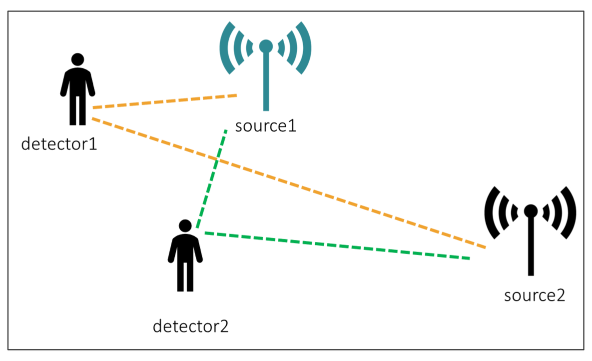
Abstract & Summary

We propose to apply independent component analysis (ICA) for extracting gravitational wave (GW) signals. This approach does not require templates of waveform, so that it can be applied for testing general relativity, and also for finding unknown GW.
Our idea is to use multiple detector data and to extract meaningful signal which should be included commonly. We demonstrate the signal extractions with injected data (both sinusoidal and inspiral waves) to the Gaussian noises, and also to the real L1/H1 data around GW150914. We also try to detect GW150914 signal from L1/H1 data, and we find the signal can be detected clearly when we shifted Livingston's data $7.5^{+0.3}_{-0.3}$ ms advanced. (LV paper suggested the shift $6.9^{+0.5}_{-0.4}$ ms).

§1 ICA Method



Independent component analysis (ICA) is a method to separate the set of data as the new set has "statistical independency" of each component. Most easiest example is to identify the voices of n person from their mixed sound files with n microphones.

Suppose we receive the time-series signal $\boldsymbol{x}(t) \equiv (x_1(t), \cdots, x_n(t))^T$ from n detectors from the n source signals $\boldsymbol{s}(t) \equiv (s_1(t), \cdots, s_n(t))^T$,

$$\boldsymbol{x}(t) = A\boldsymbol{s}(t), \tag{1}$$

where A is the time-independent matrix which represents the superposition of the source signals. Our goal is to extract the source signal $\boldsymbol{s}$ from $\boldsymbol{x}$.

ICA casts on the idea of "statistical independency" of each source signals. We write the problem as

$$\tilde{\boldsymbol{s}}(t) = W\boldsymbol{x}(t), \tag{2}$$

where $\tilde{\boldsymbol{s}}$ are statistically independent components of $\boldsymbol{s}$ obtained from this linear transformation with time-independent matrix W. Note that ICA is not appropriate for extracting Gaussian signals since their superposition is Gaussian. Therefore one strategy to find W is to look for it as

$$s_1(t) = \boldsymbol{w}_1^T V \boldsymbol{x}(t) \equiv \boldsymbol{w}_1^T \boldsymbol{z}(t) \tag{3}$$

has the most non-Gaussianity, where $\boldsymbol{w}_1^T = (w_{11}, w_{12}, \cdots)$ is a line of W, and V expresses the whiten process of $\boldsymbol{x}(t)$ (makes $\boldsymbol{x}(t)$ has no correlation and variance unity).

We apply FastICA method using kurtosis of $w^T\boldsymbol{z}$,

$$\mathrm{kurt}(\boldsymbol{w}^T\boldsymbol{z}) = E[(\boldsymbol{w}^T\boldsymbol{z})^4] - 3\{E[(\boldsymbol{w}^T\boldsymbol{z})^2]\}^2 \tag{4}$$

which measures the difference from Gaussian distribution. By requiring the norm of $\boldsymbol{w}$ is unity, $||\boldsymbol{w}||^2 = 1$, (which makes $E[(\boldsymbol{w}^T\boldsymbol{z})^2] = ||\boldsymbol{w}||^2$), from the derivative by $\boldsymbol{w}_1$, we get

$$\frac{\partial}{\partial \boldsymbol{w}_1}|\mathrm{kurt}(\boldsymbol{w}_1^T\boldsymbol{z})| = \begin{pmatrix} E[4(\boldsymbol{w}_1^T\boldsymbol{z})^3 z_1] \\ E[4(\boldsymbol{w}_1^T\boldsymbol{z})^3 z_2] \\ \vdots \end{pmatrix} - 12||\boldsymbol{w}_1||^2 \begin{pmatrix} w_{11} \\ w_{12} \\ \vdots \end{pmatrix}. \tag{5}$$

We search $\boldsymbol{w}_1$ as eq. (5) equals zero by iterative method, then identify $s_1(t)$. We then repeat the process of finding $\boldsymbol{w}_i$ to identify $s_i(t)$ as each $\boldsymbol{w}_i$ satisfy its orthogonality.
Since kurtosis is vulnerable to outliers, we actually used an approximation based on negentropy.

Previous works of ICA on GW

The usage of ICA to gravitational wave data analysis was first pointed out by De Rosa *et al*. 2012. Application of the non-Gaussian noise subtraction was suggested by Morisaki *et al*. 2016.

- De Rosa + demonstrated ICA for injected signals mimicking two interferometer data, and reported that preprocessing ICA before matched filtering technique allows to lower the level of noise (increase the SNR (signal-to-noise ratio)).
- KAGRA collaboration applied ICA to their real observation data (iKAGRA data in 2016) in 2020. By injecting a sinusoidal data to the strain channel and applying ICA together with physical environmental channels (seismic channels), then they showed that ICA recovered correct parameters with enhanced SNR.

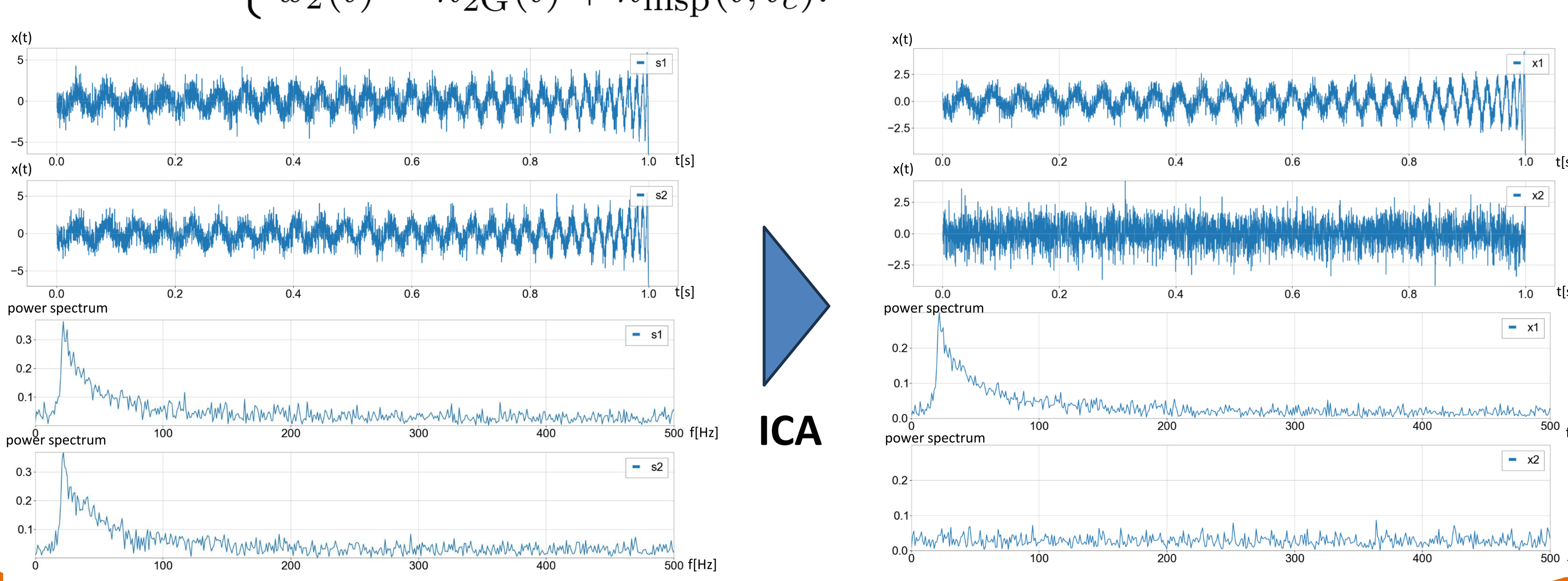
§2 Our code

1. Whitening the data $z(t)$ using PSD of detectors.
2. Normalize the data $z(t)$ (mean zero, variance one)
3. Determine the number of independent components m. Set the counter p to $p = 1$.
4. Randomly choose an initial weight vector $\boldsymbol{w}_p$.
5. Let $\boldsymbol{w}_p = E[\boldsymbol{z}g(\boldsymbol{w}_p^T\boldsymbol{z})] - E[g'(\boldsymbol{w}_p^T\boldsymbol{z})]\boldsymbol{w}_p$. Let g be $g_1(y) = \tanh y$.
6. The following orthogonalization is performed:
$$\boldsymbol{w}_p = \boldsymbol{w}_p - \sum_{j=1}^{p-1}(\boldsymbol{w}_p^T\boldsymbol{w}_j)\boldsymbol{w}_j$$
7. Let $\boldsymbol{w}_p = \boldsymbol{w}_p/||\boldsymbol{w}_p||$
8. If $\boldsymbol{w}_p$ does not converge, go back to 5.
9. Let $p = p+1$. If $p \le m$, go back to 4.

§3 Demonstrations

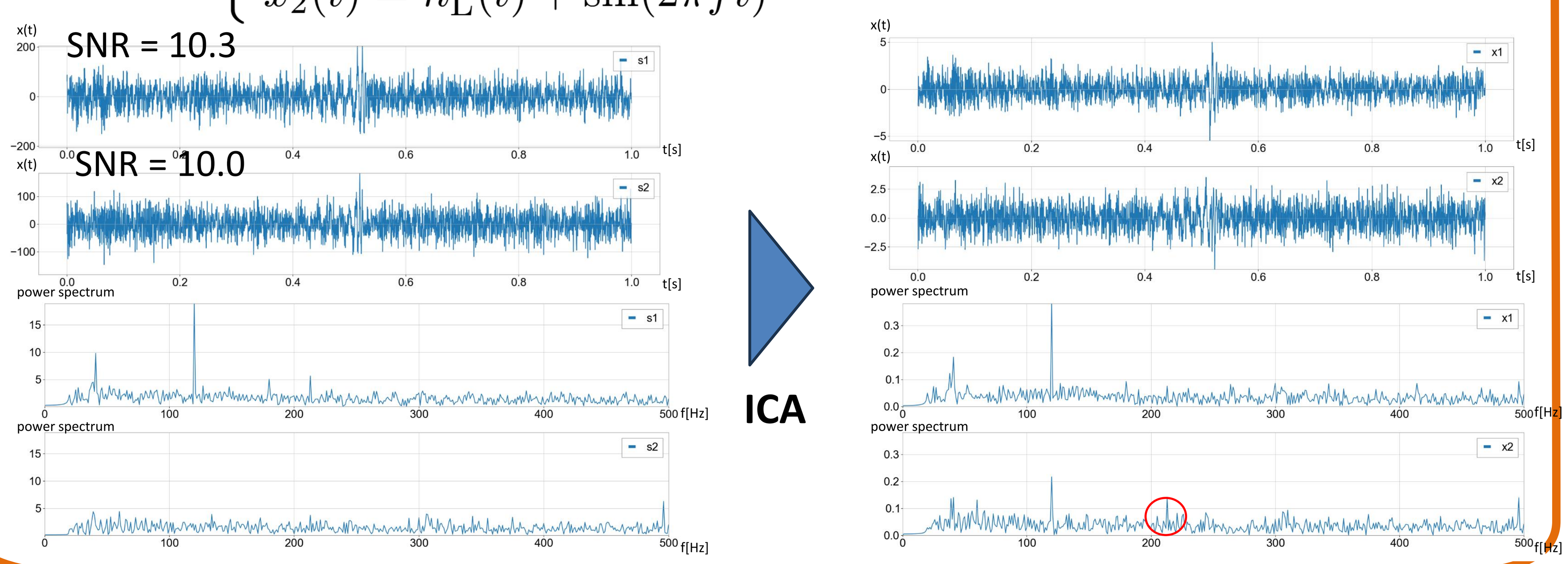
Injection to Gaussian-Noise

$$\text{Model 1 :} \begin{cases} x_1(t) = n_{1\mathrm{G}}(t) + h_{\mathrm{insp}}(t, t_c), \\ x_2(t) = n_{2\mathrm{G}}(t) + h_{\mathrm{insp}}(t, t_c). \end{cases}$$



Injection to L1/H1 data

$$\text{Model 2 :} \begin{cases} x_1(t) = n_{\mathrm{H}}(t) + \sin(2\pi f t), \\ x_2(t) = n_{\mathrm{L}}(t) + \sin(2\pi f t) \end{cases} \quad f = 213\,(\mathrm{Hz})$$



L1/H1 data

after filtering(20~200Hz)

$$\text{Model 3 :} \begin{cases} x_2(t) = h_{\mathrm{H}}(t) \\ x_2(t) = h_{\mathrm{L}}(t) \end{cases}$$

on real data at shifted time

B. P. Abbott et al. (LIGO Scientific Collaboration andVirgo Collaboration), Phys. Rev. Lett. 116, 061102(2016)